LU TP 94-9
June 1994

Colour Interference and Confinement Effects in W-pair Production

Gösta Gustafson[1], Jari Häkkinen[2]
Department of Theoretical Physics,
Sölvegatan 14A,
SE-223 62 Lund, Sweden

Abstract:
In the reaction $e^+e^- \to W^+W^- \to (q_1\overline{q}_2)(Q_1\overline{Q}_2)$ we expect that normally the colour singlet systems $(q_1\overline{q}_2)$ and $(Q_1\overline{Q}_2)$ hadronize independently into two hadron chains or strings. However also the pairs $(q_1\overline{Q}_2)$ and $(Q_1\overline{q}_2)$ form colour singlets with probability $1/N_c^2 = 1/9$. This probability could be further enhanced by gluon exchange. We therefore expect that the hadronization can give "recoupled" colour strings between these quark-antiquark pairs with some small probability, and a study of this effect will give interesting information about the vacuum structure. In this paper we discuss a possible experimental signal for recoupled events and show that such events can be identified also when the effect of gluon emission before the recoupling is taken into account.

[1]E-mail: gosta@thep.lu.se
[2]E-mail: jari@thep.lu.se

1 Introduction

At LEP2 it will be possible to study the reaction $e^+e^- \rightarrow W^+W^-$ where each of the two Ws decay into a $q\overline{q}$ system. Normally it is assumed that these two $q\overline{q}$ systems hadronize independently, thus producing two independent strings or chains of hadrons. As the lifetimes of the Ws are very short the $q\overline{q}$ systems are essentially produced on top of each other, and it is therefore conceivable that the hadronization processes interfere, and a study of this interference (or the absence of an interference) would give valuable information about the confinement mechanism and the vacuum structure. In this paper we discuss possible experimental signals of this effect.

Let's assume that the two Ws decay into quark-antiquark pairs called $q_1\overline{q}_2$ and $Q_1\overline{Q}_2$ respectively. The W lifetime is approximately 0.1fm, which implies that the distance in space and time between the decay points, at LEP2 energies, is of the order $\Delta x \sim 0.05$fm and $\Delta t \lesssim 0.1$fm. These values are small compared to the confinement scale. Therefore, when the quarks and antiquarks move away from each other, how does the quark q_1 know that its partner is $\overline{q}_2$, meaning that a string or a hadron chain should be formed between q_1 and $\overline{q}_2$?

It is true that the pair $q_1\overline{q}_2$ initially forms a colour singlet, but *also* $q_1\overline{Q}_2$ forms a singlet with probability $1/N_c^2 = 1/9$. Thus we might possibly expect that, if the pairs are produced in the same space-time point, the strings are "recoupled" and stretched between the pairs $q_1\overline{Q}_2$ and $Q_1\overline{q}_2$ with a probability of the order $1/N_c^2$. The recoupling probability can be further enhanced as the colour structure can be changed by gluon exchange. A gluon exchanged between e.g. q_1 and Q_1 would imply that the pairs $q_1\overline{q}_2$ *and* $Q_1\overline{Q}_2$ are colour octets, and strings would naturally form within the pairs $q_1\overline{Q}_2$ and $Q_1\overline{q}_2$. As discussed in refs [1, 2] hard gluons (gluons with $E \geq \Gamma_W$) are emitted independently by the decay products of the Ws. For this reason the exchange hard gluons, or interference effects from hard gluon emission, give only a small effect. The effect of *soft* gluon exchange can however not be easily estimated. It depends on the vacuum structure and cannot be calculated perturbatively.

Possible implications of the vacuum structure on the recoupling probability were pointed out in ref [3]. In a bag-type model the string between a quark and an antiquark is a colour fluxtube with width of the order 1fm and an essentially homogeneous colourelectric field inside the tube. In this case the vacuum behaves like a type I superconductor. When the two Ws decay, it is then natural to imagine that the quarks and antiquarks are surrounded by a baglike hole in vacuum, and that recoupling is likely if the distance (in space-time) between the production points for the pairs is less than 1fm.

We may also imagine that the vacuum is more similar to a type II superconductor. Here the string should consist of a thin core surrounded by an extended colourelectric field. The extension of this field ought to be of the order 1fm, but the core may be significantly more narrow. In this case we can imagine that recoupling is more unprobable and only likely if the distance between the production points is small compared to the thickness of the core.

We see that a study of the probability for recoupling (and if possible how it depends on the total energy, i.e. on the distance between the W decay points) can give interesting information about the structure of the vacuum and the confinement mechanism. Two questions are essential, i) what recoupling probability could be expected?, and ii) is it possible to identify recoupled events?

When trying to answer these questions it is important to take into account the effects of emission of (real) gluons. As mentioned above hard gluons are emitted very early [1, 2]. The space-time distance between the W decay points is of the order $1/\Gamma_W$, and therefore gluons with energies larger than Γ_W are produced incoherently by the two pairs $(q_1\overline{q}_2)$ and $(Q_1\overline{Q}_2)$. Thus rather than a possible recoupling between two $q\overline{q}$ systems we have two colour singlets, each containing a $q\overline{q}$ pair and a set of gluons, which can be recoupled, or regrouped into new colour singlet systems, in many different ways.

2 What recoupling probabilities may be expected?

When trying to estimate what recoupling probabilities could be expected it is relevant to compare with a similar situation in the B meson decay $B \rightarrow \psi + X$. If q denotes a light quark this decay can proceed via the reaction $q\overline{b} \rightarrow q\overline{c}W^+ \rightarrow q\overline{c}c\overline{s}$. Here the pairs $q\overline{c}$ and $c\overline{s}$ form colour singlets, and thus the $c\overline{c}$ pair normally forms a colour octet. Consequently the formation of a ψ meson implies the same form of colour recoupling, and some analyses of the branching ratios have indicated a recoupling probability around 15 or 20% [4]. In a B decay the energy is comparatively low and the W^+ is very much off shell and therefore the three quarks c, $\overline{c}$, and $\overline{s}$ are essentially produced in the same space-time point. Thus we see that at zero distance (between three of the quarks) and lower energies there is apparently a significant recoupling probability.

The situation in W pair production is different from B decay in two respects. First the two Ws decay in different space-time points, separated by 0.05–0.1fm. This is expected to reduce the recoupling probability, more in case vacuum is like a type II superconductor and less if it is like a type I superconductor. Secondly the emission of gluons implies that the two initial systems can be recoupled in many different ways, and this fact can in principle increase the total recoupling probability.

Before we discuss these dynamical effects it is interesting to study what the recoupling probability would be, due to only SU(3) Clebsch-Gordan coefficients, if there were *no* effects from gluon emission or the vacuum structure. Assume that two colour singlet systems, $q_1\overline{q}_2$ and $Q_1\overline{Q}_2$, are produced in the same space-time point. The probability that also $q_1\overline{Q}_2$ and $Q_1\overline{q}_2$ form colour singlets is then 1/9, as mentioned earlier. If $q_1\overline{Q}_2$ and $Q_1\overline{q}_2$ form colour singlets we may expect that they also can form separate hadron chains or strings. However, if q_1 forms a colour singlet with both $\overline{q}_2$ and $\overline{Q}_2$, the structure of the observable hadron state can *not* be determined without further dynamical assumptions. As discussed in more detail in the appendix, the algebraic group properties only give a rather weak bound that the recoupling probability, P, must lie in the range $\delta < P < 1-\delta$, where $\delta = 1/2 - \sqrt{2}/3 \approx 0.03$. Thus the recoupling probability could in principle be anything above a lower limit of 3%.

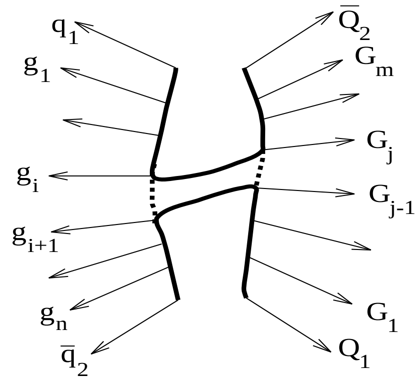

Figure 1: *The gluons $g_1 g_2 ... g_n$ (with $E > \Gamma_W$) are emitted from the original $q_1 \overline{q}_2$ pair while $G_1 G_2 ... G_m$ are emitted from the $Q_1 \overline{Q}_2$ pair. The two systems $q_1 g_1 ... g_i G_j ... G_m \overline{Q}_2$ and $Q_1 G_1 ... G_{j-1} g_{i+1} ... g_n \overline{q}_2$ are then colour singlets with probability 1/9 for* each *value of i and j. They may thus form independent hadron chains or strings.*

As mentioned in the introduction hard gluons are emitted early, and therefore gluons with energies larger than Γ_W are produced incoherently by the two pairs $q_1 \overline{q}_2$ and $Q_1 \overline{Q}_2$ [1, 2]. Thus before the hadronization and before a possible recoupling two sets of colour connected gluons $g_1 g_2 ... g_n$ and $G_1 G_2 ... G_m$ (with $E > \Gamma_W$) are emitted independently by the $q_1 \overline{q}_2$ and $Q_1 \overline{Q}_2$ systems respectively[3]. Every set of particles $q_1 g_1 g_2 ... g_i$ is now a colour triplet. Thus we can have a recoupling between the two original systems in many different ways. The two sets of particles $q_1 g_1 ... g_i G_j ... G_m \overline{Q}_2$ and $Q_1 G_1 ... G_{j-1} g_{i+1} ... g_n \overline{q}_2$ are colour singlets with probability $1/N_c^2$ for *every* choice of i and j (see Fig 1). The average number of gluons with $E > \Gamma_W$ (and $k_\perp > k_{\perp cut} = 1\text{GeV}$) is $< n >=< m > \approx 4.3$, and thus if the probability for recoupling is also of the order $1/N_c^2$ for each of these $(n+1)(m+1)$ possibilities, the *total* recoupling probability could in principle be very large.

We do not believe that all these recoupling possibilities are equally probable. Sjöstrand and Khoze [2, 5] have studied two different scenarios. In scenario II, which corresponds to a type II superconductor, it is assumed that the confining field has a very narrow core, and can be treated as an infinitely thin string. When two strings cross and have a space-time point in common, it is assumed that they can recouple with probability one (if they cross in many points only the one first in time is kept). As a result the recoupling probability is approximately 35% at $\sqrt{s} = 170\text{GeV}$. Scenario I corresponds to a type I superconductor and here the field is treated as a flux tube with radius around 0.5fm. (More precisely the field strength is proportional to $\exp(-r_\perp^2/2r_{\text{had}}^2)$, where $r_{\text{had}} = 0.5\text{fm}$). In every space-time point, where the flux tubes overlap, there is a constant recoupling probability (with a Gaussian fall off at the tube boundaries). The magnitude of this recoupling probability density is a

[3]More precisely soft gluons are emitted with a relative probability $1 - \frac{1+x(1-\epsilon)}{1+2x(1+\epsilon)+x^2(1-\epsilon)^2}$ where $\epsilon = v^2 \cos\theta$, v is the speed of the W, θ is the angle between the gluon and W momenta, and $x = \frac{\gamma(v) E_g}{\Gamma_W}$

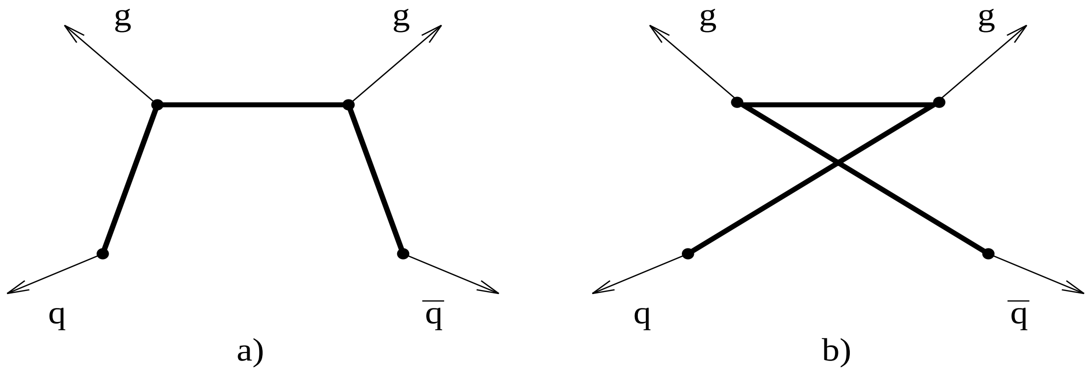

Figure 2: *The production probability for a $q\overline{q}gg$ state is, in perturbative QCD, larger for the colour connection which corresponds to a "shorter string" as in fig a).*

free parameter, but if it is chosen to be 0.9fm^{-4} the total recoupling probability is 35% just as in scenario II. In both scenarios those recouplings are favoured, in which the initial strings are cut and recoupled in the "middle", so that the two pieces have approximately the same energy. As the normal (not recoupled) systems correspond to "long" strings, stretched across the origin in the cms, it is natural that most recouplings give "shorter" strings (i.e. strings which give fewer hadrons); this is more pronounced for scenario I than for scenario II.

In a type II scenario we think that the recoupling probability of 35% might be regarded as an upper limit. This result was obtained assuming unit probability for recoupling when the strings cross, and the results from B decay may indicate that a recoupling does not always occur when the space-time structure makes it possible. In a type I scenario the flux tubes always overlap to a larger or lesser degree. The recoupling probability cannot be calculated from our present knowledge of QCD, and with all the recoupling possibilities available after the initial gluon emission, the total probability may in principle be very large. The value 0.9 fm^{-4} for the probability density which is studied in refs [2, 5], is just arbitrarily chosen in order to facilitate a comparison between the scenarios, and has no dynamical motivation.

In scenarios I and II studied by Sjöstrand and Khoze it is assumed that the recoupling probability only depends on the geometrical overlap of the field configurations in coordinate space, with no dependence on the kinematical motion of the fields. (Some dynamical effects are included in scenario II', in which only those recouplings are allowed which give "shorter strings" in the sense discussed below.) If we study $q\overline{q}gg$ four jet events, as illustrated in Fig 2, we note that the colour connection in Fig 2b, which corresponds to a "longer" string, is disfavoured. When comparing different possibilities of recoupling, as discussed above, it is conceivable that the tendency in the scenarios studied in refs [2] to favour short "strings" may be further enhanced by dynamical effects. This also means that recoupling might be more probable if it connects gluons which move in approximatively the same direction. In a type I scenario the fluxtubes always overlap in coordinate space, and it is quite conceivable that the properties in momentum space are most important for the recoupling probability.

The "length" of a string is correlated with its invariant mass and thus one possibility would be to select and study that recoupling possibility (i.e. those values of i and j above) for which

$s_1 \cdot s_2$ is minimum. (Here $\sqrt{s_1}$ and $\sqrt{s_2}$ are the invariant masses of the two systems after recoupling). A more efficient measure for the "length" of a string state is the quantity λ studied in ref [6]. It corresponds to an "effective rapidity length", and is closely correlated to the hadronic multiplicity. For hard gluons it is approximately given by $\lambda \approx \sum \ln(p_i + p_{i+1})^2/m_0^2$, where p_i are the (colourordered) parton momenta and the hadronic mass scale m_0 is around 1GeV. Thus it is conceivable that those recouplings for which $\lambda_1 + \lambda_2$ is significantly reduced may be dynamically favoured (where λ_1 and λ_2 are the λ-measures for the two recoupled strings). An extreme case corresponds to the assumption that only the recoupling for which $\lambda_1 + \lambda_2$ is minimum is likely, perhaps with a probability around 10%. It is also possible that all states, for which $\lambda_1 + \lambda_2$ is significantly reduced, have a fair recoupling probability, which could give a larger total recoupling probability.

We note that for the recoupled states with minimum $\lambda_1 + \lambda_2$, the average change $<\Delta\lambda>$ is around 2 units. (For a sample with a cut on thrust, as studied in the next section, the value of $<\Delta\lambda>$ is even larger). This can be compared with the recoupled states studied in ref [2], which are selected only from purely geometrical considerations, for which $<\Delta\lambda>$ is of order 0.7 for scenario I (and somewhat smaller for scenario II). In the same way as this geometrically selected sample, also the states with minimum $\lambda_1 + \lambda_2$ normally corresponds to recoupling where the initial states are cut in two approximately equal pieces. The average value of the quantity $d_{\text{rec}} = |E_1 - E_2|/(E_1 + E_2)$ (where E_1 and E_2 are the energies of the two pieces of a split W decay system) is given by $<d_{\text{rec}}> \approx 0.2$ which is comparable to the result in ref [2].

3 Can recoupled events be identified?

The question whether the recoupled events can be identified was addressed in ref [3]. In this older analysis the emission of gluons before recoupling was neglected, and with this simplifying treatment a very clear difference was seen between recoupled and "normal" events. If the angle between the outgoing q_1 and $\overline{\mathrm{Q}}_2$ is fairly small, e.g. around 30°, the two recoupled strings move apart in the cms with large velocities, thus producing few particles in the central region. Looking at the number of central particles, e.g. particles with $|y| < 2$, the recoupled events could easily be identified on an event by event basis.

However the gluons emitted independently by the two systems in the early phase, as discussed above, will give extra particles in the central region, which will reduce the difference between the recoupled and non-recoupled events. Sjöstrand and Khoze [2, 5] studied effects of recoupling on the determination of the W mass and on *average* distributions, and in their analysis they found small effects. Our aim here is different. We want to study if there are *some* possible recoupled systems which are sufficiently different from the "normal" systems so that they can be identified experimentally.

As mentioned above recoupled systems with two "short strings" may be particularly favoured. They are also the ones which differ most from the nonrecoupled systems, because the two short strings move away in opposite directions in a way which is most similar to the situation

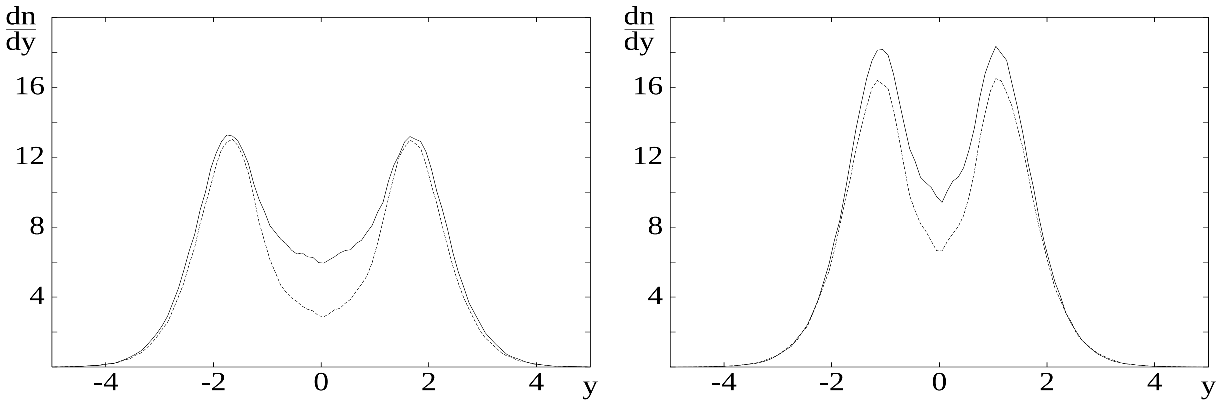

Figure 3: *Rapidity distributions for "normal" events (solid line) and recoupled events (dashed line) with thrust cuts $T > 0.92$ (fig a) and $T > 0.76$ (fig b).*

without primary gluon emission, studied in ref [3]. They will thus be more easily identified and we will in the following concentrate on those recoupling possibilities which give the shortest strings, i.e. those for which $\lambda_1 + \lambda_2$ is minimum.

In order to have a large difference between the recoupled and nonrecoupled systems, the angle between e.g. q_1 and $\overline{\mathrm{Q}}_2$ should not be too large. In ref [3] (and also in ref [2]) events where this angle was 30° were selected. In our analysis we found that the recoils from gluon emission imply that a cut on this angle is less suitable, and instead we study here events with a cut on the thrust variable T.

As mentioned above it would be interesting to study how the recoupling probability varies with the total energy, which implies a variation in the distance between the W decay points. However, as discussed in ref [2], also at the threshold energy, $\sqrt{s} = 2m_{\mathrm{W}}$, the width of the Ws implies that they are not produced at rest. Therefore the distance varies very little in the LEP2 range from threshold to 180GeV. For this reason we here present results only for 170GeV where the effect of the spread in the W mass is small and can be neglected. These results are representative also for energies close to threshold, where the average W momenta are of the same order of magnitude [2].

In our simulations gluons with $E \gtrsim \Gamma_{\mathrm{W}}$ are emitted using the Ariadne MC [7], and the recoupled events are chosen to give minimum values for the sum $\lambda_1 + \lambda_2$, as discussed above. For the hadronization process we have used the Lund string model [8] implemented in the Jetset MC [9]. In string fragmentation soft gluons with $E < \Gamma_{\mathrm{W}}$ give very little effect, and they have therefore been neglected. In Fig 3 we show the inclusive rapidity distribution for "normal" and "recoupled" events. We see that the central region is suppressed in the recoupled events, but due to the initial gluon emission it is not totally empty. Therefore this average distribution can not be used as an experimental signal for the recoupled events. It would not be possible to ascertain an admixture of recoupled event of the order 10%. (We note that the results in Fig 3 are similar to the corresponding results obtained by Sjöstrand and Khoze [2]).

However, even if the average recoupled event is not sufficiently different from the normal ones,

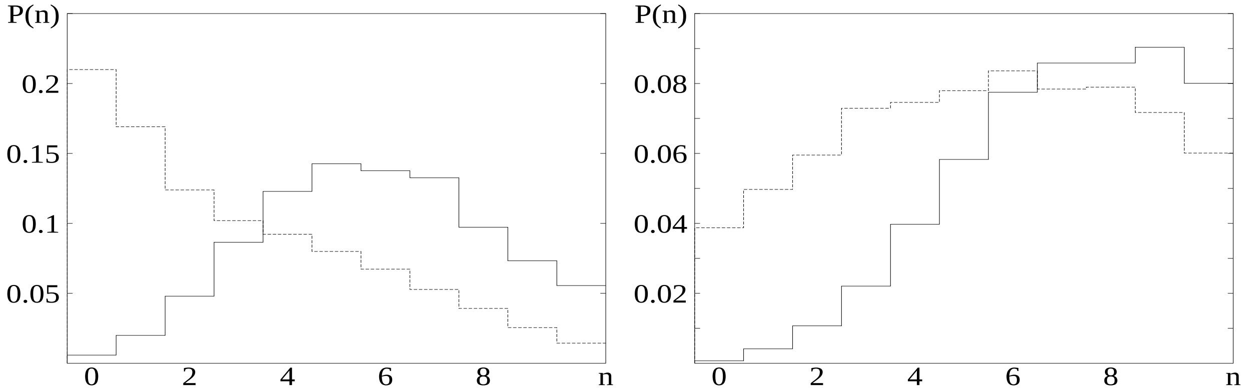

Figure 4: *Multiplicity distributions for $|y| < 0.5$ for "normal" events (solid line) and recoupled events (dashed line) with thrust cuts $T > 0.92$ (fig a) and $T > 0.76$ (fig b).*

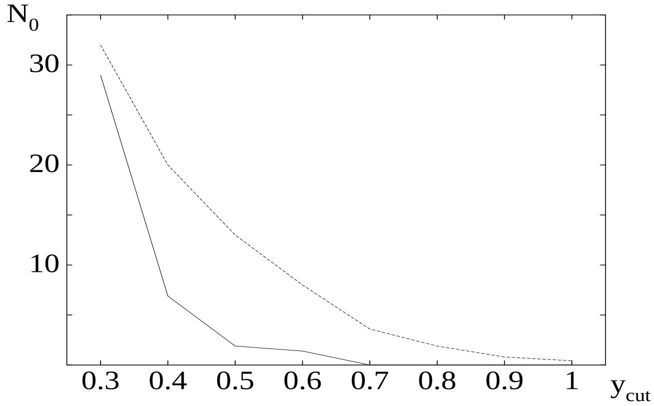

Figure 5: *Expected number of events with zero central particles, N_0, as a function of the rapidity bin size $|y| < y_{cut}$. The dashed line shows the recoupled events and the solid line shows the normal events. The total sample is assumed to be 5000 hadronic WW decays.*

it turns out that events with no central particles are much more common in the recoupled sample. Fig 4 shows the multiplicity distribution (including π^0) in the central region $|y| < 0.5$ for "normal" and recoupled events with the thrust cuts $T > 0.92$ and $T > 0.76$. With the higher thrust cut the difference between normal and recoupled events is larger but statistics become poor as only around 4% of the events pass this cut. For $T > 0.76$ we keep 60% of the events preserving a fairly large difference in the probability to have no particle in the central region. This probability is a factor 55 larger for the recoupled events than for the normal events. With 5000 events with hadronic W-decays, and with 10% probability for recoupling, we expect 13 recoupled events with $n_{central} = 0$ with a background of 1.9 events from normal events. (This signal to background ratio is essentially constant in the entire LEP2 energy range from threshold to 190GeV). The signal to background ratio can be further enhanced if one combines data from different cuts in thrust and rapidity. The dependence on the thrust cut is seen in Figs 3 and 4, and the dependence on the width of the rapidity bin is illustrated in Fig 5. From Fig 4 it is seen that also the probabilities $P(1)$ are very different between normal and recoupled events, and thus information from events with exactly *one* particle in the central rapidity bin can also be used. The best choice of observables will depend on the resolution and acceptances properties of the detectors, and for those who are interested the MC program for the recoupling is available upon request.

Note that in this example we have assumed that a recoupling occurs only to states with minimal λ. Similar results could of course be obtained if recoupling occurs also to other states with small λ-values, but with a correspondingly larger total recoupling probability.

4 Conclusions and comments

In the reaction $e^+e^- \to W^+W^- \to (q_1\overline{q}_2)(Q_1\overline{Q}_2)$ hadron chains or strings are normally expected to be stretched within the colour singlet systems $q_1\overline{q}_2$ and $Q_1\overline{Q}_2$. However also the systems $q_1\overline{Q}_2$ and $Q_1\overline{q}_2$ are singlets with probability $1/N_c^2 = 1/9$, and we could therefore expect "recoupled" chains or strings with a probability of this order of magnitude.

Due to the finite lifetime of the Ws, hard gluons (with $E > \Gamma_W$) are emitted independently by the two initial quark-antiquark pairs. This reduces the observable difference between the "normal" and the "recoupled" events. On the other hand it may also increase the recoupling probability, because the two initial colour singlet systems can be recoupled in very many ways, each in principle with a probability of order $1/N_c^2$.

The presence or absence of such recoupled events gives valuable information about the vacuum structure. Thus we would e.g. expect a larger recoupling probability if vacuum behaves like a type I superconductor than if vacuum is more like a type II superconductor. In the former case the colour fluxtube would be similar to an elongated bag, while in the latter case the confining colour field would have a more narrow central core.

Sjöstrand and Khoze [2, 5] have studied observable effects if the recoupling probability is determined purely by the geometrical structure of the confining force field, and they have found generally small effects. In this paper we have argued that it is not unlikely that dynamical effects enhance the recoupling probability for states which correspond to "short strings" producing few hadrons. These states are also the ones which differ most from the normal events, and we have seen that they can be experimentally identified if they occur with a rate of about 10%.

Sjöstrand and Khoze have also studied the effects of recoupling on the determination of the W mass [5, 2]. Assuming 35% recoupling probability they find an extra uncertainty of about 40MeV. The recoupling effects the softer hadrons, and the states studied in our analysis will give a similar uncertainty in the mass determination as those studied in refs [5, 2]. Thus as a first approximation the uncertainty in the mass determination will be proportional to the total recoupling probability. This probability cannot be calculated from our present knowledge of QCD, and could in principle be large.

It would also be interesting to study effects of recoupling in other reactions. As mentioned in ref [2] recoupling is also possible in normal Z-decay events. We can e.g. look at the reaction $e^-e^+ \to q\overline{q}gg$. If both gluons are emitted from the quark or antiquark legs (cf. Fig 6a) a

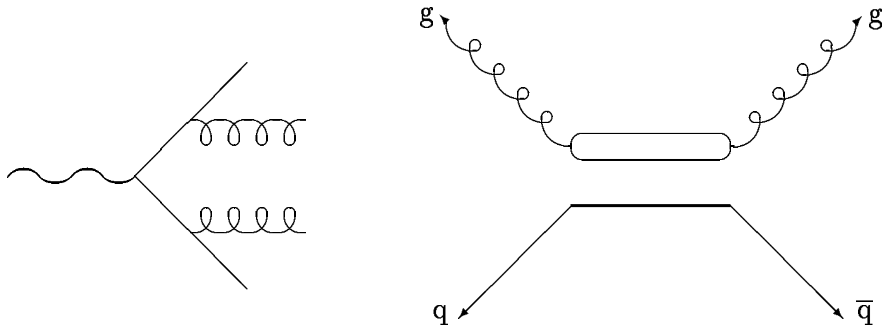

Figure 6: *$q\overline{q}$ state and ring of gluons.*

straightforward calculation shows that the probability for the two gluons to be in a colour singlet state is $1/(N_c^2 - 1) = 1/8$. In such a case the hadronization can be expected to correspond to a string between the quark and the antiquark, together with a closed string loop between the gluons, as shown in Fig 6b. (If the two gluons originate from the process $g \to gg$ they are of course always in a colour octet state. Thus the momentum distribution for the gluons in Fig 6 is the same as for two emitted photons). In a following paper we want to study observable features of such recoupled systems in this and other reactions.

Acknowledgments
We want to thank dr T. Sjöstrand for valuable discussions. This research is supported in part by the EEC Programme "Human Capital and Mobility", Network "Physics at High Energy Colliders", contract CHRX-CT93-0357 (DG 12 COMA).

A Appendix

It is interesting to study what the recoupling probability would be, due to only SU(3) Clebsch-Gordan coefficients, if there were *no* dynamical effects from soft gluons and the vacuum structure. For this we study two pairs $q_1\overline{q}_2$ and $Q_1\overline{Q}_2$ produced in the same space-time point, and we define the states $|1>$ and $|2>$ in the following way:

$$\begin{aligned} &\text{State } |1>: \quad q_1\overline{q}_2 \text{ and } Q_1\overline{Q}_2 \text{ form separate colour singlets,} \\ &\text{State } |2>: \quad q_1\overline{Q}_2 \text{ and } Q_1\overline{q}_2 \text{ form separate colour singlets.} \end{aligned} \tag{1}$$

In state $|1>$ there is a probability $1/N_c^2 = 1/9$ that also $q_1\overline{Q}_2$ and $Q_1\overline{q}_2$ form singlets. This does not imply however, that $q_1\overline{Q}_2$ and $Q_1\overline{q}_2$ form independent hadron strings with probability 1/9. We see that the states $|1>$ and $|2>$ are not orthogonal but satisfy $<1|2>= \sqrt{1/9} = 1/3$.

When the quarks and antiquarks hadronize we assume that the hadrons are formed in two separate cluster chains or strings, and we then have two possible final states, which we denote $|a>$ and $|b>$.

State $|a>$: $q_1\overline{q}_2$ and $Q_1\overline{Q}_2$ form ends of separate strings,

$$\text{State } |b\rangle\text{:} \quad q_1\overline{Q}_2 \text{ and } Q_1\overline{q}_2 \text{ form ends of separate strings.} \tag{2}$$

These states are eigenstates to observables with different eigenvalues and are therefore orthogonal to each other; $<a|b>= 0$. States $|1>$ and $|2>$ can be written as linear combinations of $|a>$ and $|b>$. Due to the symmetry of the situation they can be written as

$$\begin{aligned} |1> &= \alpha|a> +\beta|b> \\ |2> &= \beta|a> +\alpha|b> \end{aligned} \tag{3}$$

The recoupling probability is obviously given by $|\beta|^2$ and the scalar product between $|1>$ and $|2>$ gives the relation

$$<1|2>= 2\mathrm{Re}(\alpha^*\beta) = 1/3. \tag{4}$$

We can choose α real and with the notation $\alpha = \sin\theta$, $\beta = \cos\theta \exp \mathrm{i}\varphi$ we obtain

$$\sin(2\theta) \cdot \cos\varphi = 1/3 \Rightarrow |\sin(2\theta)| \geq 1/3 \tag{5}$$

Thus the recoupling probability $|\beta|^2$ is restricted by the relation

$$\delta < |\beta|^2 < 1 - \delta, \quad \delta = 1/2 - \sqrt{2}/3 \approx 0.03 \tag{6}$$

We conclude that we only obtain a rather loose restriction. The recoupling probability could be anything above a lower limit of 3% (and below 97%). (It is easy to see that the same result is obtained if $|a>$ and $|b>$ are not single states but corresponds to two groups of states).

References

[1] Yu.L. Dokshitzer, V.A. Khoze, L.H. Orr, W.J. Stirling, *Nucl. Phys.* **B403** (1993) 65

[2] T. Sjöstrand, V.A. Khoze, *Z. Physik* **C62** (1994) 281

[3] G. Gustafson, U. Pettersson, P.M. Zerwas, *Phys. Lett.* **B209** (1988) 90

[4] M. Neubert *et al.* in Heavy Flavours, Ed. A.J. Buras, M. Lindman, World Scientific 1992
S. Henderson, Proceedings of XXIX Rencontres de Moriond, 1994 and references given in [3]

[5] T. Sjöstrand, V.A. Khoze, *Phys. Rev. Lett.* **72** (1994) 28

[6] B. Andersson, P. Dahlqvist, G. Gustafson, *Phys. Lett.* **B214** (1988) 604, Z. Physik **C44** (1989) 455

[7] U. Pettersson, Lund preprint LUTP 88-5 (1988)
L. Lönnblad, DESY preprint DESY 92-046 (1992)

[8] B. Andersson, G. Gustafson, G. Ingelman, T. Sjöstrand, *Phys. Rep.* **97** (1983) 31

[9] T. Sjöstrand, *Comp. Phys. Comm.* **39** (1986) 347
T. Sjöstrand, M. Bengtsson, *Comp. Phys. Comm.* **43** (1987) 367