LU TP 93-22
September 1993

Deuteron production in e^+e^--annihilation

Gösta Gustafson[1], Jari Häkkinen[2]
Department of Theoretical Physics,
University of Lund
Sölvegatan 14A,
S-22362 Lund, Sweden

Abstract:
We argue that in e^+e^--annihilation (including Υ-decay) deuteron production should be given by the overlap of the deuteron wave function with the wave function of a pn-pair. The production rate depends sensitively upon the size of the production region. Taking into account the strong correlation between protons and neutrons, experimental results for Υ-decay are consistent with the size expected in the Lund string fragmentation model. A prediction is given for the deuteron production in Z-decay.

[1]E-mail: gosta@thep.lu.se
[2]E-mail: jari@thep.lu.se

1 Introduction

The Argus Coll. at Doris has observed a relatively sizable production of (anti-)deuterons in the decay of Υ [1]. The production rate has been compared to the production of deuterons in nucleus collisions. For this latter situation different models have been proposed. It has been suggested that deuterons are formed in the collisions between protons and neutrons [2]. It has also been proposed that the production can be described as a coalescence process, and the production rate can be determined from the overlap between the wave functions for a proton and a neutron with the wave function of a deuteron [3, 4]. This overlap depends not only on the deuteron wave function, thereby on its binding energy, but also on the interaction volume.

If the overlap is dominated by situations where $\vec{p}_\mathrm{p} \approx \vec{p}_\mathrm{n} \approx \vec{p}_\mathrm{d}/2$ we get the following relation with some constant C

$$\frac{E}{\sigma_\mathrm{tot}}\frac{\mathrm{d}^3\sigma(\mathrm{d})}{\mathrm{d}^3p} = C\,\frac{E_\mathrm{p}E_\mathrm{n}}{\sigma_\mathrm{tot}}\frac{\mathrm{d}^6\sigma(\mathrm{p,n})}{\mathrm{d}^3p_\mathrm{p}\mathrm{d}^3p_\mathrm{n}} = C\,\frac{m_\mathrm{p}}{8}\frac{E}{\sigma_\mathrm{tot}}\frac{\mathrm{d}^6\sigma}{\mathrm{d}^3p\mathrm{d}^3k}\bigg|_{\vec{k}=\vec{0}} \tag{1}$$

Here $\vec{p} \equiv \vec{p}_\mathrm{d}/2 \approx \vec{p}_\mathrm{p} \approx \vec{p}_\mathrm{n}$, $E \equiv E_\mathrm{d}/2 \approx E_\mathrm{p} \approx E_\mathrm{n}$ and

$$\vec{k} = \frac{\vec{p}_\mathrm{p} - \vec{p}_\mathrm{n}}{2}\bigg|_{\text{pn-cms}} \tag{2}$$

is the pn relative momentum in the pn cms. In a simulation process the number of deuterons corresponds to the number of pn-pairs with $|\vec{k}| < k_\mathrm{cut}$, where

$$C\,\frac{m_\mathrm{p}}{8} = \frac{4\pi}{3}k_\mathrm{cut}^3 \tag{3}$$

If the proton and neutron distributions are equal and *uncorrelated*, this can be written as

$$\frac{E}{\sigma_\mathrm{tot}}\frac{\mathrm{d}^3\sigma(\mathrm{d})}{\mathrm{d}^3p} = C\left(\frac{E}{\sigma_\mathrm{tot}}\frac{\mathrm{d}^3\sigma(\mathrm{p})}{\mathrm{d}^3p}\right)^2 \tag{4}$$

The constant C depends on the deuteron wave function and the size of the interaction volume. With the notation $C = \frac{4\pi}{3}p_0^3/m_\mathrm{p}$ $(p_0 = 2k_\mathrm{cut})$ the observed deuteron production rate in nucleus collisions is obtained for $p_0 \approx 130\mathrm{MeV}$ [5].

Applied to Υ decay, with the same value of C (corresponding to $p_0 = 130\mathrm{MeV}$), Eq (4) predicts roughly the observed number of deuterons. Ref [1] finds that the prediction is about a factor 2/3 times the experimental value, but the predicted energy distribution is somewhat too hard, with too many deuterons with high energy and too few with low energy.

In this article we want to

i) argue that in $\mathrm{e}^+\mathrm{e}^-$-annihilation reactions (including Υ decay) the deuteron production should indeed be given by the overlap of the deuteron wave function, provided the nucleon distributions are determined by a model, which does not take the attractive pn potential into account.

ii) demonstrate that a careful analysis, including the correlation between protons and neutrons, reproduces the observed energy distribution. The observed rate is consistent with estimates of the interaction volume based on the Lund string fragmentation model.

iii) make predictions for deuteron production in e^+e^--annihilation into $q\overline{q}$-pairs, e.g. at LEP.

2 Wave function overlap

In an e^+e^--annihilation event a $q\overline{q}$ or 3g system is transformed into hadrons. This transformation is determined by the Hamiltonian

$$H = H_0 + H_I \tag{5}$$

where H_I describes an attractive force between a proton and a neutron with a deuteron bound state, and H_0 describes the remaining strong interaction. We assume that the final state interaction H_I can be treated as a perturbation.

Thus H_0 describes the transition to hadronic states (denoted Ψ_k) containing free protons and neutrons. A model for this transition is given by the Lund fragmentation model [6]. The full Hamiltonian H gives transitions to states Ψ'_i, which contain deuterons as well as pn states in the continuum, orthogonal to the deuteron wave function. To the lowest order in a perturbation expansion the probability to obtain a definite state Ψ'_i is in this case simply given by the overlap with the unperturbed states Ψ_k.

We note that this case is different from the (more complicated) situation in nucleus collisions, where the initial state is an eigenstate to the full Hamiltonian H, i.e. a state of type Ψ'_i. If this state contains an unbound pn-pair, the wave function for this pair is orthogonal to that of a deuteron. Thus such a pair can be transformed into a deuteron only by the interaction with some other particle.

The overlap with the deuteron wave function is estimated in ref [4]. Here we will use a different formalism, but our results are very similar. The binding energy of the deuteron is small and therefore the wave function is quite extended. To get an intuitive feeling for the result we first study the situation when the range of the attractive potential is very small compared to the extension of the wave function. Results from a more accurate calculation will be presented afterwards.

Separating the cms motion and the relative motion we introduce the variables $\vec{r} = \vec{r}_\mathrm{p} - \vec{r}_\mathrm{n}$ and $\vec{k} = (\vec{p}_\mathrm{p} - \vec{p}_\mathrm{n})/2$ (from Eq (2)) as measured in the pn cms. The approximate deuteron wave function has the following form

$$\Psi_\mathrm{d}(r) \approx N\frac{\mathrm{e}^{-\kappa r}}{r} \tag{6}$$

$$\kappa = \sqrt{m_\mathrm{p}\mathcal{E}}; \quad N = \sqrt{\frac{\kappa}{2\pi}}$$

Here $\mathcal{E}$ is the deuteron binding energy and the reduced mass equals $m_p/2$. For the free pn-pair we could imagine a plane wave ansatz

$$\Psi_{\rm pn} = \frac{1}{\sqrt{V}}\,{\rm e}^{{\rm i}\vec{k}\vec{r}} \tag{7}$$

where V is the quantization volume. The square of the overlap is then

$$A(k) = |<\Psi_{\rm d}|\Psi_{\rm pn}>|^2 = \frac{8\pi\kappa}{V}\frac{1}{(\kappa^2+k^2)^2} \tag{8}$$

This expression falls off rapidly with increasing momentum k, and the probability for a pn-pair to join into a deuteron is large only for $k \lesssim \kappa$. If the production cross section is approximately constant within this region the result in Eq (1) is justified, and we obtain

$$C\,\frac{m_{\rm p}}{8} = \frac{3}{4}\int A(k)\,{\rm d}^3k = \frac{3}{4}\frac{(2\pi)^3}{V} \tag{9}$$

The factor 3/4 is a spin factor representing the probability to form a total spin 1 state.

This result depends on the quantization volume, which indicates that it is sensitive to the production volume for the pn-pair. Therefore we replace the plane wave equation in Eq (7) by a wave package

$$\tilde{\Psi}_{\rm pn} = \tilde{N}{\rm e}^{-r^2/2\sigma^2+{\rm i}\vec{k}\vec{r}} \qquad \tilde{N}^{-2} = \sigma^3\sqrt{\pi^3} \tag{10}$$

We then get

$$C\,\frac{m_{\rm p}}{8} = \frac{3}{4}\int \left|<\tilde{\Psi}_{\rm pn}|\Psi_{\rm d}>\right|^2 {\rm d}^3k = 6\sqrt{\pi}\,\kappa\,\sigma^{-2}{\rm e}^{\kappa^2\sigma^2}{\rm erfc}(\kappa/\sigma) \underset{\kappa\sigma\ll 1}{\approx} 6\sqrt{\pi}\,\kappa\,\sigma^{-2} \tag{11}$$

Here erfc is an error function and the last approximation is obtained when κ is small compared to σ^{-1}. Expressed in the variable $k_{\rm cut}$ or p_0, defined above, this approximation implies

$$(2k_{\rm cut})^3 = p_0^3 = \frac{36}{\sqrt{\pi}}\,\kappa\sigma^{-2} \tag{12}$$

To improve this approximate result we replace the wave function in Eq (6) by a wave function corresponding to a square well potential with radius $R = 1.7$ fm and depth $V_0 = 36.4$ MeV. This function has the form

$$\Psi_{\rm d} = \begin{cases} N_1\dfrac{\sin k'r}{r} & r < R \qquad k' = \sqrt{m_p(V_0-\mathcal{E})} \\[2ex] N_2\dfrac{{\rm e}^{-\kappa r}}{r} & r > R \qquad \kappa = \sqrt{m_{\rm p}\mathcal{E}} \end{cases} \tag{13}$$

The overlap with the wave package in Eq (10) is calculated numerically, and the result is shown in Fig 1. We have also studied a different wave package, with separate production points for the nucleons

$$\tilde{\Psi}'_{\rm pn} = N'{\rm e}^{-(\vec{r}-\vec{\mu})^2/2\sigma^2+{\rm i}\vec{k}\vec{r}} \tag{14}$$

It turns out that the result is very similar to that in Fig 1 if only σ^2 is replaced by $\sigma^2+\mu^2$.

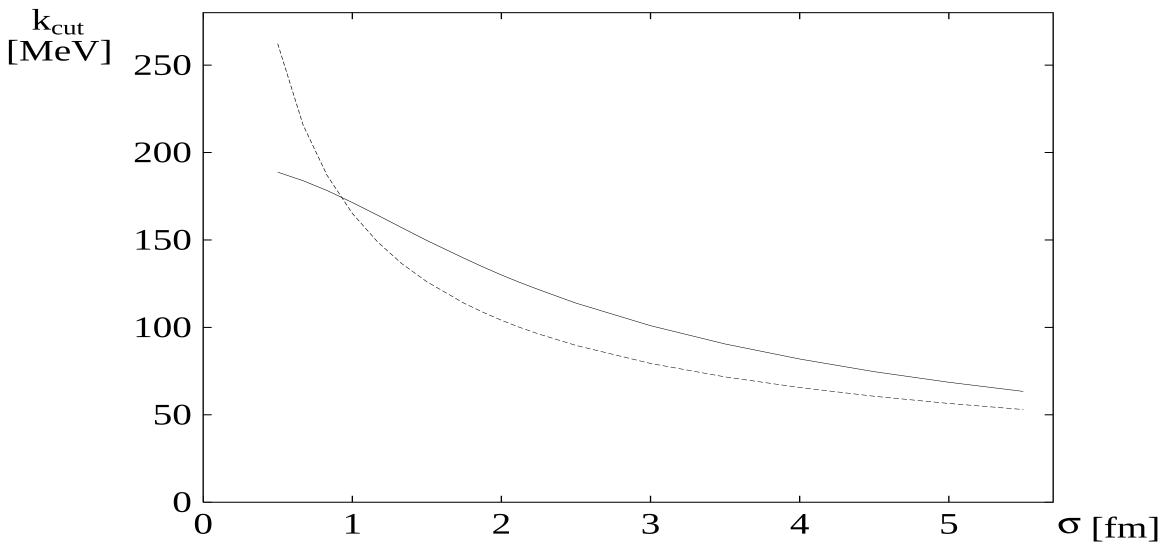

Figure 1: *Wave function overlap expressed in the parameter k_{cut} as a function of the size σ of the production region. The solid line corresponds to the wave function in Eq* (13)*, and the dashed line to the approximation in Eq* (12)*. Note that the production probability is proportional to k_{cut}^3.*

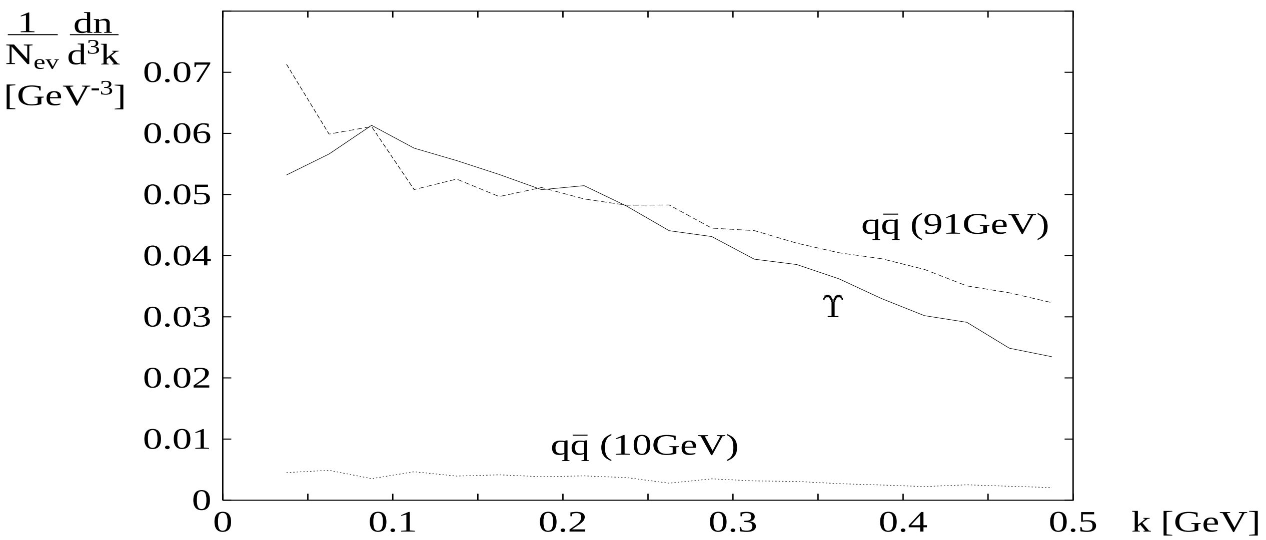

Figure 2: *Distribution of pn pairs as a function of the relative momentum k. (Included nucleons are either directly produced or Δ-decay products). Results are shown for Υ-decay (solid line), $q\bar{q}$ continuum at* 10 *GeV (dotted line), and Z-decay (dashed line).*

3 Correlations and resonances

To take into account the pn-correlations and the effect of resonances we simulate events with the Lund fragmentation model as implemented in the JETSET MC [7]. We have first checked that the distribution in $\vec{k}$ is almost constant for $k \lesssim 200$MeV. This is demonstrated in Fig 2. The result implies that only the integral over the overlap, $\int A(k) \mathrm{d}^3 k$, is important

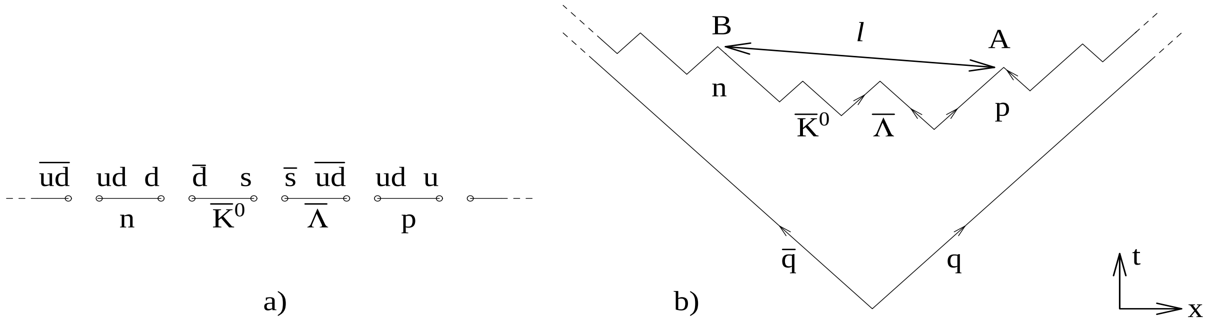

Figure 3: **a)** *Possible string breakup process with a pn pair.* **b)** *Space-time structure of the breakup.*

and that the cross section is given by Eqs (1) and (9).

When estimating the number of pn pairs, which can combine to form a deuteron, it is important to note that not all nucleons are directly produced in the hadronization process. Thus nucleons from the decay of strange baryons with very long lifetimes, are not expected to be able to form deuterons. For Δ-resonances the lifetime is so short that a decay product should have a possibility to join another nucleon into a deuteron, although with a larger production region.

In our simulations we have found that for 24% of the pn pairs both nucleons are directly produced. For 38% at least one nucleon is a Δ-decay product, and the other nucleon is either directly produced or also a Δ-decay product. (In most of these pairs the other nucleon is directly produced, and in only 1/4 of them both nucleons originate from Δ-particles.) For the remaining 38% at least one nucleon is a decay product of a strange baryon. In most of these pairs, 30% of the total number, at least one nucleon comes from a Λ. These fractions are practically independent of the total energy of the pair. We note that only in a relatively small fraction of the pn pairs both nucleons are directly produced.

To evaluate the number of deuterons we must first estimate the production volume for the pn pairs. In string fragmentation particles, which are close in momentum space, are also produced close in coordinate space. Thus it is possible to understand that the pion pair production region, obtained in HBT-analyses, is of the order of 1 fm, also at high energies when the string length may be tenths of fm or more. For pn pairs the production volume must be somewhat larger than for pions, because nucleons are more massive, and when the string breaks there must be at least one antibaryon between the proton and the neutron. A typical example is shown in Fig 3. A space-time diagram for the string motion and breakup is shown in Fig 3b.

It is not obvious how to define the production points for the different hadrons. The simplest alternative is the point where the constituents meet for the first time. This corresponds to the points A and B in Fig 3b, for the proton and the neutron respectively. We have used the MC to calculate the distribution in the invariant distance, l, between these points for a straight string. Fig 4 shows the result for pairs with a small relative momentum

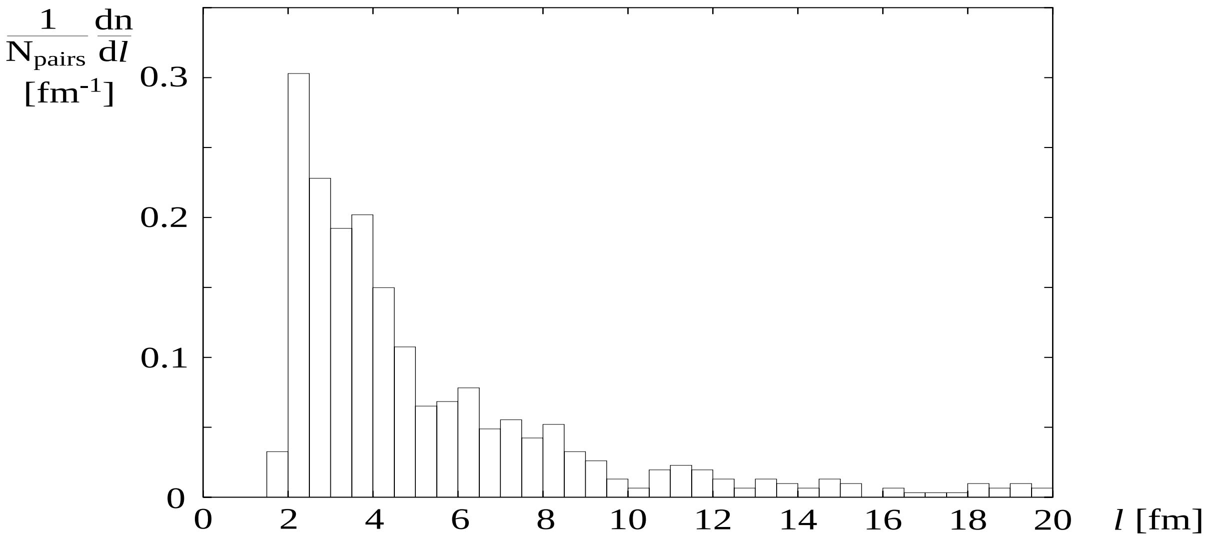

Figure 4: *Distribution in invariant distance between the production points for pn pairs with low relative momentum. (For 5% of the pairs the distance is larger than 20fm.)*

($k < 150$MeV). (In this figure only directly produced nucleons are included). We note that there is a large peak around 3fm, with a rather limited tail stretching to large volumes, and very few pairs below 2fm. The result presented corresponds to 91GeV, which implies that the maximum distance is 91fm (we assume a string tension $\kappa = 1$GeV/fm). As there are very few pairs with low relative momentum which have distances larger than 15fm, we see that these results essentially also correspond to an infinitely long string.

The situation in Υ-decay does not correspond to an infinitely long string. In the Lund string model it is represented by a closed string, which is stretched out by the three gluons to a triangular shape. In view of the difficulties to define the production points in an unambiguous way, we have not found it meaningful to perform a complicated simulation calculating the location of the production points for such a triangular string. For a closed string with total energy 10GeV the maximum distance is 5fm. Thus the large distance tail in Fig 4 is not relevant for the Υ-decay, and we think that a reasonable estimate for the average distance between the production points is around 3fm.

The Δ-resonance has a lifetime of 1.7fm. If this is added in quadrature to an original distance of $\sim$3fm it will increase the distance by about 0.5fm.

4 Results

4.1 Υ-decay

The experimental results for deuteron production in ref [1] are well reproduced with σ about 3fm. In Fig 5 we have presented results from MC simulations obtained using $\sigma = 3.5$fm for directly produced pn pairs and $\sigma = 4$fm when one nucleon comes from a Δ-decay and

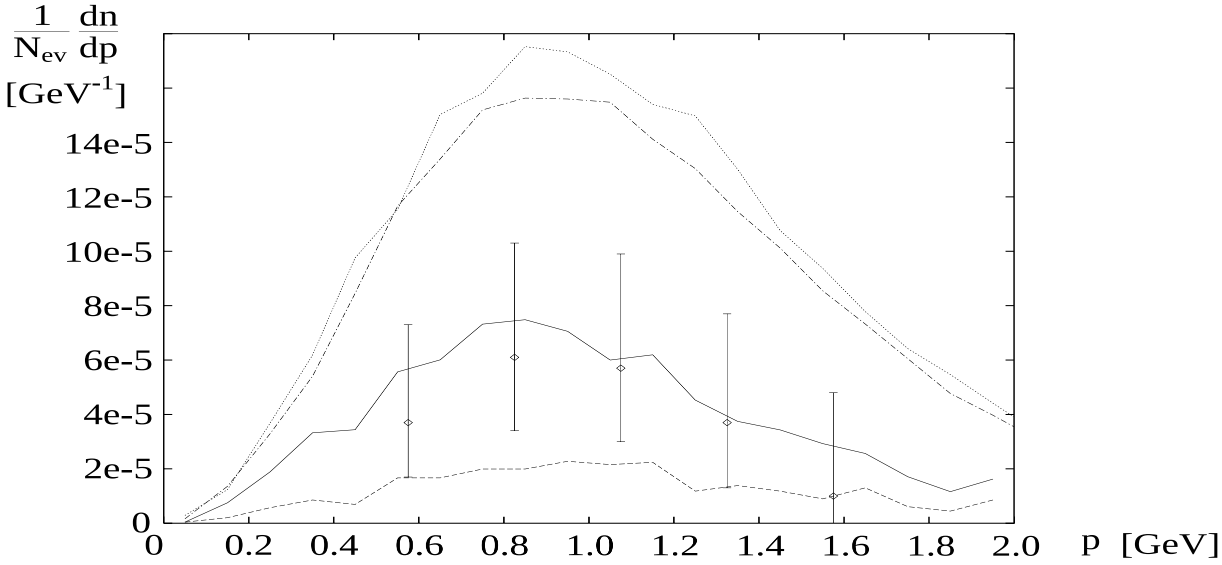

Figure 5: *The momentum distribution for deuterons in* Υ*-decay. The solid line is the result including directly produced nucleons and* Δ*-decay products as described in the text. The dashed line corresponds to only directly produced nucleons. The other curves are the results from Eq* (4) *assuming uncorrelated nucleons, including only direct nucleons and* Δ*-decays (dashed-dotted line) and including also strange baryons other than* Λ *(dotted line).*

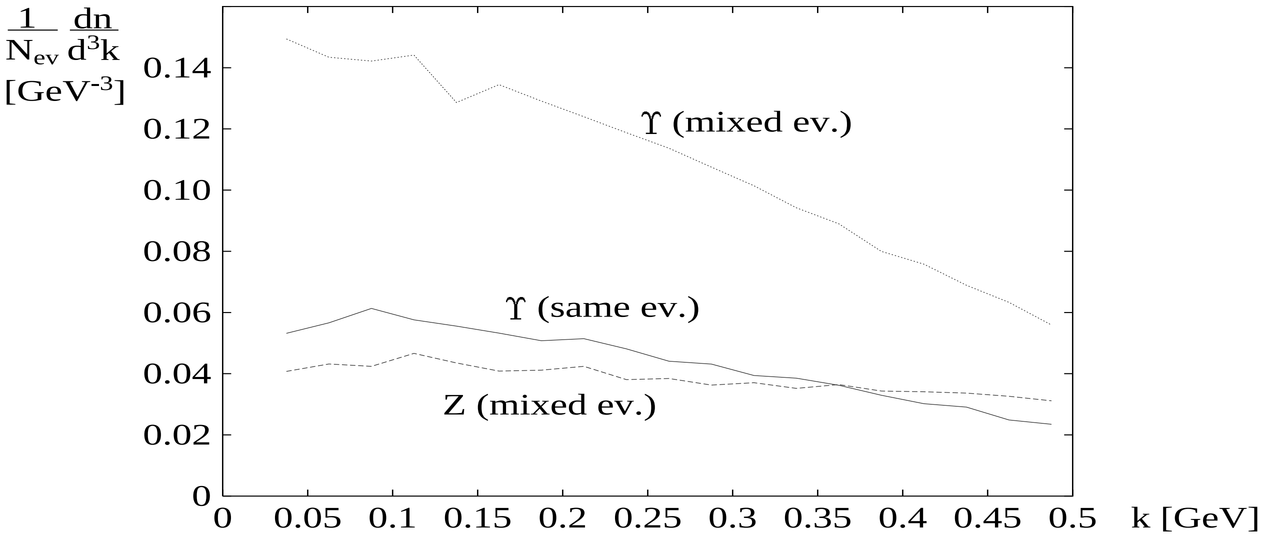

Figure 6: *Distribution in relative momentum k of pn pairs, where the nucleons in the pair are taken from different events. The dotted line is for* Υ*-decay, and the dashed line is for Z-decay. For comparison the solid line shows the distribution for correlated pairs in* Υ*-decay from Fig* 2.

the other is either directly produced or another Δ-decay product. According to Fig 1 these values correspond to $k_{\rm cut} = 90$Mev and $k_{\rm cut} = 82$MeV respectively. We note that also σ-values as low as 2.8 and 3.3, instead of 3.5 and 4, would give results within the experimental errors.

In Fig 5 is also shown results obtained if the nucleons in a pair were uncorrelated, which corresponds to Eq (4). If we do not include nucleons from strange baryon decays and use an average value of k_{cut} (86Mev), the result is about a factor 2.5 larger. This shows that the nucleons are strongly correlated. The correlation is not so much because a proton and a neutron in the same event avoid having the same momentum. They rather avoid being in the same event. This is seen in Fig 6, which shows the distribution in relative momentum, k, for pn-pairs from different events. This curve is similar in shape to the distribution in Fig 2, but the normalization is much higher. We interpret this as a consequence of the limited energy available. A proton and a neutron must be accompanied by two anti-baryons, and together these four particles would take a large part of the total Υ-energy. (If also nucleons from strange baryons other than Λ's were included the result would be even larger, as indicated by the dotted line in Fig 5. Including also Λ-decays would increase the result by another factor of 1.5).

If the nucleons were uncorrelated we would thus need a much smaller k_{cut}-value, and hence a larger production region. In ref [1] it is pointed out that in this case a fair agreement with data is obtained for $k_{\mathrm{cut}} = p_0/2 = 65\mathrm{MeV}$. Not including strange baryon decays this would also be the result from our simulations. From Fig 1 we see that this corresponds to a σ-value around 5.3fm, which seems to be an unrealistically large production region.

We note that in our simulation the shape of the spectrum is rather similar if we use the approximation in Eq (4) with uncorrelated nucleons. The main difference lies in the normalization. This was not the case for the analysis in ref [1], using experimental data for proton production. The main discrepancy is however for the first bin, where the experimental uncertainty in the proton spectrum is very large.

4.2 $\mathrm{q\overline{q}}$ continuum

In the process $\mathrm{e^+e^-} \rightarrow \mathrm{q\overline{q}}$ the particle density is lower than for $\mathrm{e^+e^-} \rightarrow \Upsilon \rightarrow 3\mathrm{g}$. Thus the production rate for deuterons is also lower. From Fig 2 we see that the number of low mass pn pairs in the $\mathrm{q\overline{q}}$ continuum at 10GeV is only about 4% of the number at Υ. This is consistent with the upper limit in ref [1], which corresponds to $1.7 \cdot 10^{-5}$ d/event or about 25% of the production rate in Υ-decay.

At LEP energies the total particle multiplicity is larger, and therefore also the number of pn pairs. To, in an approximate way, take into account the tail in the distribution of production point distances in Fig 4, we have used the larger σ-values 4 and 4.5fm for directly produced pairs and pairs including Δ-decay. Fig 7 shows the predicted momentum distribution at 91GeV. The total production rate is $5 \cdot 10^{-5}$ deuterons per event.

In Fig 6 we also show results for pairs from mixed Z-decay events. Here we do not get a very large result using mixed events. This is a natural consequence of the pronounced jet structures in the events from Z-decays. Thus if we would like to use Eq (4) to estimate the deuteron production, the larger σ-value in Z-decay compared to Υ-decay would imply a smaller value for C or p_0, but this is counteracted by the smaller anti-correlation (cf. Fig 6 and 2) which gives a larger value for C.

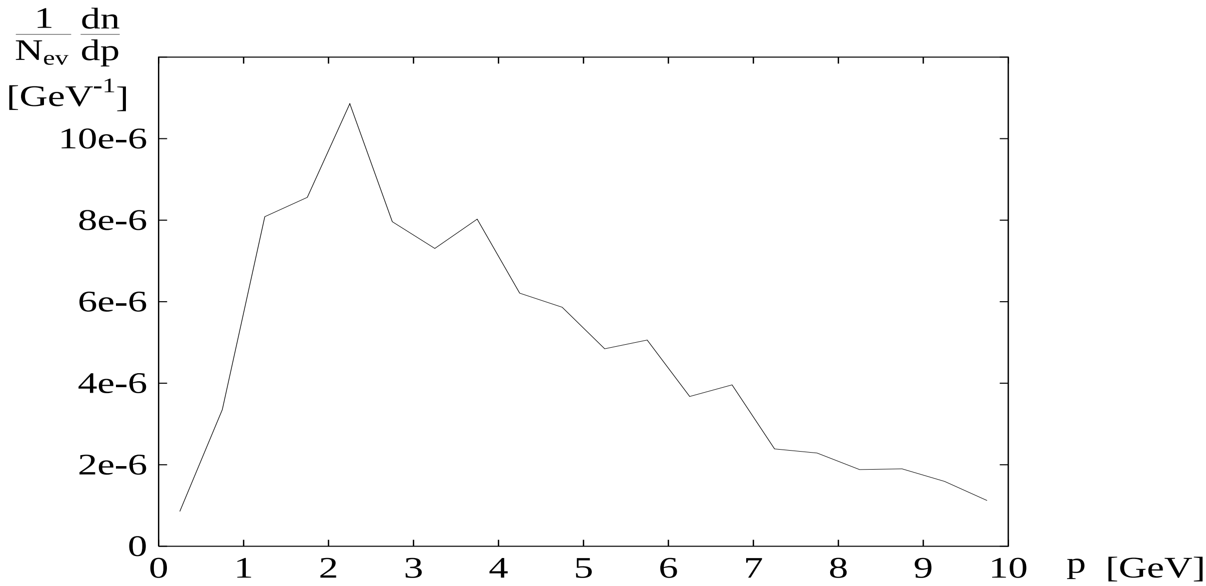

Figure 7: *The predicted momentum distribution for deuterons at* 91*GeV.*

5 Conclusions

We have argued that in e^+e^--annihilation (including Υ-decay) deuteron production should be given by the overlap of the deuteron wave function with the wave function of a pn-pair, provided the nucleon distributions are determined by a model which does not take the attractive pn potential into account. This situation is simpler than the situation in nucleus collision, for which a coalescence process has been proposed in refs [3] and [4].

When calculating the deuteron production rate it is very essential to take the pn correlations into account. Protons and neutrons are strongly anti-correlated so that the number of pn-pairs (with low invariant mass) in an Υ event is about a factor 2.5 smaller than the number obtained by mixing events.

The production rate depends on the size of the production region. The experimental results for Υ-decay from ref [1] are consistent with the size expected in the Lund string fragmentation model. (Also the momentum distribution is well reproduced by the model calculations). If the anti-correlation was not taken into account an unrealistically large production region would have been needed. We have here taken into account that nucleons from Δ-decay correspond to a larger production region, and that the lifetime of strange baryons is so long that their contributions can be neglected.

We have made a prediction for deuteron production in Z-decay. The integrated production rate corresponds to $5 \cdot 10^{-5}$ deuterons per event.

References

[1] ARGUS Collab., H. Albrecht *et al.*, *Phys. Lett.* **236B** (1990) 102

[2] R. Hagedorn, *Phys. Rev. Lett.* **5** (1960) 276
W.D. Myers, *Nucl. Phys.* **296A** (1978) 177
J. Gosset, J.J. Kapusta, G.D. Westfall, *Phys. Rev.* **18C** (1978) 844

[3] S.T. Butler, C.A. Pearson, *Phys. Rev. Lett.* **7** (1961) 69
S.T. Butler, C.A. Pearson, *Phys. Lett.* **1** (1962) 77
A. Schwarzschild, Č. Zupančič, *Phys. Rev.* **129** (1963) 854

[4] H. Sato and K. Yazaki, *Phys. Lett.* **98B** (1981) 153

[5] H.H. Gutbrod *et al.*, *Phys. Rev. Lett.* **37** (1976) 667
J. Gosset *et al.*, *Phys. Rev.* **16C** (1977) 629
M.C. Lemaire *et al.*, *Phys. Lett.* **85B** (1979) 38
V.B Gavrilov *et al.*, *Z. Phys.* **324A** (1986) 75

[6] B. Andersson, G. Gustafson, G. Ingelman, and T. Sjöstrand, *Phys. Rep.* **97** (1983) 31

[7] T. Sjöstrand, M. Bengtsson *Comp. Phys. Comm.* **43** (1987) 367
T. Sjöstrand, JETSET 7.3, program and manual