Theoretical status of ε'/ε

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Abstract. We briefly overview the historical controversy around Standard Model predictions of ε'/ε and clarify the underlying physics. A full update of this important observable is presented, with all known short- and long-distance contributions, including isospin-breaking corrections. The current Standard Model prediction, $\mathrm{Re}(\varepsilon'/\varepsilon) = (14 \pm 5) \cdot 10^{-4}$ [1, 2], is in excellent agreement with the experimentally measured value.

1. Historical prelude

The first evidence of CP non-invariance in particle physics was the non-zero value of the ratios

$$\eta_{+-} \equiv \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} \equiv \varepsilon + \varepsilon' , \qquad \eta_{00} \equiv \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} \equiv \varepsilon - 2\,\varepsilon' , \tag{1}$$

which mainly originates in the $\Delta S = 2$ weak transition between the K^0 and $\bar{K}^0$ states [3, 4]: $|\varepsilon| = |\eta_{00} + 2\,\eta_{+-}|/3 = (2.228 \pm 0.011) \cdot 10^{-3}$. A tiny difference between the two ratios was reported for the first time in 1988 by the CERN NA31 collaboration [5], and later established at the 7.2σ level with the full data samples of NA31 [6], NA48 [7–9] and the Fermilab experiments E731 [10] and KTeV [11–13]:

$$\mathrm{Re}\left(\varepsilon'/\varepsilon\right) = \frac{1}{3}\left(1 - \left|\frac{\eta_{00}}{\eta_{+-}}\right|\right) = (16.6 \pm 2.3) \cdot 10^{-4} . \tag{2}$$

This important measurement demonstrated the existence of direct CP violation in the $K^0 \to 2\pi$ decay amplitudes, confirming, therefore, the Standard Model (SM) quark-mixing mechanism where CP violation is associated with a $\Delta S = 1$ transition.

The pioneering leading-order (LO) estimates of the strong-penguin (Q_6) amplitude predicted values of $\varepsilon'/\varepsilon \sim 2 \cdot 10^{-3}$ [14]. However, since the large top quark mass enhances the electroweak penguin (Q_8) correction that has the opposite sign, the first next-to-leading-order (NLO) calculations [15–19] found results one order of magnitude smaller than (2). Larger values around 10^{-3} were nevertheless obtained (also at NLO) with model-dependent estimates of non-perturbative hadronic contributions [20–23].

[4] Speaker

It was soon realised that those calculations claiming small values of ε'/ε were missing the important role of the final pion dynamics [24, 25]. The proper inclusion of long-distance contributions with chiral-perturbation-theory (χPT) techniques gave $\mathrm{Re}(\varepsilon'/\varepsilon) = (17 \pm 9) \cdot 10^{-4}$ [26]. Taking also into account a more refined analysis of isospin-breaking (IB) corrections [27–29], induced by electromagnetic interactions and the light-quark mass difference, led finally to $\mathrm{Re}(\varepsilon'/\varepsilon) = (19 \pm 10) \cdot 10^{-4}$ [30], in good agreement with the experimental value but with a rather large uncertainty.

The controversy around ε'/ε resurrected in 2015, when the lattice RBC-UKQCD collaboration reported $\mathrm{Re}(\varepsilon'/\varepsilon) = (1.38 \pm 5.15 \pm 4.59) \times 10^{-4}$ [31, 32], 2.1σ below the experimental measurement. This has triggered a revival of the old naive estimates [33, 34], some of them making also use of the lattice data [35–37], and a large amount of new-physics (NP) explanations (a list of references is given in Refs. [1, 2]). However, the current lattice simulation needs to be taken with a grain of salt because it fails to reproduce the $I = J = 0$ $\pi\pi$ scattering phase shift δ_0^0, which plays a key role in the calculation. The lattice value of $\delta_0^0(m_K)$ is 2.9σ below the experimental result, a much larger discrepancy than the one quoted for ε'/ε, and nobody suggests any NP explanation for this phase-shift anomaly. RBC-UKQCD is obviously working hard to fix the problem [38].

In view of the situation, we have performed a complete update of the SM calculation of ε'/ε, with analytical χPT techniques, taking into account our current knowledge of all relevant inputs, such as quark masses and non-perturbative low-energy constants (LECs). Our final result [1, 2],

$$\mathrm{Re}(\varepsilon'/\varepsilon) \;=\; (14 \pm 5) \cdot 10^{-4}\,, \tag{3}$$

is in good agreement with the experimental world average in Eq. (2).

2. Basic dynamical features of $\mathbf{K \to \pi\pi}$

The $K^0 \to \pi\pi$ decays can be characterized through the amplitudes $\mathcal{A}_I = A_I\, \mathrm{e}^{\delta_I}$, where $I = 0$ or $I = 2$ denote the isospin state of the two final pions ($I = 1$ is forbidden by Bose symmetry) and the strong phases δ_I equal the S-wave $\pi\pi$ scattering phase shifts $\delta_0^I(m_K)$, in the limit of isospin conservation. Assuming isospin symmetry, a direct fit to the $K \to \pi\pi$ rates gives [39]

$$A_0 = (2.704 \pm 0.001) \cdot 10^{-7}\,\mathrm{GeV}, \qquad A_2 = (1.210 \pm 0.002) \cdot 10^{-8}\,\mathrm{GeV}, \qquad \delta_0 - \delta_2 = (47.5 \pm 0.9)^\circ, \tag{4}$$

where the tiny (CP-odd) imaginary parts of A_I have been neglected. Thus, the kaon data exhibit two important properties:

(i) A spectacular enhancement of the isoscalar amplitude ($\Delta I = \frac{1}{2}$ rule), generated by the strong forces, that suppresses the ratio $\omega \equiv \mathrm{Re}(A_2)/\mathrm{Re}(A_0) \approx 1/22$ by a factor sixteen with respect to its naive SM expectation (without QCD) of $1/\sqrt{2}$.

(ii) A huge phase-shift difference, due to the strong final-state interactions (FSI) of the two pions. Therefore, the amplitudes $\mathcal{A}_I = \mathrm{Dis}(\mathcal{A}_I) + i\,\mathrm{Abs}(\mathcal{A}_I)$ have large absorptive components $\mathrm{Abs}(\mathcal{A}_I)$, specially the isoscalar one. Neglecting the small CP-odd parts, the known $\pi\pi$ scattering phase shifts at $\sqrt{s} = m_K$, $\delta_0^0(m_K) = (39.2 \pm 1.5)^\circ$ and $\delta_0^2(m_K) = (-8.5 \pm 1.5)^\circ$ [40], imply that

$$\mathrm{Abs}(\mathcal{A}_0)/\mathrm{Dis}(\mathcal{A}_0) = \tan\delta_0 \approx 0.82\,, \qquad\qquad \mathrm{Abs}(\mathcal{A}_2)/\mathrm{Dis}(\mathcal{A}_2) = \tan\delta_2 \approx -0.15\,. \tag{5}$$

The short-distance perturbative calculations claiming small SM values of ε'/ε are unable to generate the physical phase shifts, *i.e.*, they predict $\delta_I = 0$ and, therefore, $\mathrm{Abs}(\mathcal{A}_I) = 0$, failing completely to understand the empirical ratios (5). Since $A_0 = \sqrt{1 + \tan^2\delta_0}\; \mathrm{Dis}(\mathcal{A}_0) \approx 1.3 \times \mathrm{Dis}(\mathcal{A}_0)$, missing the absorptive contribution leads to a gross underestimation of the isoscalar amplitude. This unitarity pitfall implies also incorrect predictions for the dispersive

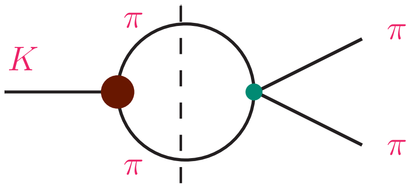

Figure 1. Feynman topology generating an absorptive contribution to the $K \to \pi\pi$ amplitudes. The dashed vertical line indicates the corresponding unitarity cut.

components, since they are related by analyticity with the absorptive parts: a large absorptive contribution generates a large dispersive correction that is obviously missed in those naive estimates.

Figure 1 displays the one-loop diagrammatic topology that generates the absorptive contribution through an on-shell intermediate $\pi\pi$ state. Taking into account the correct chiral structure of the $K\pi\pi$ vertex, the implications of this loop correction are easily computed [26]:

$$\Delta\mathcal{A}_0/\mathcal{A}_0^{\rm tree} \;=\; (2m_K^2 - m_\pi^2)\, B_{\rm loop} + \ldots \qquad \Delta\mathcal{A}_2/\mathcal{A}_2^{\rm tree} \;=\; -(m_K^2 - 2m_\pi^2)\, B_{\rm loop} + \ldots \tag{6}$$

where the dots stand for contributions from other topologies without absorptive parts, and

$$B_{\rm loop} \;=\; \frac{1}{32\pi^2 F_\pi^2} \left\{ \sigma_\pi \left[\log\left(\frac{1-\sigma_\pi}{1+\sigma_\pi}\right) + i\pi \right] + \log\left(\frac{\nu_\chi^2}{m_\pi^2}\right) + 1 \right\} \tag{7}$$

with $\sigma_\pi \equiv \sqrt{1 - 4m_\pi^2/m_K^2}$. The two isospin amplitudes get corrections of opposite signs, and the mass-dependent prefactors in Eq. (6) make the effect larger by a factor 2.3 in the isoscalar case. The finite one-loop absorptive amplitudes induced by the $i\pi$ term in $B_{\rm loop}$ are model independent. They represent universal corrections that only depend on the $\pi\pi$ quantum numbers:

$$\mathrm{Abs}(\mathcal{A}_0)/\mathcal{A}_0^{\rm tree} \;=\; 0.47\,, \qquad \mathrm{Abs}(\mathcal{A}_2)/\mathcal{A}_2^{\rm tree} \;=\; -0.21\,. \tag{8}$$

It is worth stressing that these absorptive contributions are present for any effective $K\pi\pi$ vertex in Figure 1, generating an on-shell intermediate $\pi\pi$ state with the appropriate quantum numbers. Any hypothetical NP contribution at short distances would just modify the denominators in (8), leading to some $\Delta\mathcal{A}_I^{\rm tree} \sim g_I^{\rm SD}\mathcal{O}_I$ with some low-energy four-quark operator $\mathcal{O}_I$. Owing to unitarity, the $I=0$ or $I=2$ quantum numbers of this operator determine the same absorptive corrections given in Eq. (8).[6] Moreover, these corrections are identical for the CP-even and CP-odd amplitudes, since they just originate from the real and imaginary parts, respectively, of the short-distance coupling $g_I^{\rm SD}$ (both in the SM and NP cases).[7] The size of these unitarity corrections slightly increases at higher loop orders [25, 26].

3. Anatomy of ε'/ε

Direct CP violation appears through the interference between the two isospin amplitudes,

$$\mathrm{Re}(\varepsilon'/\varepsilon) \;=\; -\frac{\omega}{\sqrt{2}\,|\varepsilon|}\left[\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} - \frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}\right] \;=\; -\frac{\omega_+}{\sqrt{2}\,|\varepsilon|}\left[\frac{\mathrm{Im}A_0^{(0)}}{\mathrm{Re}A_0^{(0)}}\,(1-\Omega_{\rm eff}) - \frac{\mathrm{Im}A_2^{\rm emp}}{\mathrm{Re}A_2^{(0)}}\right]. \tag{9}$$

The observable effect is suppressed by the small value of ω and is very sensitive to IB contributions [27–29], parametrized by [2]

$$\Omega_{\rm eff} \;=\; 0.11 \pm 0.09\,, \tag{10}$$

[6] Unfortunately, most analyses of NP contributions to K decays ignore the presence of absorptive amplitudes.

[7] The absence of FSI in the CP-odd penguin amplitude has been claimed in Ref. [34], on the basis of an incorrect calculation that violates both chiral symmetry and unitarity.

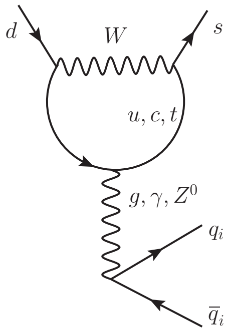

Figure 2. Strong and electroweak penguin diagrams, involving the three up-type quarks needed to have CP violation. The CP-odd amplitudes are proportional to the combination of CKM parameters $\mathrm{Im}\,(V_{td}V_{ts}^*) = -\mathrm{Im}\,(V_{cd}V_{cs}^*) \approx \eta\lambda^5 A^2$.

because small corrections to A_0 feed into the small amplitude A_2 enhanced by the large factor $1/\omega$. In the right-hand side of Eq. (9), $\omega_+ = \mathrm{Re}(A_2^+)/\mathrm{Re}(A_0)$ where A_2^+ is directly extracted from the $K^+ \to \pi^+\pi^0$ rate, the (0) superscript denotes the isospin limit, and A_2^{emp} contains the electromagnetic-penguin contribution to A_2 (the remaining contributions are included in Ω_{eff}).

Since $\mathrm{Im}A_2$ is already an IB effect, $\mathrm{Re}A_2^{(0)} \approx \mathrm{Re}A_2$ can be directly obtained from Eq. (4). The CP-even amplitude $\mathrm{Re}A_0^{(0)}$ is also basically fitted to experimental data [2]. Thus, besides the IB parameter Ω_{eff}, one only needs a theoretical prediction for the CP-odd amplitudes $\mathrm{Im}A_0^{(0)}$ and $\mathrm{Im}A_2^{\mathrm{emp}}$, which to a very good approximation are dominated by the strong (Q_6) and electroweak (Q_8) penguin operators, respectively:

$$Q_6 \;=\; -8 \sum_{q=u,d,s} (\bar{s}_L q_R)\,(\bar{q}_R d_L)\,, \qquad\qquad Q_8 \;=\; -12 \sum_{q=u,d,s} e_q\,(\bar{s}_L q_R)\,(\bar{q}_R d_L)\,. \qquad (11)$$

The hadronic matrix elements of these operators have a sizeable chiral enhancement, due to their scalar/pseudoscalar structure. In the limit of a large number of QCD colours the two colour-singlet quark currents factorize at the hadron level, allowing for an easy determination in terms of their χPT counterparts. Neglecting the small contributions from all other four-quark operators, one gets

$$\mathrm{Re}(\varepsilon'/\varepsilon) \;\approx\; 2.2\cdot 10^{-3}\left\{B_6^{(1/2)}\,(1-\Omega_{\mathrm{eff}}) - 0.48\,B_8^{(3/2)}\right\}, \qquad (12)$$

where the factors $B_6^{(1/2)}$ and $B_8^{(3/2)}$ parametrize the deviations of the true matrix elements from their large-N_C approximations. At $N_C \to \infty$, $B_6^{(1/2)} = B_8^{(3/2)} = 1$ and there is a sizeable cancellation between the three terms in (12). Taking Ω_{eff} from Eq. (10), one gets $\mathrm{Re}(\varepsilon'/\varepsilon) \approx 0.9\cdot 10^{-3}$, a factor 2.4 smaller than the naive Q_6 contribution.[8] However, this rough estimate does not yet include any chiral loop corrections because they are suppressed by a factor $1/N_C$. In particular, the important logarithmic contributions generating the absorptive components of the amplitudes are totally missed at large N_C. These χPT corrections increase the Q_6 contribution by about 35% and suppress the Q_8 one by 45% [26], destroying the numerical cancellation in Eq. (12) and bringing back the prediction to larger values.

4. χPT calculation of ε'/ε

In order to perform a reliable prediction of ε'/ε one needs a well-defined effective field theory (EFT) framework, able to control the large logarithmic corrections generated by the presence of widely separated mass scales: $m_\pi < m_K < \nu_\chi \le \mu \ll M_W$. Figure 3 shows the chain of EFTs needed to describe the relevant physics at the different scales involved.

[8] The inputs advocated in Ref. [35], $B_6^{(1/2)} = 0.57$, $B_8^{(3/2)} = 0.76$ and $\Omega_{\mathrm{eff}} = 0.15$, imply a much larger cancellation leading to $\mathrm{Re}(\varepsilon'/\varepsilon) \approx 2.6\cdot 10^{-4}$.

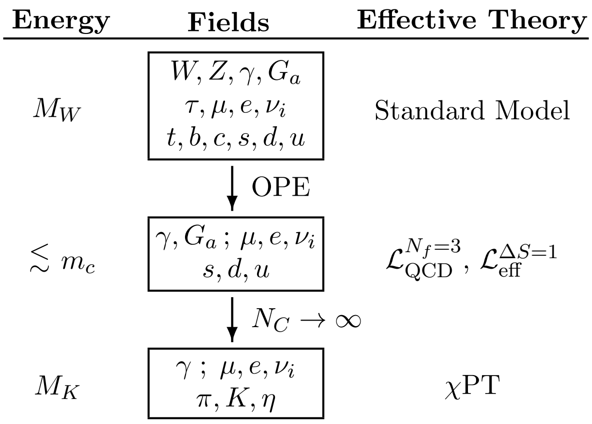

Figure 3. Evolution from M_W to the kaon mass scale.

The short-distance QCD logarithmic corrections are very efficiently summed up, all the way down from M_W to $\mu \geq 1$ GeV, with the operator product expansion (OPE). One gets then a $\Delta S = 1$ effective Lagrangian $\mathcal{L}_{\mathrm{eff}}^{\Delta S=1} \sim \sum_i C_i(\mu)\, Q_i(\mu)$, which is a sum of four-fermion (light-quark) operators Q_i, modulated by Wilson coefficients $C_i(\mu)$ that contain all the dynamical information from scales heavier than μ. They are fully known at the NLO [41–44] and preliminary NNLO results have been already presented at this conference [45–47].

Dynamics at the kaon mass scale can be rigorously described with χPT, the effective field theory of the QCD Goldstone particles (π, K, η). Using chiral symmetry, one can build the most general effective realization of $\mathcal{L}_{\mathrm{eff}}^{\Delta S=1}$ in terms of Goldstone fields, organised as a systematic expansion in powers of momenta over the chiral symmetry breaking scale (~ 1 GeV). Chiral symmetry determines the structure of the χPT operators with the same symmetry properties as the corresponding four-quark operators Q_i, while the short-distance dynamical information is encoded in LECs. The determination of these LECs requires to match the two EFTs in their common region of validity, around 1 GeV. Currently, this can be easily done in the limit $N_C \to \infty$, which turns out to be a very good approximation for Q_6 and Q_8 because their anomalous dimensions survive the large-N_C limit [1].

The great advantage of the χPT Lagrangian is that it allows for an accurate prediction of the long-distance logarithmic corrections, fulfilling all requirements of unitarity and analyticity. In addition to the logarithms generating the absorptive contributions, there are other large chiral logarithms, such as the $\log(\nu_\chi^2/m_\pi^2)$ term in Eq. (7), that encode the ultraviolet (ν_χ) and infrared (m_π) properties of the EFT and need to be properly taken into account.

Using all this EFT technology, a complete update of the SM prediction of ε'/ε has been recently performed in Refs. [1, 2]. More technical details of this calculation are presented in a separate talk at this conference [48]. The final result, given before in Eq. (3), agrees within errors with the experimental world average. Possible improvements in order to further reduce the current theoretical uncertainty have been discussed in Ref. [1].

Acknowledgements
This work has been supported in part by the Spanish Government and ERDF funds from the EU Commission [grant FPA2017-84445-P], the Generalitat Valenciana [grant Prometeo/2017/053], the Swedish Research Council [grants 2015-04089 and 2016-05996] and the European Research Council (ERC) under the EU Horizon 2020 research and innovation programme (grant 668679). The work of H.G. is supported by the Bundesministerium für Bildung und Forschung (BMBF).

V.C. acknowledges support by the US DOE Office of Nuclear Physics.

References

[1] Gisbert H and Pich A 2018 *Rept. Prog. Phys.* **81** 076201 (*Preprint* `1712.06147`)
[2] Cirigliano V, Gisbert H, Pich A and Rodríguez-Sánchez A 2019 (*Preprint* `1911.01359`)
[3] Cirigliano V, Ecker G, Neufeld H, Pich A and Portoles J 2012 *Rev. Mod. Phys.* **84** 399 (*Preprint* `1107.6001`)
[4] Tanabashi M *et al.* (Particle Data Group) 2018 *Phys. Rev.* **D98** 030001
[5] Burkhardt H *et al.* (NA31) 1988 *Phys. Lett.* **B206** 169–176
[6] Barr G D *et al.* (NA31) 1993 *Phys. Lett.* **B317** 233–242
[7] Fanti V *et al.* (NA48) 1999 *Phys. Lett.* **B465** 335–348 (*Preprint* `hep-ex/9909022`)
[8] Lai A *et al.* (NA48) 2001 *Eur. Phys. J.* **C22** 231–254 (*Preprint* `hep-ex/0110019`)
[9] Batley J R *et al.* (NA48) 2002 *Phys. Lett.* **B544** 97–112 (*Preprint* `hep-ex/0208009`)
[10] Gibbons L K *et al.* 1993 *Phys. Rev. Lett.* **70** 1203–1206
[11] Alavi-Harati A *et al.* (KTeV) 1999 *Phys. Rev. Lett.* **83** 22–27 (*Preprint* `hep-ex/9905060`)
[12] Alavi-Harati A *et al.* (KTeV) 2003 *Phys. Rev.* **D67** 012005 [Erratum: Phys. Rev.D70,079904(2004)] (*Preprint* `hep-ex/0208007`)
[13] Abouzaid E *et al.* (KTeV) 2011 *Phys. Rev.* **D83** 092001 (*Preprint* `1011.0127`)
[14] Hagelin J S and Gilman F J 1983 *Phys. Lett.* **126B** 111–116
[15] Buras A J, Jamin M and Lautenbacher M E 1993 *Nucl. Phys.* **B408** 209–285 (*Preprint* `hep-ph/9303284`)
[16] Buras A J, Jamin M and Lautenbacher M E 1996 *Phys. Lett.* **B389** 749–756 (*Preprint* `hep-ph/9608365`)
[17] Bosch S, Buras A J, Gorbahn M, Jager S, Jamin M, Lautenbacher M E and Silvestrini L 2000 *Nucl. Phys.* **B565** 3–37 (*Preprint* `hep-ph/9904408`)
[18] Ciuchini M, Franco E, Martinelli G and Reina L 1993 *Phys. Lett.* **B301** 263–271 (*Preprint* `hep-ph/9212203`)
[19] Ciuchini M, Franco E, Martinelli G, Reina L and Silvestrini L 1995 *Z. Phys.* **C68** 239–256 (*Preprint* `hep-ph/9501265`)
[20] Bertolini S, Fabbrichesi M and Eeg J O 2000 *Rev. Mod. Phys.* **72** 65–93 (*Preprint* `hep-ph/9802405`)
[21] Hambye T, Kohler G O, Paschos E A and Soldan P H 2000 *Nucl. Phys.* **B564** 391–429 (*Preprint* `hep-ph/9906434`)
[22] Bijnens J and Prades J 2000 *JHEP* **06** 035 (*Preprint* `hep-ph/0005189`)
[23] Hambye T, Peris S and de Rafael E 2003 *JHEP* **05** 027 (*Preprint* `hep-ph/0305104`)
[24] Pallante E and Pich A 2000 *Phys. Rev. Lett.* **84** 2568–2571 (*Preprint* `hep-ph/9911233`)
[25] Pallante E and Pich A 2001 *Nucl. Phys.* **B592** 294–320 (*Preprint* `hep-ph/0007208`)
[26] Pallante E, Pich A and Scimemi I 2001 *Nucl. Phys.* **B617** 441–474 (*Preprint* `hep-ph/0105011`)
[27] Ecker G, Muller G, Neufeld H and Pich A 2000 *Phys. Lett.* **B477** 88–92 (*Preprint* `hep-ph/9912264`)
[28] Cirigliano V, Pich A, Ecker G and Neufeld H 2003 *Phys. Rev. Lett.* **91** 162001 (*Preprint* `hep-ph/0307030`)
[29] Cirigliano V, Ecker G, Neufeld H and Pich A 2004 *Eur. Phys. J.* **C33** 369–396 (*Preprint* `hep-ph/0310351`)
[30] Pich A 2004 *Proceedings, 32nd International Conference on High Energy Physics (ICHEP 2004): Beijing, China, August 16-22, 2004. Vol. 1+2* (*Preprint* `hep-ph/0410215`)
[31] Bai Z *et al.* (RBC, UKQCD) 2015 *Phys. Rev. Lett.* **115** 212001 (*Preprint* `1505.07863`)
[32] Blum T *et al.* 2015 *Phys. Rev.* **D91** 074502 (*Preprint* `1502.00263`)
[33] Buras A J and Gérard J M 2015 *JHEP* **12** 008 (*Preprint* `1507.06326`)
[34] Buras A J and Gerard J M 2017 *Eur. Phys. J.* **C77** 10 (*Preprint* `1603.05686`)
[35] Buras A J, Gorbahn M, Jäger S and Jamin M 2015 *JHEP* **11** 202 (*Preprint* `1507.06345`)
[36] Kitahara T, Nierste U and Tremper P 2016 *JHEP* **12** 078 (*Preprint* `1607.06727`)
[37] Aebischer J, Bobeth C and Buras A J 2019 (*Preprint* `1909.05610`)
[38] Christ N H 2019 '$K \to \pi\pi$ decay, ϵ' and the RBC-UKQCD kaon physics program', talk at Kaon 2019
[39] Antonelli M *et al.* (FlaviaNet Working Group on Kaon Decays) 2010 *Eur. Phys. J.* **C69** 399–424 (*Preprint* `1005.2323`)
[40] Colangelo G, Gasser J and Leutwyler H 2001 *Nucl. Phys.* **B603** 125–179 (*Preprint* `hep-ph/0103088`)
[41] Buras A J, Jamin M, Lautenbacher M E and Weisz P H 1992 *Nucl. Phys.* **B370** 69–104 [Addendum: Nucl. Phys.B375,501(1992)]
[42] Buras A J, Jamin M, Lautenbacher M E and Weisz P H 1993 *Nucl. Phys.* **B400** 37–74 (*Preprint* `hep-ph/9211304`)
[43] Buras A J, Jamin M and Lautenbacher M E 1993 *Nucl. Phys.* **B400** 75–102 (*Preprint* `hep-ph/9211321`)
[44] Ciuchini M, Franco E, Martinelli G and Reina L 1994 *Nucl. Phys.* **B415** 403–462 (*Preprint* `hep-ph/9304257`)
[45] Cerdà-Sevilla M 2019 'Higher-order effects in ϵ'/ϵ', talk at Kaon 2019
[46] Buras A J, Gambino P and Haisch U A 2000 *Nucl. Phys.* **B570** 117–154 (*Preprint* `hep-ph/9911250`)
[47] Gorbahn M and Haisch U 2005 *Nucl. Phys.* **B713** 291–332 (*Preprint* `hep-ph/0411071`)
[48] Rodríguez-Sánchez A 2019 'Isospin-breaking corrections to ϵ'/ϵ', talk at Kaon 2019