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Mirror QCD and Cosmological Constant

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Abstract

An analog of Quantum Chromo Dynamics (QCD) sector known as mirror QCD (mQCD) can affect the cosmological evolution and help in resolving the Cosmological Constant problem. In this work, we explore an intriguing possibility for a compensation of the negative QCD vacuum contribution to the ground state energy density of the universe by means of a positive contribution from the chromomagnetic gluon condensate in mQCD. The trace anomaly compensation condition and the form of the mQCD coupling constant in the infrared limit have been proposed by analysing a partial non-perturbative solution of the Einstein–Yang-Mills equations of motion.

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I. INTRODUCTION

The ground state of Yang-Mills (YM) theories plays a critical role in both Particle Physics and Cosmology. In particular, the gluon condensate in Quantum Chromo Dynamics (QCD) largely determines non-trivial properties of the topological QCD vacuum and is responsible e.g. for the color confinement effects and hadron mass generation which can be understood beyond the Perturbation Theory (PT) only (for a comprehensive review on the QCD vacuum, see e.g. Refs. [1–4] and references therein). The gluon condensate directly influences properties of the quark-gluon plasma and its hadronisation, as well as dynamics of the QCD phase transition. On the other hand, YM condensates have various implications in the cosmological evolution ranging from the Cosmic Inflation [5–7] to the phenomenon of late-time acceleration and the Dark Energy (DE) [8–10] (see also Refs. [11–16]).

Currently, the Cosmological Constant (CC) with the vacuum equation of state $w \equiv p/\epsilon = -1$ is a preferred scenario for the late-time acceleration epoch supported by a wealth of recent observations provided that $w = -1.006 \pm 0.045$ (see e.g. Refs. [17, 18]). Despite of many DE/CC models existing in the literature, there is not a compelling resolution of the CC problem i.e. why the CC term is small and positive as well as why the CC term is non-zeroth and exists at all. From the Quantum Field Theory (QFT) viewpoint, the ground state energy density of the universe should account for a bulk of various contributions from existing quantum fields at energy scales ranging from the Quantum Gravity (Planck) scale, $M_{\rm PL} \simeq 1.2 \cdot 10^{19}$ GeV, down to the QCD confinement scale, $\Lambda_{\rm QCD} \simeq 0.1$ GeV. Even such relatively well-known vacuum subsystems of the Standard Model (SM) as the Higgs and quark-gluon condensates exceed by far the observed cosmological constant which is often considered as a severe problem [19, 20] (for recent reviews on this topic, see e.g. Refs. [21–23] and references therein). Also it is well known, that an every field in the universe forms a divergent perturbative vacuum contribution, which is usually cut off at the Planck scale. The cancellation of these contributions may need the introduction of additional bosonic and fermionic fields putting important constraints for the particle spectrum [24–26].

In this work, we discuss the problem connected with formation of big non-perturbative vacuum contributions on the hadronic scale after the QCD phase transition, assuming that the contributions from higher scales are already compensated. In the case of confined QCD with color $SU(3)$ gauge symmetry, there is a rather unique (negative-valued) contribution to the ground state energy of the universe originating from the non-perturbative quantum fluctuations of the quark and gluon fields [1, 2, 27, 28], $\epsilon^{\rm QCD} < 0$. Given the fact that the CC term observed in astrophysical measurements is very small (and positive-valued),

$$\epsilon_{\rm CC} > 0\,, \qquad \left|\frac{\epsilon_{\rm CC}}{\epsilon^{\rm QCD}}\right| \simeq 10^{-44}\,, \tag{1.1}$$

one must eliminate the QCD vacuum contribution, $\epsilon^{\rm QCD}$, with an unprecedented accuracy over forty decimal digits. A dynamical mechanism for such a gross cancellation of vacua terms is yet theoretically unknown although several possible scenarios have been discussed so far e.g. in Refs. [9, 21, 29–31]. This work is devoted to making a further step in exploring this possibility in quantum YM theories with a non-trivial ground state.

Clearly, in order to cancel the QCD vacuum contribution, $\epsilon^{\rm QCD}$, formed during the QCD phase transition epoch, a positive contribution to the vacuum energy density should be formed at the same QCD energy scale $\Lambda_{\rm QCD}$. Where could such an extra contribution originate from?

Here we suggest a new scenario of compensation realized by means of a hidden (mirror) sector of particles [32] which correspond to the extra non-Abelian gauge group and whose possible interaction with the visible SM sectors is strongly suppressed.

In particular, a class of models known as Neutral Naturalness theories has been proposed in the literature [33–35] as a promising solution of the naturalness problem in the SM protecting the weak scale from large radiative corrections. Various phenomenological implications of such a "Mirror World" concept have been discussed e.g. in Ref. [36]. In particular, the mirror color $SU(3)$ gauge group is typically assumed to be a symmetry describing the confined phase in full analogy with ordinary QCD revealing interesting signatures at the Large Hadron Collider due to e.g. a mixing of mirror glueballs with the Higgs boson [37].

Quite naturally, the quantum vacua contributions from the "Mirror World" should contribute to the CC on the same footing as known vacua since "mirror" particles are expected to gravitate in the same way as the usual ones. We argue that mirror QCD (mQCD) sector, if exists, should affect the cosmological expansion, in particular, via an extra non-trivial "mirror gluon" contribution into the ground state energy of the universe. Below, we will demonstrate that under certain conditions the mirror gluon condensate can contribute to the energy density of the universe with positive sign and thus may, in principle, eliminate the negative QCD vacuum effect yielding a vanishingly small CC term. Attributing the positive vacuum energy contribution to the mQCD sector non-interacting with quarks and gluons in ordinary QCD, one may therefore resolve the issue of why such a positive-valued condensate energy density does not emerge in QCD hadron physics and affects the CC-term only. Then the observed CC can, in principle, be formed as a remnant of the gluon condensate cancellation in expanding universe (e.g. due to an uncompensated quantum gravity correction to the QCD ground state energy) [9], which appears to be remarkably consistent with the observed CC and with the Zeldovich scaling relation [38]. The exact compensation of the QCD vacuum effect by means of the mirror gluon condensate is therefore the central point to the observable smallness of the CC.

II. QCD AND MIRROR QCD VACUA COMPENSATION

The condensate in QCD is formed by the contributions of gluon and quark non-perturbative quantum fluctuations

$$\epsilon^{\rm QCD} = \epsilon^{\rm g} + \epsilon^{\rm q} \simeq -(5\pm1)\times 10^9\ {\rm MeV}^4\,, \qquad \epsilon^{\rm q} = \frac{1}{4}\langle 0|m_u\bar{u}u + m_d\bar{d}d + m_s\bar{s}s|0\rangle\,. \quad (2.1)$$

Usually, the dominant gluon contribution is given by means of the trace anomaly relation in QCD

$$\epsilon^{\rm g} \equiv \frac{1}{4}\,\langle 0|T^{\mu,{\rm g}}_{\mu}|0\rangle\,, \qquad T^{\mu,{\rm g}}_{\mu} = \frac{\beta(\bar{g}_s^2)}{2}\,F^a_{\mu\nu}F^{\mu\nu}_a\,, \quad (2.2)$$

which to one-loop order reads [1, 2]

$$\epsilon^{\rm g} = -\frac{b}{32}\langle 0|\frac{\alpha_s}{\pi}F^a_{\mu\nu}F^{\mu\nu}_a|0\rangle\,, \qquad \alpha_s = \frac{\bar{g}_s^2}{4\pi}\,, \quad (2.3)$$

where $b = 9$ is the first (one-loop) coefficient of the negative perturbative β-function in $SU(3)$ gluodynamics with three light flavors

$$\beta(\bar{g}_s^2) = -\frac{b\bar{g}_s^2}{16\pi^2} < 0\,, \qquad \bar{g}_s^2 = \frac{16\pi^2}{b\ln(Q^2/\Lambda^2_{\rm QCD})}\,, \quad (2.4)$$

with $\Lambda_{\rm QCD}$ being the QCD scale parameter (the normalization of β corresponds to Ref. [39]). The formation of the chromomagnetic gluon condensate $\langle F^2\rangle > 0$ is typically considered at characteristic momentum scales μ_g inverse to the correlation length l_g, i.e. $\mu_g \sim l_g^{-1} \simeq 1.2$ GeV [1, 2], where the perturbative QCD still provides a realistic estimate. This validates the use of one-loop approximated expression (2.3).

One would like to explain why such a big negative contribution (2.1), which is responsible for a variety of well-known phenomena in hadron physics and is rather unique for QCD, does not affect the cosmological expansion at late times. Provided that the observed CC-term density

$$\epsilon_{\rm CC} \simeq 3\times 10^{-35}\,{\rm MeV}^4\,, \tag{2.5}$$

is tiny compared to the QCD vacuum density (2.3), the latter should be almost totally eliminated during the QCD phase transition epoch. Which mechanism could be responsible for that?

A mirror copy of QCD may generate a similar gluon contribution to the trace anomaly proportional to the corresponding β-function in mQCD,

$$\epsilon^{\rm mQCD}_{\rm gluon} \equiv \frac{1}{4}\,\langle 0|T_\mu^{\mu,{\rm mQCD}}|0\rangle \propto \beta(\bar{g}^2)\,. \tag{2.6}$$

In mQCD framework mirror quarks can be much heavier than in ordinary QCD [35]. Applying the idea, that mQCD is similar in main features to usual QCD, this means, that in mQCD the vacuum is formed only by mirror gluon contribution with pure gluonic β-function, as long as the heavy quark condensates [40] are compensated by quark part of β-function, i.e.

$$\epsilon^{\rm mQCD} = \epsilon^{\rm mQCD}_{\rm gluon}\,. \tag{2.7}$$

A possible cancellation of QCD and mQCD vacuum may ensure a required smallness of the observable CC density

$$\epsilon^{\rm QCD} \simeq -\epsilon^{\rm mQCD}\,, \tag{2.8}$$

which means that the corresponding mirror gluon condensate should provide a positive contribution to the vacuum density, i.e. $\epsilon^{\rm mQCD} > 0$. We suppose, that mQCD gluon condensate can compensate both gluon and quark condensates of usual QCD.

Adopting the traditional hypothesis that the mQCD sector of mirror quarks and gluons is confined but is not (or very weakly) interacting with the observed SM sectors [37] and considering only chromomagnetic condensates, the compensation condition (2.8) can be satisfied if and only if the mQCD β-function is positive, i.e.

$$\epsilon^{\rm mQCD} > 0\,, \qquad \langle F^2_{\rm mQCD}\rangle > 0\,, \qquad \beta(\bar{g}^2) > 0\,, \tag{2.9}$$

which is not realized in the perturbative mQCD regime due to Eq. (2.4). It is, however, possible to achieve the positivity of the non-perturbative β-function provided that at the characteristic scale of the QCD gluon condensate formation, μ_g, the mQCD sector is in deeply non-perturbative regime. The latter condition can be satisfied if the mQCD scale parameter is large, i.e.

$$\Lambda_{\rm mQCD} \gg \mu_g \simeq 1.2~{\rm GeV}\,, \tag{2.10}$$

such that the mirror gluon condensate would be in deeply non-perturbative regime by the moment in the cosmological evolution when its density gets precisely cancelled with the QCD contribution. Note that the compensation conditions for the QCD and mQCD contributions (2.8) and (2.9), if indeed realized in nature, may be one of the most important implications of the mirror QCD in Cosmology yielding a vanishing CC-term and thus providing a dynamical way to resolve the CC problem.

Can the sign of the β-function in mQCD become positive in the non-perturbative regime? In order to answer this question, one has to employ a proper formalism which extends the effective action approach in a gauge theory beyond the perturbativity domain. Indeed, the compensation (2.8) emerges as a long-distance phenomenon and thus should hold beyond the PT.

III. EFFECTIVE YANG-MILLS THEORY IN EXPANDING UNIVERSE

The effective action of the quantum YM gauge $SU(N)(N = 2, 3, \dots)$ theory consistently accounting for the trace anomaly and properly generalised to the FLRW background reads [41, 42] (see also Ref. [10])

$$S_{\rm eff}[\mathcal{A}] = \int \mathcal{L}_{\rm eff}\sqrt{-g}d^4x\,, \qquad \mathcal{L}_{\rm eff} = \frac{J}{4\bar{g}^2(J)}\,, \qquad J = -\frac{\mathcal{F}^2}{\sqrt{-g}}\,, \qquad \mathcal{F}^2 \equiv \mathcal{F}^a_{\mu\nu}\mathcal{F}^{\mu\nu}_a\,,$$
$$g \equiv \det(g_{\mu\nu})\,, \qquad g_{\mu\nu} = a(\eta)^2{\rm diag}(1,\,-1,\,-1,\,-1)\,, \qquad t = \int a(\eta)d\eta\,, \tag{3.1}$$

where the YM field and the corresponding stress tensor are defined as usual:

$$\mathcal{A}^a_\mu \equiv \bar{g}\,A^a_\mu\,, \qquad \mathcal{F}^a_{\mu\nu} \equiv \bar{g}\,F^a_{\mu\nu}\,, \qquad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \bar{g}\,f^{abc}A^b_\mu A^c_\nu$$

with internal (in adjoint representation) $a, b, c = 1, \dots N^2 - 1$ and Lorentz $\mu, \nu = 0, 1, 2, 3$ indices and the gauge coupling $\bar{g} = \bar{g}(J)$ satisfying the RG evolution equation [41, 42]

$$2J\frac{d\bar{g}^2}{dJ} = \bar{g}^2\,\beta(\bar{g}^2)\,. \tag{3.2}$$

Depending on the sign of the invariant J, one distinguishes the chromoelectric $J > 0$ and chromomagnetic $J < 0$ YM fields.

The effective YM equation of motion in a non-trivial background metric reads

$$\left(\frac{\delta^{ab}}{\sqrt{-g}}\partial_\nu\sqrt{-g} - f^{abc}\mathcal{A}^c_\nu\right)\left[\frac{\mathcal{F}^{\mu\nu}_b}{\bar{g}^2\sqrt{-g}}\left(1 - \frac{1}{2}\beta\big(\bar{g}^2\big)\right)\right] = 0\,. \tag{3.3}$$

and can be employed beyond the PT as long as the non-perturbative β-function is known. Remarkably enough, this equation has a simple manifestly non-perturbative ground-state solution with positive β-function

$$\beta\big(\bar{g}^2(J)\big) = 2\,, \tag{3.4}$$

which is a complete analog of similar solution (see Ref. [42], Eqs. (13) and (17)) found in the Euclidian case with the negative coupling and the β-function corresponding to the ferromagnetic vacuum. It is also a non-perturbative analog of the perturbative solution [5, 29] eliminating the traceless part of the energy-momentum tensor

$$T_\mu^\nu \;=\; \frac{1}{\bar{g}^2}\Big[1-\frac{1}{2}\beta(\bar{g}^2)\Big]\Big(-\frac{\mathcal{F}^a_{\mu\lambda}\mathcal{F}_a^{\nu\lambda}}{\sqrt{-g}}-\frac{1}{4}\delta_\mu^\nu\,J\Big)-\frac{\delta_\mu^\nu\beta(\bar{g}^2)}{8\bar{g}^2}\,J\,,$$

which then takes the following form

$$T_{\mu,0}^\nu \;=\; -\frac{J}{4\bar{g}^2(J)}\delta_\mu^\nu\,. \tag{3.5}$$

It is important to point out, that the equations (3.4-3.5) were obtained in the pure YM case when the interaction with other fields can be neglected. In particular, we neglect the mQCD quark current in the right hand side of Eq. (3.3). Let us also stress that we consider the effective YM Lagrangian (3.1) and energy-momentum tensor (3.5) as a classical model [42] which possesses well-known properties of the full quantum theory such as (i) local gauge invariance, (ii) RG evolution and asymptotic freedom, (iii) correct quantum vacuum configurations, and (iv) trace anomaly given by the last term in Eq. (3.5). These provide a sufficient motivation and physics interest in cosmological aspects of the considering effective model.

As the solution (3.4) leads to the energy-momentum tensor of vacuum type (3.5), it immediately follows from the Friedmann equations that the corresponding energy density is constant

$$-\frac{J}{4\bar{g}^2(J)} = \mathrm{const}\,, \tag{3.6}$$

thus, the contribution of the YM fields has a cosmological constant form. In particular, this can be realised if $J = \mathrm{const}$. Indeed, the solution (3.4) fixes the invariant J to its constant initial value

$$J(t) \equiv J(t=0) = J_0\,. \tag{3.7}$$

Such solutions were also considered in Refs. [5, 29, 42] in connection with the spontaneous vacuum magnetisation and in the domain concept of the QCD vacuum [43] (see also the recent paper [44] and references therein).

Further, we will apply the solution (3.4) and (3.7) to the the mQCD theory in the non-perturbative regime. The energy-momentum tensor in this case becomes constant as expected

$$T_\mu^{\nu *} = \epsilon^{\rm mQCD}\delta_\mu^\nu\,, \qquad \epsilon^{\rm mQCD} \equiv -\frac{J_0^{\rm mQCD}}{4\bar{g}_0^2}\,, \tag{3.8}$$

where $\bar{g}_0^2 = \bar{g}^2(J_0)$. The coupling $\bar{g}^2(J)$ touches the linear function $f(J) = \bar{g}_0^2\cdot(J/J_0)$ at the point J_0 (indeed, it has the same value and derivative). And vice versa, if $\bar{g}^2(J)$ touches $\mathrm{const}\cdot J$ at some point J_0, then $\frac{d\bar{g}^2}{dJ}|_{J=J_0} = \frac{\bar{g}_0^2}{J_0}$, which means that Eq. (3.4) is satisfied at J_0. Indeed, the existence of such a contact point is a necessary and sufficient condition for the solution (3.4) with fixed $J = J_0$. This allows us to constrain generic non-perturbative behavior of the corresponding $\bar{g}^2(J)$. An illustration of the corresponding infrared behavior

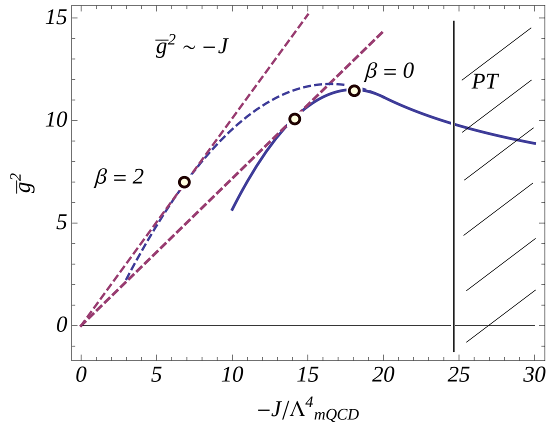

FIG. 1: An example of the non-perturbative mQCD coupling constant $\bar{g}^2 = \bar{g}^2(J)$ behavior as a function of J in consistency with the non-perturbative solution found in Eq. (3.4).

of the mQCD coupling in consistency with both the non-perturbative asymptotics for the β-function (3.4) and the conventional perturbative regime of asymptotic freedom (2.4) is shown in Fig. 1. A desirable non-monotonic shape of the coupling was earlier discussed in the case of usual QCD [45, 46].

From Eq. (3.6) one notices that the mQCD gauge field give a constant vacuum contribution to the energy-momentum tensor in the Einstein equations. Since the QCD and mQCD contributions to the ground-state energy density have opposite signs there is a compelling possibility that they can, in principle, cancel each other at some moment in the cosmological history provided that

$$\epsilon^{\rm mQCD} \to -\epsilon^{\rm QCD} \tag{3.9}$$

in the infrared regime of mQCD. Keeping $\bar{g}_0^2 > 0$ in both QCD and mQCD, we arrive at the following form of the compensation condition (2.8)

$$\epsilon^{\rm QCD} \simeq \frac{J_0^{\rm mQCD}}{4\bar{g}_0^2} < 0\,, \qquad J_0^{\rm mQCD} < 0\,, \tag{3.10}$$

which means, that mQCD condensate has to be chromomagnetic.

After such a compensation is achieved, only a very small $\epsilon_{\rm CC}$ contribution, which could be formed by other vacuum sources $\epsilon_{\rm vac}$ and possibly by a non-compensated part of mQCD and QCD vacua contributions $\epsilon^{\rm QCD} + \epsilon^{\rm mQCD}$, remains. Of course, a fine-tuning is unavoidable to match the observations, although would not be entirely unreasonable due to the same order magnitude of the QCD and mQCD contributions. Such a vacua alignment, if realised in Nature, can be suggested e.g. by their common Anthropic origins [47]. Then the standard Friedmann equation in the non-stationary FLRW universe

$$\frac{3}{\varkappa}\frac{(a')^2}{a^4} = \epsilon_{\rm mat} + \epsilon_{\rm CC}\,, \qquad \epsilon_{\rm CC} \equiv \epsilon^{\rm QCD} + \epsilon^{\rm mQCD} + \epsilon_{\rm vac}\,, \tag{3.11}$$

determines the cosmological evolution, $a = a(\eta)$, driven by the gluon and mirror gluon condensate densities (compensating each other exactly or in part), the matter contribution, $\epsilon_{\rm mat}$, and other possible vacua contributions of a different kind, $\epsilon_{\rm vac}$.

Let us notice for completeness, that Eq. (3.4) allows for a more specific solution apart from constant fields (3.7). This another solution appears, if the non-perturbative β-function becomes constant satisfying Eq. (3.4) in some finite domain which corresponds to the strong coupling regime (saturated behaviour). In this case, Eq. (3.4) can be substituted into the RG equation

$$\frac{d\ln\bar{g}^2}{d\ln\big(-J/(\xi\Lambda_{\rm mQCD})^4\big)} = \frac{1}{2}\beta\big(\bar{g}^2\big) = 1\,, \tag{3.12}$$

(ξ is a numerical parameter) which implies that the mQCD coupling is proportional to J in the infrared limit, e.g.

$$\bar{g}^2(J) = \bar{g}_0^2\,\frac{J}{J_0}\,, \qquad \bar{g}_0^2 \equiv \bar{g}^2(J_0)\,. \tag{3.13}$$

Such behaviour clearly guarantees a constant vacuum energy (see Eq. (3.5)) as well as a possibility for the QCD/mQCD vacua compensation. According to Eq. (3.13) the gauge coupling squared has to approach the linear $\bar{g}^2 \sim J$ asymptotics in the non-perturbative regime (note, for the constant field solution (3.7) $\bar{g}^2$ has to satisfy a much less restrictive constraint and just touches the linear asymptotics $\bar{g}^2 \sim J$ at a fixed point).

So, within the QCD/mQCD vacua compensation hypothesis, both vacua subsystems should be generated in early universe at close (but different) energy scales and then get compensated during the cosmological QCD phase transition epoch. As was shown above, this can be realised in a deeply non-perturbative regime for the mirror gluon condensate which asymptotically acquires the same absolute value of energy density and opposite sign compared to the QCD gluon one, such that they almost exactly eliminate each other at macroscopically large space-time separations.

IV. SUMMARY

In this Letter, we have considered cosmological consequences of a possible exact cancellation of vacuum energies between QCD and a mirror high-scale copy of QCD in the confined regime. One of the most important implications of such a cancellation is a possibility for the dynamical elimination of the QCD vacuum density contribution to the cosmological constant resulting in its observed smallness (provided that all perturbative vacua are eliminated by some other mechanism, see e.g. Ref. [24–26]). By an appropriate fine-tuning of QCD and mirror QCD vacua parameters, the compensation is provided by a partial non-perturbative solution of the Yang-Mills equation of motion corresponding to a positive constant β-function in deeply infrared regime of mirror QCD. Such a dynamical vacua compensation due to different signs of β-functions in QCD and mirror QCD can be yet another implication of the Anthropic Principle [47] realised for QCD and mirror gluon condensates.

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