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Heavy quark production in pA collisions: the double parton scattering contribution.

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In this paper we estimate the double parton scattering (DPS) contribution for the heavy quark production in pA collisions at the LHC. The cross sections for the charm and bottom production are estimated using the dipole approach and taking into account the saturation effects, which are important for high energies and for the scattering with a large nucleus. We compare the DPS contribution with the single parton scattering one and demonstrate that both are similar in the kinematical range probed by the LHC. Predictions for the rapidity range analysed by the LHCb Collaboration are also presented. Our results indicate that the study of the DPS contribution for the heavy quark production in pPb collisions at the LHC is feasible and can be useful to probe the main assumptions of the approach.

I. INTRODUCTION

In hadronic collisions at high energies the occurrence of multi-parton interactions (MPI) is a consequence of the high density of partons in the hadron wave functions. In this kinematic regime the huge number of gluons increases the probability that two or more hard gluon-gluon fusion in a single hadron – hadron collision take place. The single gluon-gluon fusion in this kind of process is usually called Single Parton Scattering (SPS) and its contribution is in general the dominant process in perturbative QCD (pQCD) calculations. Recently, several theoretical and experimental studies have shown that Double Parton Scattering (DPS) processes cannot be neglected at LHC energies (For a recent review see, e.g. Ref. [1]). In particular, the experimental results from the LHCb Collaboration on four D meson production in pp collisions [2] can only be described with the inclusion of the DPS contribution. Besides accounting for a significant part of the cross section, the study of DPS processes is also important for other reasons. It can, for example, help us to understand the spatial structure of hadrons [3], the multi-parton correlations in the hadronic wave function [3–8] and is expected to help in the search for new physics (See, e.g., Ref. [9]).

One of the promising processes to probe the DPS mechanism is heavy quark production. At high energies, this process probes the hadron wave function at very small values of the Bjorken - x and its cross section can be calculated perturbatively. This process is dominated by gluon - gluon scatterings and a large cross section is predicted at the LHC by the single scattering mechanism. As a consequence of the large luminosity of small - x gluons in the initial state, we expect a significant contribution of the DPS mechanism to heavy quark production. This expectation has been confirmed by the analysis performed in Refs. [10–12] (See also Refs. [7, 13]). In particular, in Ref. [12] we have investigated the impact of saturation effects in DPS production of heavy quarks. The results from Refs. [10, 12] demonstrated that for charm production in pp collisions at LHC energies the double parton scattering contribution becomes comparable with the single parton scattering one. Moreover, in Ref. [12] we also demonstrated that the production of $c\bar{c}b\bar{b}$ contributes significantly to bottom production.

Another possibility to probe the DPS mechanism is the analysis of different final states in nuclear collisions. The studies performed in Ref. [5, 14–17] have shown that the DPS mechanism is strongly enhanced in pA and AA collisions. These studies encourage us to extend our previous analysis to pA collisions and investigate the DPS contribution to heavy quark production. In particular, we will estimate the magnitude of the DPS cross section for pPb collisions at $\sqrt{s} = 5.02$ TeV, which can be measured at the LHC. As at small-x and a large nucleus we expect a large contribution of saturation effects to heavy quark production [18], we also include these effects in our calculations.

This paper is organized as follows. In the next Section we present the basic assumptions and formulas derived in Refs. [14, 15], which we use to calculate the DPS cross sections for the heavy quark production in pA collisions. In Section III we estimate the total cross section for the $c\bar{c}c\bar{c}$, $b\bar{b}b\bar{b}$ and $c\bar{c}b\bar{b}$ production for different nuclei and analyse its energy dependence. The DPS and SPS contributions are compared and the magnitude of the DPS contribution for pPb collisions at $\sqrt{s} = 5.02$ TeV is presented. Predictions for the kinematical range probed by the LHCb experiment also are show. Finally, in Section IV we summarize our main conclusions.

II. THE FORMALISM

Initially let us present a brief review of the formalism used to treat single and double parton scattering in a generic hadron - hadron collision. In the case of a SPS process, we assume that only one hard interaction occurs per collision. The basic idea, which justifies this approach, is that the probability of a hard interaction in a collision is very small, which makes the probability of having two or more hard interactions in a collision highly suppressed with respect to the single interaction probability. As discussed in Refs. [10–12] such assumption is reasonable in the kinematical regime in which the flux of incoming partons is not very high. However, at LHC energies there is a high probability of scattering of more than one pair of partons in the same hadron - hadron collision. Consequently, it is important to take into account the contribution of the DPS processes. Following the same factorization approximation assumed for processes with a single hard scattering, it is possible to derive the DPS contribution for the heavy quark cross section considering two independent hard parton sub-processes. It is given by (See, e.g. Ref. [3])

$$\begin{aligned}\sigma^{DPS}_{h_1h_2\rightarrow Q_1\bar{Q}_1Q_2\bar{Q}_2} = \left(\frac{m}{2}\right)\int \Gamma^{gg}_{h_1}(x_1,x_2;\boldsymbol{b}_1,\boldsymbol{b}_2;\mu_1^2,\mu_2^2)\hat{\sigma}^{gg}_{Q_1\bar{Q}_1}(x_1,x_1',\mu_1^2)\hat{\sigma}^{gg}_{Q_2\bar{Q}_2}(x_2,x_2',\mu_2^2)\\ \times\Gamma^{gg}_{h_2}(x_1',x_2';\boldsymbol{b}_1-\boldsymbol{b},\boldsymbol{b}_2-\boldsymbol{b};\mu_1^2,\mu_2^2)dx_1dx_2dx_1'dx_2'd^2b_1d^2b_2d^2b\ ,\end{aligned} \tag{1}$$

where we assume that the quark-induced sub-processes can be disregarded at high energies, $\Gamma^{gg}_{h_1}(x_1,x_2;\boldsymbol{b}_1,\boldsymbol{b}_2;\mu_1^2,\mu_2^2)$ are the two-gluon parton distribution functions which depend on the longitudinal momentum fractions x_1 and x_2, and on the transverse positions $\boldsymbol{b}_1$ and $\boldsymbol{b}_2$ of the two gluons undergoing hard processes at the scales μ_1^2 and μ_2^2. The functions $\hat{\sigma}$ are the parton level sub-processes cross sections and $\boldsymbol{b}$ is the impact parameter vector connecting the centres of the colliding hadrons in the transverse plane. Moreover, $m/2$ is a combinatorial factor which accounts for indistinguishable and distinguishable final states. For $Q_1 = Q_2$ one has $m = 1$, while $m = 2$ for $Q_1 \neq Q_2$. It is common in the literature to assume that the longitudinal and transverse components of the double parton distributions can be decomposed and that the longitudinal components can be expressed in terms of the product of two independent single parton distributions. As in [12] we will also assume the validity of these assumptions and consider that the DPS contribution to the heavy quark cross section can be expressed in a simple generic form given by

$$\sigma^{DPS}_{h_1h_2\rightarrow Q_1\bar{Q}_1Q_2\bar{Q}_2} = \left(\frac{m}{2}\right)\frac{\sigma^{SPS}_{h_1h_2\rightarrow Q_1\bar{Q}_1}\sigma^{SPS}_{h_1h_2\rightarrow Q_2\bar{Q}_2}}{\sigma_{eff}}\ , \tag{2}$$

where σ_{eff} is a normalization cross section representing the effective transverse overlap of partonic interactions that produce the DPS process. It is related with the impact parameter integral of the overlap function $t(\mathbf{b})$: $\sigma_{eff} = [\int d^2b\, t^2(\mathbf{b})]^{-1}$, where $t(\mathbf{b}) = \int f(\mathbf{b_1})f(\mathbf{b_1}-\mathbf{b})d^2b_1$ and $f(\mathbf{b})$ describes the transverse parton density in a given hadron. In general, it has been considered as a free parameter to be determined through fits to experimental $pp/p\bar{p}$ data, being given by: $\sigma_{eff,pp} \approx 13 \pm 2$ mb. Eq. (2), usually called “pocket formula”, expresses the DPS cross section

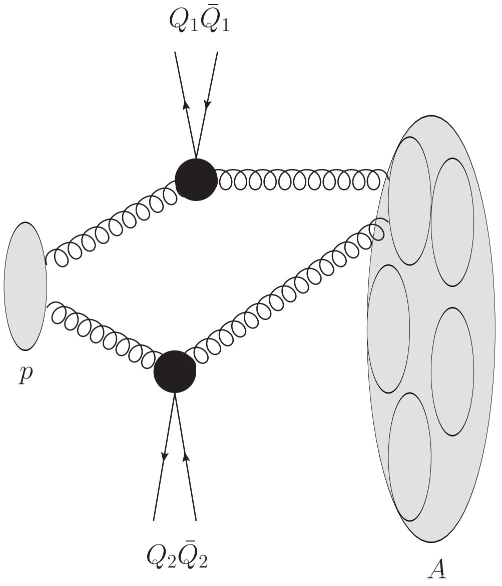

FIG. 1: Heavy quark production through DPS in pA collisions. Left: Two gluons coming from the proton projectile scatter with two gluons coming from the same nucleon in the target nucleus; Right: Two gluons coming from the proton projectile scatter with two gluons coming from different nucleons in the target nucleus.

as the product of two individual SPS cross sections assuming that the two SPS sub-processes are uncorrelated and do not interfere. The validity of these strong assumptions at LHC and higher energies is still an open question, which has motivated several theoretical studies (See, e.g. Refs. [3, 7]). However, the phenomenological analysis of different processes indicates that Eq. (2) can be considered a reasonable first approximation for the treatment of DPS processes.

In order to extend the treatment of DPS processes to proton - nucleus collisions we need to take into account that the parton flux associated to the nucleus is enhanced by a factor $\propto A$ and that in the interaction the two gluons associated to the proton can interact with two gluons coming from the same nucleon from the nucleus or with two gluons coming from different nucleons from the nucleus. Both possibilities are represented in the left and right panels of the Fig 1. Hereafter, we will denote the cross sections associated to these two contributions by $\sigma_{pA}^{DPS,1}$ and $\sigma_{pA}^{DPS,2}$, respectively. A way to treat these contributions was proposed in Ref. [14] and applied in Ref. [15] to the production of same - sign WW in pA collisions, which was suggested to be a signal for DPS. In what follows we extend the framework presented in Refs. [14, 15] to the calculation of heavy quark production. The basic assumptions are the following: (a) the contribution associated to $\sigma_{pA}^{DPS,1}$ can be estimated scaling the proton - nucleon pN cross section by the number A of nucleons inside the nucleus. i.e. $\sigma_{pA}^{DPS,1} = A \cdot \sigma_{pN}^{DPS}$; (b) the contribution associated to $\sigma_{pA}^{DPS,2}$ can be estimated using the Abramovsky - Gribov - Kancheli cutting rules [19]. Consequently, the authors of Ref. [14] obtain $\sigma_{pA}^{DPS,2} = \sigma_{pN}^{DPS} \cdot \sigma_{eff,pp} \cdot F_{pA}$, where $F_{pA} = [(A-1)/A] \int T_{pA}^2(\mathbf{r}) d^2r$ and T_{pA} is the nuclear thickness function which depends on the impact parameter $\mathbf{r}$ between the colliding proton and nucleus. The factor $(A-1)/A$ was introduced to take into account the difference between the number of nucleon pairs and the number of different nucleon pairs. These two contributions for the DPS pA cross section add up to the following final compact formula [15]:

$$\sigma_{pA\to ab}^{DPS} = \sigma_{pA}^{DPS,1} + \sigma_{pA}^{DPS,2} = A\sigma_{pN\to ab}^{SPS}\left[1 + \frac{1}{A}\sigma_{eff,\,pp}F_{pA}\right] \tag{3}$$

which implies

$$\sigma_{pA\to ab}^{DPS} \;=\; \left(\frac{m}{2}\right)\frac{\sigma_{pN\to a}^{SPS}\cdot\sigma_{pN\to b}^{SPS}}{\sigma_{eff,\,pA}}\;, \tag{4}$$

with the normalization effective cross section given by:

$$\sigma_{eff,\,pA} = \frac{\sigma_{eff,\,pp}}{A + \sigma_{eff,\,pp}\,F_{pA}}\;. \tag{5}$$

In the simplest approximation that the nucleus has a spherical form (with uniform nucleon density) of radius $R_A = r_0 A^{1/3}$, and $r_0 = 1.25$ fm, the integral of the nuclear thickness factor becomes:

$$F_{pA} = \frac{9A(A-1)}{8\pi R_A^2}\,. \tag{6}$$

Considering $A = 208$ in the above equations one finds that $\sigma_{eff,\,pp}/\sigma_{eff,\,pA} \approx 3\,A$ instead of the simple scale factor A that one would naively expect. Moreover, this also implies that the pPb DPS cross section is enhanced by a factor $3A$ (≈ 600) in comparison to the DPS contribution in pp processes.

The main input in the calculation of the DPS pA cross section, Eq. (4), is the pN cross section associated to the SPS process. As in our previous study [12], we will estimate this quantity using the dipole approach, which effectively takes into account higher-order QCD corrections [20] and allows to easily include saturation effects, which are expected to contribute significantly at the small values of x probed in heavy quark production at the LHC. As demonstrated in [12] (See also Ref. [21] for a recent analysis), this approach is able to describe the RHIC and LHC data. In the dipole approach the total cross section for the process $pN \to Q\bar{Q}X$ is given by [22, 23]:

$$\sigma(pN \to \{Q\bar{Q}\}X) = 2\int_0^{-ln(2m_Q/\sqrt{s})} dy\, x_1\, G_p(x_1,\mu_F)\,\sigma(GN \to \{Q\bar{Q}\}X) \tag{7}$$

where $x_1 G_p(x_1,\mu_F)$ is the projectile gluon distribution, the cross section $\sigma(GN \to \{Q\bar{Q}\}X)$ describes heavy quark production in a gluon - nucleon interaction, y is the rapidity of the pair and

μ_F is the factorization scale. The basic idea of this approach is that before interacting with the hadron target N a gluon is emitted by the projectile p, which fluctuates into a color octet pair $Q\bar{Q}$. As in the low-x regime the time of fluctuation is much larger than the time of interaction, and color dipoles with a defined transverse separation $\vec{\rho}$ are eigenstates of the interaction. The cross section for the process $G + N \rightarrow Q\bar{Q}X$ is given by:

$$\sigma(GN \rightarrow \{Q\bar{Q}\}X) = \int_0^1 d\alpha \int d^2\rho \; |\Psi_{G\rightarrow Q\bar{Q}}(\alpha,\rho)|^2 \; \sigma^N_{Q\bar{Q}G}(\alpha,\rho) \tag{8}$$

where $\Psi_{G\rightarrow Q\bar{Q}}$ is the light-cone (LC) wave-function of the transition $G \rightarrow Q\bar{Q}$ and $\sigma^N_{Q\bar{Q}G}$ is the scattering cross section of a color neutral quark-antiquark-gluon system on the hadron target N [20, 22–24]. As discussed in Ref. [12, 18], this cross section can be expressed in terms of the dipole - proton cross section which is determined by the QCD dynamics at high energies and is probed in the deep inelastic scattering ep processes studied at HERA. Eq. (7) can be directly generalized to describe the total cross section of heavy quark production in pA collisions [18] considering the fact that color dipoles are eigenstates of the interaction. Therefore the $Q\bar{Q}G$-nucleus interaction can be expressed in terms of the cross section on a nucleon target using the Glauber-Gribov formalism:

$$\sigma^A_{Q\bar{Q}G}(x,\rho) = 2\int d^2\boldsymbol{b}\left\{1 - \exp\left[-\frac{1}{2}\,\sigma^N_{Q\bar{Q}G}(x,\rho^2)\,T_A(\boldsymbol{b})\right]\right\}\;, \tag{9}$$

where $T_A(\boldsymbol{b})$ is the nuclear profile function, which is obtained from a 3-parameter Fermi distribution for the nuclear density normalized to A. As in our previous studies [12, 18], we will assume that the dipole - nucleon cross section can be described by the phenomenological saturation model proposed by Golec-Biernat and Wusthoff (GBW) in Ref. [25]. As demonstrated in Refs. [12, 18], the predictions for heavy quark production using this simplified model are very similar to those obtained using as input the solution of the running coupling Balitsky - Kovchegov equation [26], which is the current state of the art of the treatment of the non-linear and quantum effects in the hadron wave function. Moreover, following Ref. [12] we will assume that $\mu_F = 2m_Q$ and that xG is given in terms of the CTEQ10 parametrization [27].

III. RESULTS AND DISCUSSION

In what follows we will present our predictions for the integrated DPS pA cross section of $c\bar{c}c\bar{c}$, $b\bar{b}b\bar{b}$ and $c\bar{c}b\bar{b}$ production. We will estimate $\sigma^{DPS}_{pA\rightarrow Q_1\bar{Q}_1Q_2\bar{Q}_2}$ considering the full rapidity range covered by the LHC as well as the rapidity range probed by the LHCb experiment ($2.0 < y < 4.5$). The single parton scattering cross section associated to the process $pN \rightarrow Q\bar{Q}X$ will be calculated

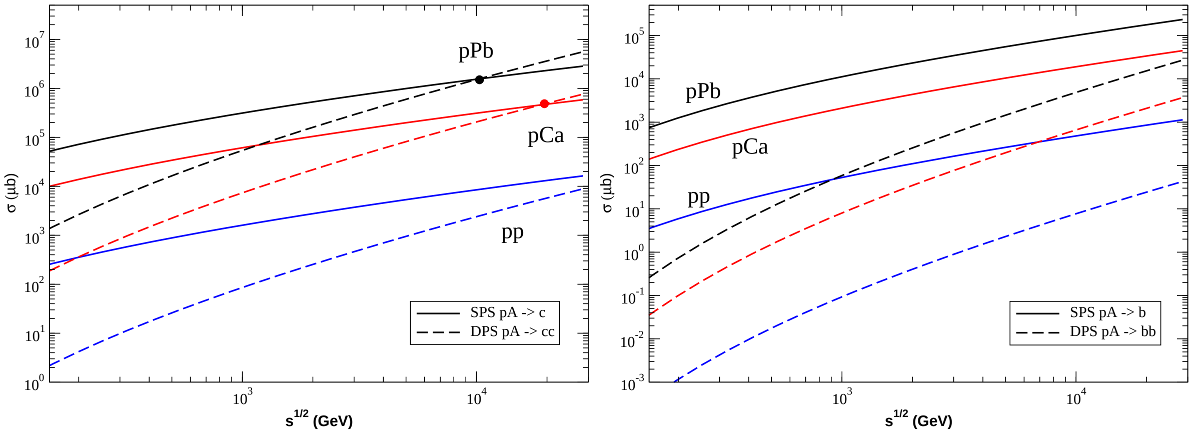

FIG. 2: Energy dependence of the SPS and DPS cross sections for charm (left panel) and bottom (right panel) production in pp, pCa and pPb collisions. The SPS (DPS) predictions are represented by solid (dashed) lines.

Final state	Mechanism	$\sqrt{s} = 2.76$ TeV	$\sqrt{s} = 5.02$ TeV	$\sqrt{s} = 8.8$ TeV
$c\bar{c}$	SPS	664 mb	994 mb	1420 mb
$c\bar{c}c\bar{c}$	DPS	258 mb	602 mb	1280 mb
$b\bar{b}$	SPS	32 mb	55 mb	90 mb
$b\bar{b}b\bar{b}$	DPS	0.5	1.5 mb	3.9 mb

TABLE I: Predictions for the SPS and DPS contributions for charm and bottom production in pPb collisions at different center - of - mass energies considering the full kinematical range covered by the LHC.

using Eq. (7). For the case of a nuclear target, we will use Eq. (9) as input in our calculations. Moreover, in our analysis the contribution of the single parton scattering processes associated to the $gg \to Q_1\bar{Q}_1Q_2\bar{Q}_2$ diagram will not be included, since its magnitude is very small in the kinematical range considered [28].

Initially let us analyse the nuclear dependence of SPS and DPS cross sections. As emphasized in the previous section, the DPS contribution in nuclear collisions is enhanced in comparison to pp collisions. This can be observed in the results presented in Fig. 2, where we show our predictions for charm (left panel) and bottom (right panel) production. In the case of charm production we can see that the energy where the SPS and DPS contributions becomes identical (indicated by a small circle in the figure) decreases at larger values of A. We can see that for $A = 1$ the equality takes place above the considered energy range, whereas $\sigma^{DPS}_{pA\to c\bar{c}c\bar{c}} = \sigma^{SPS}_{pA\to c\bar{c}}$ occurs at $\sqrt{s} \approx 19.6$ and 10.4 TeV for $A = 40$ and 208, respectively. In the case of bottom production, the SPS and DPS contributions are identical only for energies beyond the range considered in the figure.

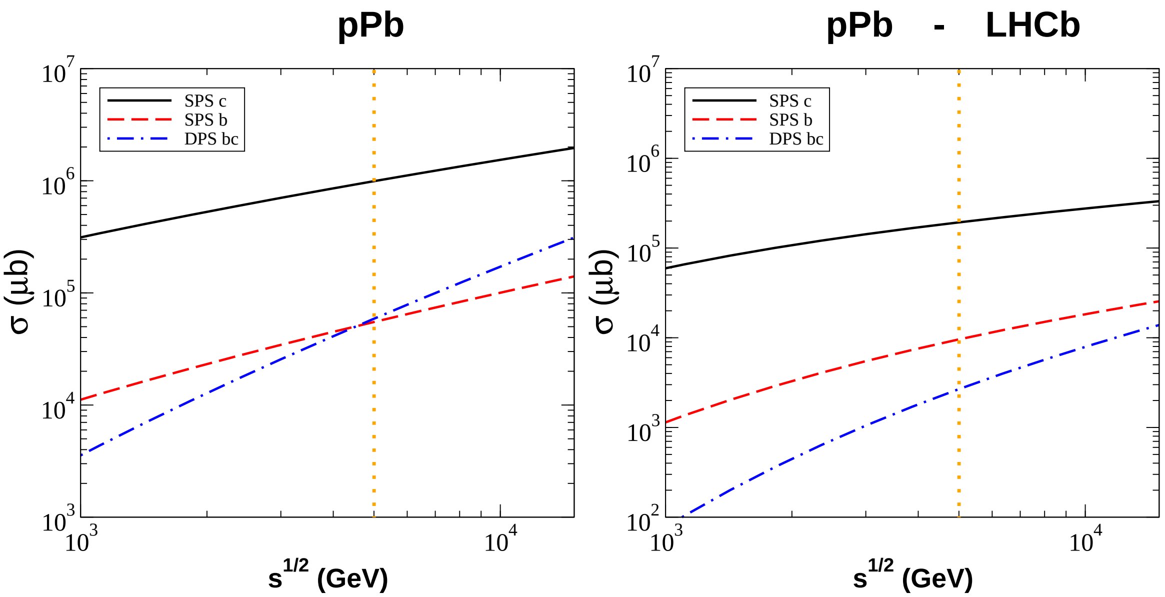

FIG. 3: Comparison between the SPS predictions for charm (solid line) and bottom (dashed line) production and the DPS one for the production of the $b\bar{b}c\bar{c}$ final state (dot - dashed line) in pPb collisions. In the left panel we present our predictions obtained considering the full rapidity range covered by the LHC, while in the right panel the cross sections were integrated over the rapidity range covered by the LHCb experiment ($2 < y < 4.5$).

In Table I we present our predictions for the SPS and DPS cross sections for pPb collisions at different center-of-mass energies. Our results indicate that the DPS contribution for charm production is non - negligible in the range of energies probed by the LHC in pPb collisions, as it already was in pp collisions [12]. In the case of the bottom production, the DPS contribution is smaller than 5% at LHC energies.

Another possible final state that can be produced considering the DPS mechanism is the $b\bar{b}c\bar{c}$ system, which can be generated when one gluon - gluon interaction creates a $b\bar{b}$ and the other a $c\bar{c}$ pair. As demonstrated in Ref. [12], the DPS production of $b\bar{b}c\bar{c}$ can be responsible for approximately half of the total amount of bottom quarks produced in pp collisions at the LHC. In what follows we will analyse how this conclusion is modified in pPb collisions. In Fig. 3 we compare the SPS production cross sections of $c\bar{c}$ and of $b\bar{b}$ pairs, denoted respectively by " SPS c " and " SPS b ", with the DPS production cross section for the $b\bar{b}c\bar{c}$ final state (denoted " DPS bc " in the figure). In the left panel we present our predictions obtained considering the full rapidity range covered by the LHC, while in the right panel the cross sections were integrated over the rapidity range covered by the LHCb experiment ($2 < y < 4.5$). The vertical dotted - lines indicates the center - of - mass energy of 5.02 TeV. In the case that the cross sections are integrated over

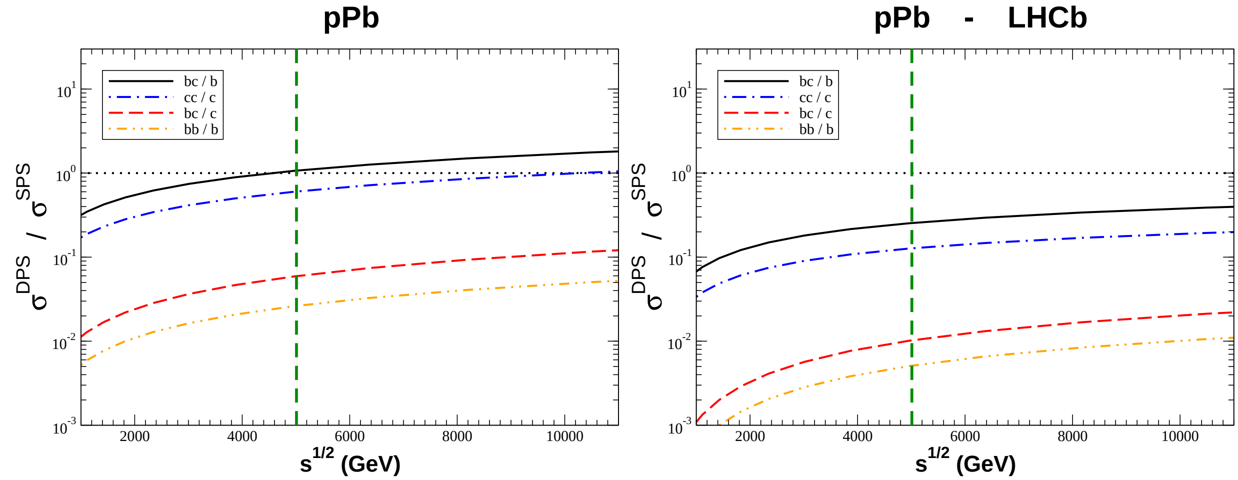

FIG. 4: Energy dependence of the ratio between the DPS and SPS cross sections for different combinations of final states. Left panel: Cross sections integrated over the full LHC rapidity range. Right panel: The cross sections are integrated over the rapidity range of the LHCb experiment ($2 < y < 4.5$).

the full rapidity range, one has that the associated production of a $b\bar{b}$ with a $c\bar{c}$ becomes of the same order of the SPS production of a $b\bar{b}$ in pPb collisions for energies of the order of 4 TeV, being dominant at larger energies. As expected, it occurs at smaller energies than in pp collisions, where we have estimated that $b\bar{b}c\bar{c}$ and $b\bar{b}$ cross sections are similar only at $\sqrt{s} \approx 10$ TeV. On the other hand, if the LHCb rapidity range is considered, the $b\bar{b}c\bar{c}$ cross section is a factor four smaller than the $b\bar{b}$ one.

In order to obtain a more precise estimate of the DPS contributions relative to the SPS ones, in Fig. 4 we present the energy dependence of the ratio $\sigma^{DPS}/\sigma^{SPS}$ for different final states. We denote by "bc/b" the ratio between the DPS production of $b\bar{b}c\bar{c}$ final state and the SPS production of $b\bar{b}$ pair, with analogous notation for the other combinations. In the left panel we present the predictions for the full LHC rapidity range, while in the right panel we integrated over the rapidity range covered by the LHCb experiment. The vertical dashed line indicates $\sqrt{s} = 5.02$ TeV. Our results for the full rapidity range indicate that the ratios "bc/b" and "cc/c" are of order of unity in pPb collisions at $\sqrt{s} = 5.02$ TeV, while the ratios "bc/c" and "bb/b" are smaller than 0.05. In contrast, all ratios are smaller than 0.3 in the LHCb rapidity range. In Ref. [12] we estimated these same ratios for pp collisions. Comparing the above results obtained for pPb collisions with those presented in Fig. 4 of Ref. [12], we have that these are considerably greater. Therefore, even at the rapidity range of the LHCb, heavy quark production in DPS processes is more likely to be experimentally detected in pPb collisions than in pp collisions. As pointed in Ref. [15], this can be useful to constrain the value of $\sigma_{eff,pp}$, since F_{pA} is reasonably well determined from the nuclear geometry [See Eqs. (4) and (5)].

IV. CONCLUSION

Recent experimental and theoretical studies of different final states that can be produced in pp collisions at the LHC have demonstrated that the contribution of double parton scattering processes can be non - negligible and should be taken into account. Such contribution becomes large at high energies due to the large parton luminosity in the initial state and is enhanced in nuclear collisions. In this paper we have extended our previous study of DPS production of heavy quarks in pp collisions to pA collisions. We have used the dipole approach and we have taken into account the saturation effects which are expected to be important for small x and large nuclei. We estimated the A dependence of the SPS and DPS cross sections and demonstrated that the DPS contribution for charm production is similar to the SPS one for pPb collisions at $\sqrt{s} = 5.02$ TeV and dominates at larger energies. Additionally, we have shown that the associated production of a $b\bar{b}$ with a $c\bar{c}$ has a cross section similar to the SPS cross section for the production of a $b\bar{b}$. Our results indicate that the analysis of the $c\bar{c}c\bar{c}$ and $b\bar{b}c\bar{c}$ final states in pPb collisions at the LHC can be useful to constrain the double parton scattering mechanism.

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