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LEFT-RIGHT-SYMMETRIC MODEL BUILDING

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Abstract

We have studied left-right-symmetric (LR) model building in two specific instances: the Minimal Left-Right-Symmetric Model (MLRM), with gauge group $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ and parity as the LR symmetry; and a non-supersymmetric, trinified theory, with gauge structure $SU(3)_L \otimes SU(3)_R \otimes SU(3)_C \otimes \mathbb{Z}_3$ and an additional, novel, $SU(3)$ family symmetry. For the MLRM, we have rederived the Lagrangian in the gauge and mass eigenbases, partly using the SARAH [11] model building framework. We have demonstrated how the gauge symmetry is broken to the Standard Model, and explicitly found the corresponding Goldstone bosons. For the trinified model, we have constructed the Lagrangian, spontaneously broken the gauge and global symmetries, and calculated the masses and charges of the resulting particle spectra. We show that the addition of the $SU(3)$ family symmetry reduces the amount of free parameters to less than ten. We also demonstrate a possible choice of vacuum which breaks the trinified gauge group down to $SU(3)_C \otimes U(1)_Q$, and find particularly simple minimum for this choice of potential. We conclude that the MLRM deserves its place as a popular LR extension, with several appealing features, such as naturally light neutrinos. The trinified model with $SU(3)$ family symmetry, meanwhile, is an economic and exciting new theory. Our first, simple version seems phenomenologically viable, using very few parameters. Furthermore, several other theoretical variations are possible, many of which seem worthy of study.

Populärvetenskaplig sammanfattning

Det är i princip omöjligt att överskatta vikten av begreppet symmetri för modern fysik. Redan när Maxwell på 1800-talet förenade elektricitet och magnetism till en enda kraft fanns en underlig egenskap i hans teori. Hans fysikaliska system kännetecknas av *potentialer*, som är relaterade till de elektriska och magnetiska fälten. Det visade sig att om man förändrar dessa potentialer enligt specifika regler så får man samma system, samma fysik, tillbaka. Identiska fysikaliska resultat ges alltså av flera olika konfigurationer av potentialerna. Man säger att teorin är *invariant* under en *intern symmetri*, där symmetritransformationerna är de ovan nämnda reglerna. Samma koncept styr idag hur fysiker konstruerar teorier som beskriver de fundamentala krafterna och partiklarna; om man vet exakt vilka symmetrier som teorin är invariant under, så kan man räkna ut exakt hur de olika partiklarna växelverkar. Naturens fundamentala krafter ges allts av de interna symmetrierna!

Det är alltså inte konstigt att mycket av arbetet i att konstruera en teori för det subatomära Universum ligger i att försöka hitta vilka symmetrier den bör besitta. En specifik typ av intern symmetri är s.k. vänster-högersymmetri. Med vänster och höger avses inte det man brukar mena i dagligt tal, utan snarare egenskaper som vissa partiklar har; sådana partiklar kan vara antingen vänster- eller högerhänta. Standardmodellen för partikelfysik beskriver naturen på den väldigt lilla skalan bättre än någon annan teori någonsin har gjort. Den behandlar dock s.k. vänster- och högerhänta partiklar ojämlikt, och det står inte klart varför, eller om det måste vara så. De flesta fysiker tycker det hade varit mest naturligt om Naturen behandlade dem lika.

Här granskar jag två teorier som faktiskt behandlar vänster och höger jämlikt, vilket leder till en mängd nya egenskaper och förutsägelser. Förhoppningen är att en sådan teori ska visa sig beskriva naturen ännu bättre, förklara saker som standardmodellen inte kan, och på så sätt ge oss en djupare förståelse för verklighetens mest grundläggande struktur.

Contents

Foreword

This thesis is the result of roughly one year of work, done at Lund University, under Roman Pasechnik. In the early summer of 2014, when discussing my project, Roman presented me with a long list of possible projects. Among them was a fascinating Grand-Unified Theory, called Trinification, which seemed to offer a wide range of predictions; regenerating the rich theoretical structure of the Standard Model, using a highly unified structure, with very few free parameters. I was intrigued, and the choice was easy. In order for me to learn about model building before tackling this new theory, we thought it wise for me to treat a more well-studied case, where results would be available in the literature for me to reference. This model ended up being the Minimal Left-Right-Symmetric Model (MLRM), which is of additional interest as an intermediate step when Trinification is broken down to the Standard Model.

The experience has been highly educational. I have learned how field theories are constructed from the ground up, and become intimately familiar with symmetry breaking. I have had to handle large, messy calculations using several of the computational tools of the trade. I have also learned about more about renormalization theory, effective potentials, anomalies and other field theory. Thinking back to what I knew, or didn't, a year ago, seems almost surreal. This project has been my first taste of actual research and I feel well-prepared to continue my career in particle physics.

In hindsight, it's tempting to think that I would have liked to have done more with the Trinification theory. Furthermore, there are several avenues of research which we started to pursue, like studying the vacuum structure at the 1-loop level for both theories, but were unable to complete due to computational limitations or lack of time. However, I realize that reproducing the known results for the MLRM was a necessary step in order for me to learn how to treat Trinification, and that studying the MLRM with tools like SARAH has opened interesting new possibilities, like studying the vacuum at 1-loop, as mentioned. To my knowledge, this has not been done before. Thus, several of the things that I would have liked to address in the thesis we are now planning to study; things I have worked on which are not included here, like code for the effective potential and a Trinification SARAH model file, will be of use as my work continues in the coming months.

1 Introduction

1.1 Background and rationale

Having been almost continually tested through the latter half of the last century, the Standard Model (SM; for a thorough description, see Ref. [1]) is one of the most complete and accurate theories in the history of physics. Its gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ encompasses all known fundamental interactions except gravity (a consistent quantum field theory of which currently eludes theoretical physics). Its success ranges from remarkable numerical predictions of electroweak parameters—some, like that of the fine structure constant α, accurate to around ten parts in a billion [2]—to the no less remarkable discovery of the Higgs boson in 2012 [3, 4].

However, the SM is, in several regards, theoretically incomplete and arbitrary. There are no appealing Dark Matter candidates; the particle spectra contain huge hierarchies which are theoretically not well-motivated (why is the top quark 35,000 times heavier than the down quark? Why are the neutrinos so extremely light?); the large number of free parameters (around 30) allows fine-tuning which detracts from the impressiveness of some predictions.

Since the SM so well describes Nature at energies for which it is phenomenologically valid, further theoretical developments should approximate the SM, at least to some degree, at these energies (in the same way that modern physics approaches classical results when quantum effects are small). Thus, when new theories are constructed, in the hopes of solving some of the theoretical inconsistencies of the SM, they are commonly designed as extensions to the SM, with larger symmetry groups or particle content.

One class of such theories is left-right-symmetric (LR) models, where left and right chiralities are treated equally at high energy scales. This is in contrast to the SM, where, for example, the charged electroweak current only couples to left-handed fermions. These models resolve a number of unsatisfactory features of the SM, such as

- The fact that the SM prefers one handedness over the other is not theoretically well-understood. LR models are often seen as more beautiful since they restore this symmetry.
- Since these models commonly feature heavy Majorana right-handed neutrinos, small left-handed neutrino masses are naturally introduced via see-saw mechanism [5].
- In the SM, the hypercharge Y is an arbitrary quantum number. In left-right-symmetric models this generator arises in a more coherent way from the less arbitrary quantity $B - L$ (the baryon number minus the lepton number).

This leaves unresolved, of course, several other problems with the SM: The arbitrariness of the large mass hierarchies, the unappealingly large number of free parameters, and the fact that charge is quantized (meaning the charge of the electron and the charge of the proton satisfy $Q_e = -Q_P$), among others. These issues are addressed in Grand-Unified Theories (GUTs); theories where the SM, or extensions to it, are embedded in more symmetric theories. These large symmetry groups are then broken down to the SM group at some high scale, often in steps, producing the correct phenomenology and, preferably, new predictions. Desirable qualities include a group structure which, through the way it is broken, produces the aforementioned charge quantization; fewer free parameters by means of unification; dark matter candidates among its particle spectrum, et cetera.

In this study, we have considered two LR theories: The first is a straight-forward extension to the SM, possessing a $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ symmetry, commonly referred to as the Minimal Left-Right-Symmetric Model (MLRM) [6–8]. The second is a non-supersymmetric (non-SUSY) version of a GUT with the gauge group $SU(3)_C \otimes SU(3)_L \otimes SU(3)_R$, called Trinification [9]. In addition to these gauge symmetries, Trinification also possesses a global family or flavour symmetry, $SU(3)_f$, a novel feature. We have investigated symmetry breaking, particle spectra and mixings, and other features of the models. This has been achieved in part with the help of the Mathematica package SARAH [10, 11].

The study is essentially divided into three parts: Chapter 1 contains an introduction and the theoretical basis used throughout the thesis.

Chapter 2 is dedicated to the MLRM. We first introduce the model and its symmetries and particle content, and go on to study the spontaneous gauge symmetry breaking mechanism, including the identification of the Goldstone bosons. We also derive the tree level Lagrangian in the physical basis, and give a short phenomenological overview. Furthermore, we have laid the basis for future research, most notably by constructing the corresponding SARAH model file, and other code, which may be used to find the renormalization group equations and, subsequently, to study the vacuum properties at the 1-loop level.

Chapter 3 introduces the Trinification model. We have shown that the group $SU(3)_L \otimes SU(3)_R \otimes SU(3)_C \otimes SU(3)_f$, where the last group is a global family symmetry, breaks down to the SM group, and derived the group representations, masses and charges of the particles at tree level. We discuss the features of the theory, most importantly the novel addition of the global $SU(3)$ family symmetry.

1.2 The effective potential in classical and quantum-corrected scalar theories

We will now briefly review the *effective potential* and spontaneous symmetry breaking (SSB) for simple scalar theories, classically and with quantum corrections. Our derivations roughly follow Refs. [12] and [13].

In a classical field theory, in order to find the vacuum expectation value of some field $\langle\phi\rangle$, we simply minimize the classical potential. When quantum effects are turned on (that is, when higher-order loop diagrams are also taken into account), however, this value may be shifted [12]. We seek a function which, when minimized, yields the quantum loop-corrected value of $\langle\phi\rangle$.

Consider a classical theory of a single real scalar field ϕ with the Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}\mu^2\phi^2 - \frac{1}{4!}\lambda\phi^4. \tag{1.1}$$

The theory is generated by the functional

$$Z[J] = e^{iW[J]} = \int D\phi e^{i(S[\phi]+J\phi)}$$

where

$$J\phi \equiv \int d^4x J(x)\phi(x).$$

is the source term. Let us define the classical field as

$$\phi_{\text{cl}}(x) \equiv \langle\Omega|\phi(x)|\Omega\rangle$$

i.e. the VEV of the quantum field (in the presence of the external source). Then

$$\phi_{\text{cl}}(x) = \frac{\delta W[J]}{\delta J(x)} = \frac{1}{Z[J]}\int D\phi e^{i(S[\phi]+J\phi)}\phi(x). \tag{1.2}$$

Note that ϕ_{cl} is given as a functional of J.

We wish to construct a Legendre transform, moving from $W(J)$, to some $\Gamma(\phi_{\text{cl}})$. Let us define

$$\Gamma[\phi_{\text{cl}}] \equiv W[J] - \int d^4x J(x)\phi_{\text{cl}}(x) \tag{1.3}$$

The idea is that J be eliminated from the RHS in favour of ϕ_{cl}, through the dependence of J on ϕ_{cl} given in Eqn. (1.2). Γ is called the *effective action*. Calculating the functional derivative of the effective action with respect to J, we find the simple result

$$\frac{\delta\Gamma[\phi_{\text{cl}}]}{\delta\phi_{\text{cl}}(x)} = \int d^4y \frac{\delta J(y)}{\delta\phi_{\text{cl}}(x)}\frac{\delta W[J]}{\delta J(y)} - \int d^4y \frac{\delta J(y)}{\delta\phi_{\text{cl}}(x)}\phi_{\text{cl}}(y) - J(x) = J(x).$$

In other words, if $J(x) = 0$ and we have no external source, we have

$$\frac{\delta\Gamma[\phi_{\rm cl}]}{\delta\phi_{\rm cl}(x)} = 0. \tag{1.4}$$

Now, since the effective action should be a spatially extensive quantity [12], we can write it as some coefficient $V_{\rm eff}$ times the four-volume $\mathcal{V}$ of the system,

$$\Gamma[\phi_{\rm cl}] = -\mathcal{V}V_{\rm eff}(\phi_{\rm cl}).$$

Plugging this into (1.4), we immediately see that

$$\frac{\partial V_{\rm eff}(\phi_{\rm cl})}{\partial\phi_{\rm cl}} = 0$$

for $J = 0$; in other words, the *effective potential* $V_{\rm eff}$ gives $\langle\phi\rangle$ when minimized in the absence of external sources. Since $J = 0$, the definition (1.3) implies that $\Gamma = -W$; the effective potential is simply the energy density of the state.

Let us now derive the form of $V_{\rm eff}$, following [13], to the first loop order. We start by computing $W[J]$. From here on, we omit the "cl" index and write $\phi_{\rm cl} \equiv \phi$. Let us denote by $\phi_s(x)$ the solution to the equation

$$\frac{\delta(S[\phi] + \int d^4y J(y)\phi(y))}{\delta\phi(x)} = 0,$$

implying

$$\partial^2\phi_s(x) + V'(\phi_s(x)) = J(x).$$

Letting $\phi = \phi_s + \tilde{\phi}$, expanding in orders of $\tilde{\phi}$ and restoring $\hbar$, we have

$$\begin{aligned} Z[J] = e^{(i/\hbar)W[J]} &= \int D\phi e^{(i/\hbar)(S[\phi]+J\phi)} \\ &\approx e^{(i/\hbar)(S[\phi_s]+J\phi_s)} \int D\tilde{\phi} e^{(i/\hbar)\int d^4x((\partial\tilde{\phi})^2/2 - V''(\phi_s)\tilde{\phi}^2/2)} \\ &= e^{(i/\hbar)(S[\phi]+J\phi) - \operatorname{tr}\log(\partial^2+V''(\phi_s))/2} \end{aligned}$$

or

$$W[J] = S[\phi_s] + J\phi_s + \frac{i\hbar}{2}\operatorname{tr}\log(\partial^2 + V''(\phi_s)) + \mathcal{O}(\hbar^2).$$

Using (1.2),

$$\phi = \frac{\delta W}{\delta J} = \frac{\delta(S[\phi_s] + J\phi_s)}{\delta\phi_s}\frac{\delta\phi_s}{\delta J} + \phi_s + \mathcal{O}(\hbar) = \phi_s + \mathcal{O}(\hbar).$$

Plugging this into Eqn. (1.3), the Legendre transform defining the effective action, we find

$$\Gamma[\phi] = S[\phi] + \frac{i\hbar}{2}\operatorname{tr}\log(\partial^2 + V''(\phi)) + \mathcal{O}(\hbar^2). \tag{1.5}$$

which follows from the relation $\det M = e^{\operatorname{tr}\log M}$ for the matrix exponential. Assuming ϕ is constant in x (the vacuum is invariant under translation) allows us to evaluate the trace in momentum space:

$$\begin{aligned}
\operatorname{tr}\log(\partial^2 + V''(\phi)) &= \int d^4x \langle x|\log(\partial^2 + V''(\phi))|x\rangle \\
&= \int d^4x \int \frac{d^4k}{(2\pi)^4}\langle x|k\rangle\langle k|\log(\partial^2 + V''(\phi))|k\rangle\langle k|x\rangle \\
&= \int d^4x \int \frac{d^4k}{(2\pi)^4}\log(-k^2 + V''(\phi)).
\end{aligned}$$

Let us write as an Ansatz for Γ

$$\Gamma[\phi] = \int d^4x(-A(\phi) + B(\phi)(\partial\phi)^2 + C(\phi)(\partial\phi)^4 + \dots)). \tag{1.6}$$

Then, under our assumptions of ϕ constant in x and no external sources,

$$\Gamma[\phi] = \int d^4x(-A(\phi)).$$

Eqn. (1.4) thus implies that $A'(\phi) = 0$; minimizing A gives $\langle\phi\rangle$, and we identify $A = V_{\text{eff}}$.[1]

Using the Ansatz (1.6) with (1.5), recalling that $S[\phi] = \int d^4x(-V(\phi))$ under our assumptions, we finally obtain

$$V_{\text{eff}}(\phi) = V(\phi) - \frac{i\hbar}{2}\int \frac{d^4k}{(2\pi)^4}\log\left(\frac{-k^2 - V''(\phi)}{k^2}\right) + \mathcal{O}(\hbar^2), \tag{1.7}$$

known as the *Coleman-Weinberg effective potential.* We have supplied the constant factor k^2 to make the logarithm dimensionally sensible. Eqn. (1.7) takes the form of the classical potential V plus quantum corrections parametrized by $\hbar$.

The momentum integral in (1.7) diverges due to its quadratic dependence on the implicitly imposed cutoff. To remedy this, we add counterterms to the original Lagrangian (1.1):

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}\mu^2\phi^2 - \frac{1}{4!}\lambda\phi^4 + A(\partial\phi)^2 + B\phi^2 + C\phi^4.$$

Adding these to (1.7) we obtain

$$V_{\text{eff}}(\phi) = V(\phi) + \frac{\hbar}{2}\int^{\Lambda^2} \frac{d^4k_E}{(2\pi)^4}\log\left(\frac{k_E^2 - V''(\phi)}{k_E^2}\right) B\phi^2 + C\phi^4 + \mathcal{O}(\hbar^2)$$

where we have also performed a Wick rotation into Euclidean space and imposed a cutoff $k_E^2 = \Lambda^2$. Integrating, we find

$$V_{\text{eff}}(\phi) = V(\phi) + \frac{\Lambda^2}{32\pi^2}V''(\phi) - \frac{(V''(\phi))^2}{64\pi^2}\log\frac{e^{1/2}\Lambda^2}{V''(\phi)} + B\phi^2 + C\phi^4. \tag{1.8}$$

[1] Note that the argument of A is ϕ_{cl}, equal to the VEV $\langle\phi\rangle$ of the quantum operator ϕ, with the subscript dropped.

$V(\phi) \sim \phi^4$, so $V''(\phi) \sim \phi^2$ and $(V''(\phi))^2 \sim \phi^4$, and each term containing the cut-off can be absorbed into a corresponding counterterm, leaving the expression for the effective potential containing only physical parameters.

We will now demonstrate how this occurs for the simplified case of $\mu^2 = 0$. This scenario is of physical interest; when $\mu^2 > 0$, the vacuum lies at the origin and everything is symmetric; when $\mu^2 < 0$, the $\phi \to -\phi$ symmetry is spontaneously broken. It is not clear, however, what happens when we take $\mu^2 = 0$ and include quantum corrections. Is the symmetry spontaneously broken or not?

Plugging $V(\phi) = -\frac{\lambda}{4!}\phi^4$ into (1.8), we get

$$V_{\text{eff}} = \left(\frac{\Lambda^2\lambda}{64\pi^2} + B\right)\phi^2 + \left(\frac{\lambda}{4!} + \frac{\lambda^2}{(16\pi)^2}\log\left(\frac{\phi^2}{\lambda^2}\right) + C\right)\phi^4 + \mathcal{O}(\lambda^3) \tag{1.9}$$

where C has been redefined to absorb constants in ϕ. To determine the two counterterm coefficients B and C we need two renormalization conditions. To obtain the first, we note that setting $\mu^2 = 0$ means that the so-called *renormalized mass-squared*, defined as the coefficient of ϕ^2 in V, is zero. We wish to preserve this in V_{eff}, so we take

$$\left.\frac{d^2V_{\text{eff}}}{d\phi^2}\right|_{\phi=0} = 0$$

as the first condition. Referring to (1.9), this implies $B = -\Lambda^2\lambda/(64\pi^2)$. We cannot employ the same idea for the ϕ^4 term, since the corresponding coefficient in V_{eff} contains $\log\phi$, which is not defined for $\phi = 0$. Instead, we evaluate this derivative at some scale Q. Thus

$$\left.\frac{d^4V_{\text{eff}}}{d\phi^4}\right|_{\phi=Q} = \lambda(Q), \tag{1.10}$$

where $\lambda(Q)$ is a coupling that runs depending on the scale Q, is our second renormalization condition. Solving (1.10) for C and plugging back into (1.9), we get, finally,

$$V_{\text{eff}} = \frac{\lambda(Q)}{4!}\phi^4 + \frac{\lambda^2(Q)}{(16\pi)^2}\phi^4\left(\log\left(\frac{\phi^2}{Q^2}\right) - \frac{25}{6}\right) + \mathcal{O}(\lambda^3(Q)), \tag{1.11}$$

where we have noted that $\lambda = \lambda(Q) + \mathcal{O}(\lambda^2)$. Thus, we have the renormalized (depending only on purely physical parameters) effective potential for our ϕ^4 theory with $\mu^2 = 0$.

We are now equipped to answer the question regarding the spontaneous symmetry breaking, or lack thereof, in a quantum-corrected theory with $\mu^2 = 0$. Let us consider the effective potential (1.11) close to the origin. The leading, "classical", term $\propto \phi^4$, vanishes. The next-to-leading order term, however, contains the factor $\log(\phi^2/Q^2)$, which approaches negative infinity in the $\phi \to 0$ limit. The origin, then, is a local maximum, which means that there is a local minimum at some $\langle\phi\rangle \neq 0$.

The function $a\phi^4 + b\phi^4\log(\phi^2)$, for some coefficients a and b, has been plotted against ϕ in Fig. 1.1 to illustrate this. Thus, clearly, the reflection $\phi \to -\phi$ symmetry is spontaneously broken by the minima generated by quantum corrections. This is known as *radiative symmetry breaking* [15].

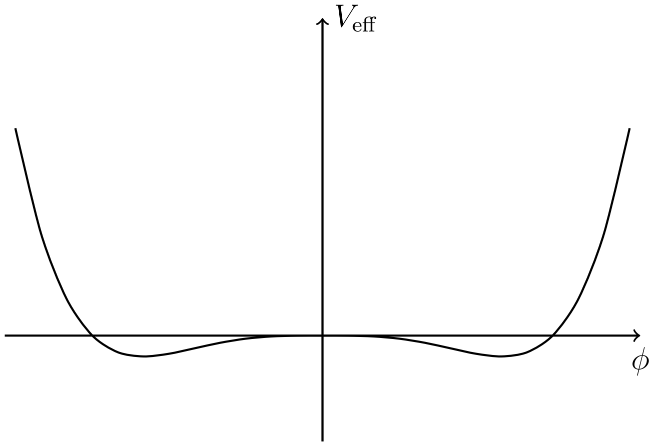

Figure 1.1: *The function $V_{\text{eff}} = a\phi^4 + b\phi^4 \log(\phi^2)$, for some constants a, b, plotted against ϕ. This function is of the form of the effective potential in Eqn. (1.11). The two minima are generated by quantum corrections, and spontaneously break the original reflection symmetry of the theory, despite the lack of a classical μ^2 term.*

Even for more complicated theories, the expressions remain simple. In this paper, we will use the form given in [16], for a general, renormalizable theory:

$$V_{\text{eff}} \approx V + \frac{1}{(16\pi)^2} V^{(1)},$$
$$V^{(1)} = \sum_n \frac{m_n^4}{4} (-1)^{2s_n} (2s_n + 1) \left(\log\left(\frac{m_n^2}{Q^2} \right) - \frac{3}{2} \right).$$

Here, V is the classical (tree-level) potential; the sum over n runs over all particles in the theory; s_n and m_n are the spins and tree-level masses for each particle. This form holds for the so-called $\overline{DR}$ renormalization scheme.

1.3 Goldstone's theorem

1.3.1 Spontaneous symmetry breaking in the linear sigma model

Following [12], we will give a brief introduction to the concept of spontaneous symmetry breaking (SSB). Consider the N-field linear sigma model,

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi^i)^2 + \frac{1}{2}\mu^2 (\phi^i)^2 - \frac{\lambda}{4}((\phi^i)^2)^2,$$

where the factor $(\phi^i)^2$ is always summed over $i = 1, \ldots, N$. Note the rescaling from the usual convention $\lambda/4! \to \lambda/4$. This Lagrangian is invariant under the group $O(N)$ of rotations in N-dimensional space, the operations of which can be written as

$$\phi^i \to R_{ij} \phi^j,$$

where R is an orthogonal $N \times N$ matrix.

Since, at this stage, this is a fully classical field theory, we obtain the vacuum by simply minimizing the potential

$$V(\phi^i) = -\frac{1}{2}\mu^2(\phi^i)^2 + \frac{\lambda}{4}((\phi^i)^2)^2. \tag{1.12}$$

This is satisfied by any vector of fields ϕ_0^i for which

$$(\phi_0^i)^2 = \frac{\mu^2}{\lambda}.$$

Thus, we are free to choose the direction in which this vector of fixed length points. Once common such choice is

$$\phi_0 = (0, \ldots, 0, v),$$

i.e. letting the vacuum point purely in the ϕ_N direction in field space, for some number v. From (1.12), $v^2 = \mu^2/\lambda$. We are obviously free to write the theory in terms of new fields π, σ, shifting from the origin to v in the ϕ_N direction:

$$\phi^i(x) = (\pi^k(x), v + \sigma(x)),$$

where now $k = 1, \ldots, N-1$. Let us plug our new fields into the Lagrangian, keeping only the interesting terms (quadratic and higher), and simplify:

$$\begin{aligned}\mathscr{L} = &\frac{1}{2}(\partial_\mu \pi^k)^2 + \frac{1}{2}(\partial_\mu \sigma)^2 - \frac{1}{2}(\sqrt{2}\mu)^2\sigma^2 \\ &- \sqrt{\lambda}\mu\sigma^3 - \frac{\lambda}{4}\sigma^4 - \sqrt{\lambda}\mu\sigma(\pi^k)^2 - \frac{\lambda}{2}\sigma^2(\pi^k)^2 - \frac{\lambda}{4}((\pi^k)^2)^2.\end{aligned}$$

Looking closer at this result, we interpret the $1/2(\sqrt{2}\mu)^2\sigma^2$ term as a mass term for the boson σ. The mass terms $\sim (\pi^k)^2$ are absent, so the π^k's are massless. Since there are $N-1$ of these directions in field space to rotate among, the Lagrangian's original $O(N)$ symmetry is hidden, replaced by the subgroup $O(N-1)$. Our choice of a specific vacuum, the vector ϕ_0^i which points in the Nth direction, breaks the $O(N)$ symmetry of the vacuum.

A rotation in N-dimensional space can take place in $N(N-1)/2$ planes; cf. the familiar case of $N = 3$, where there are 3 planes in which a rotation can be made (or, three Euler angles which together completely specify any rotation). Thus, a theory which is symmetric under the group $O(N)$ has $N(N-1)/2$ continuous symmetries. We end up with a $O(N-1)$-symmetric theory, and so $N(N-1)/2 - (N-1)(N-2)/2 = N-1$ symmetries have been broken, the same number as the massless π bosons. This is a very general result, encased in Goldstone's theorem, which we will treat in more detail below.

Considering Fig. 1.2, we may also argue geometrically. The potential is spherically symmetric, and we can excite the system from the vacuum in two directions; radially, climbing the slope, or tangentially, by moving around the minimal circle. Moving along this equipotential circle corresponds to a massless excitation; the Goldstone mode.

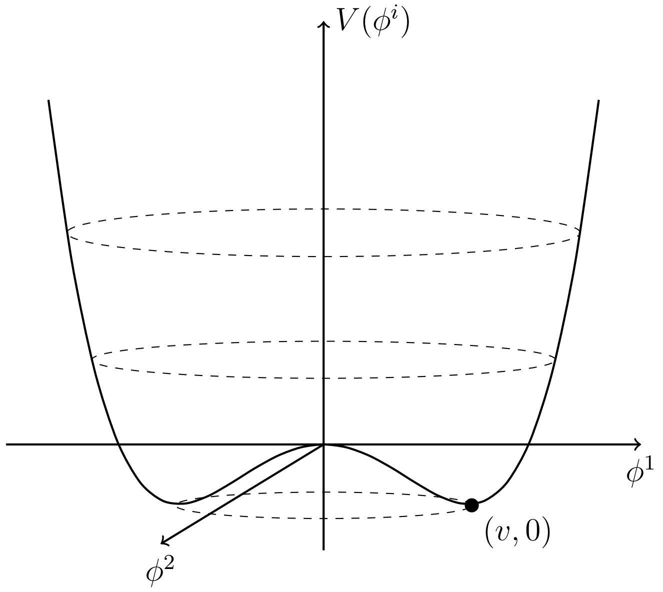

Figure 1.2: *The linear sigma model potential* $V = -\mu^2(\phi^i)^2 + \lambda((\phi^i)^2)^2$, *for* $N = 2$, *plotted against the fields* $\phi^{1,2}$. *The infinitely degenerate vacuum states lie along the circle with radius* $\mu^2/\lambda = v$, *among them our choice of ground state* $(v, 0)$.

1.3.2 The Goldstone theorem

We will now give a general overview and proof of the Goldstone theorem [14], of which we saw an example in the previous section. The theorem states that when a continuous symmetry of a theory is spontaneously broken, there will result a massless boson corresponding to each generator of the broken symmetry. The proof given here follows Ref. [12]. Consider a classical theory of N scalar fields $\phi^i(\mathrm{x})$, abbreviated ϕ. The Lagrangian is

$$\mathscr{L} = \text{kinetic terms} - V(\phi). \tag{1.13}$$

Let ϕ_0^i be a constant vector of fields such that it minimizes the potential V:

$$\left.\frac{\partial V(\phi)}{\partial \phi}\right|_{\phi(x)=\phi_0} = 0.$$

The expansion about this minimum is

$$V(\phi) = V(\phi_0) + \frac{1}{2}(\phi - \phi_0)^i(\phi - \phi_0)^j \left(\frac{\partial^2 V(\phi)}{\partial \phi^i \partial \phi^j}\right)_{\phi=\phi_0} + \dots,$$

where the matrix

$$\left(\frac{\partial^2 V(\phi)}{\partial \phi^i \partial \phi^j}\right)_{\phi=\phi_0} \equiv m_{ij}^2$$

is symmetric and has as its eigenvalues the squared mass of each particle. Since V has a minimum at ϕ_0, the curvature is positive there, and thus the eigenvalues of m^2_{ij} are non-negative. We wish to show that any spontaneously broken symmetry (that is, a symmetry of $\mathscr{L}$ in (1.13) but not of the vacuum ϕ_0) results in a zero eigenvalue of m^2_{ij}.

That $\mathscr{L}$ is invariant under a continuous symmetry means that it is unchanged under a transformation

$$\phi^i \to \phi^i + \alpha\Delta^i(\phi) \tag{1.14}$$

of all the fields. Here, α parametrizes the transformation, and Δ^i depends on all the ϕ^i's. If we consider only constant fields, $\mathscr{L} = -V$, and so the potential must be invariant under (1.14). Another way of writing this invariance is

$$V(\phi^i) = V(\phi^i + \alpha\Delta^i(\phi)) \implies \Delta^i(\phi)\frac{\partial V(\phi)}{\partial\phi^i} = 0.$$

Differentiating once w.r.t. ϕ^j and evaluating at the vacuum, we get

$$\left(\frac{\partial\Delta^i}{\partial\phi^j}\right)_{\phi_0}\left(\frac{\partial V}{\partial\phi^i}\right)_{\phi_0} + \Delta^i(\phi_0)\left(\frac{\partial^2 V}{\partial\phi^i\partial\phi^j}\right)_{\phi_0} = 0.$$

Since ϕ_0 minimizes V, the first term is zero. It follows therefore that the second term be must also be zero. The relation then reads

$$0\cdot\Delta_j(\phi_0) = \Delta^i(\phi_0)\left(\frac{\partial^2 V}{\partial\phi^i\partial\phi^j}\right)_{\phi_0} = \Delta^i(\phi_0)m^2_{ij}$$

which is an eigenvalue problem: when the vector $\Delta^i(\phi_0)$ is nonzero, which means that vacuum is not invariant under our symmetry transformation, it is the eigenvector of m^2_{ij} corresponding to the eigenvalue 0. Thus, a spontaneously broken symmetry engenders a massless scalar, which we wished to prove.

Let us now look more closely at a non-Abelian gauge theory, containing a set of real scalar fields which transform as

$$\phi^i \to (1 + i\omega^a t^a)_{ij}\phi^j, \tag{1.15}$$

while keeping the Lagrangian unchanged. The ω^a's are infinitesimal parameters (with non-trivial spacetime dependence, which we suppress) and the t^a's generate the transformation. The gauge-covariant derivative acting on ϕ^i is then

$$D_\mu\phi^i = (\partial_\mu - igt^aW^a_\mu)_{ij}\phi^j$$

The gauge boson kinetic terms are half the square of this, while letting the ϕ^i's obtain a nontrivial VEV $\langle\phi^i\rangle = \phi^i_0$. The term of interest to us is then

$$\mathscr{L}_{\text{GB mass}} = \frac{1}{2}m^2_{ab}W^a_\mu W^{b\mu}$$

where the mass matrix is given by

$$m_{ab}^2 = g^2(it^a\phi_0)^i(it^b\phi_0)^i \tag{1.16}$$

If a symmetry remains unbroken, it means that the corresponding gauge boson remains massless. This is encoded in (1.16): the vacuum transforms as (1.15), so invariance under the symmetry generated by some t^a is equivalent to $t^a\phi_0 = 0$. Then, the corresponding entry of m_{ab}^2 is zero.

We can now also compare the transformations (1.14) and (1.15). As we saw, the vector $\Delta^i(\phi_0)$ is an eigenvector of the scalar mass matrix with mass zero (using the simplification of constant fields). Thus, $(it^a\phi_0)_i$ is, in fact, a vector which at the vacuum is parallel to the Goldstone mode. In other words, i times some broken generator transforms the vacuum (infinitesimally) in the corresponding Goldstone direction. This important fact will be useful to us when aiming to identify the Goldstone bosons in the scalar spectra treated in later chapters.

1.3.3 Goldstone's theorem in the presence of quantum effects

In Section 1.3.2, we showed that when a symmetry of a scalar theory is spontaneously broken, the matrix of second derivatives of the potential V w.r.t. the fields has a corresponding zero eigenvalue for each broken generator. We now wish to argue that the situation is completely mirrored in a theory where quantum corrections are taken into account. As discussed in Section 1.2, the effective potential V_{eff}, when minimized, gives the classical expectation value just as V does without quantum effects. In addition, it necessarily obeys the same symmetries as V [12]. Thus, the first part of the original proof may be immediately repeated for the present case; for every continuous symmetry of the theory that is spontaneously broken, the matrix

$$\frac{\partial^2 V_{\text{eff}}(\phi_{\text{cl}})}{\partial\phi_{\text{cl}}^i\partial\phi_{\text{cl}}^j}$$

(recalling that $\phi_{\text{cl}} \equiv \langle\Omega|\phi|\Omega\rangle$; we will once again use the shorthand $\phi_{\text{cl}} \equiv \phi$) obtains one zero eigenvalue. It remains to be shown that this means that there is a massless scalar boson in the spectrum.

In Ref. [12], it is shown that the second derivative of the effective action Γ is equal to the inverse propagator (that is, the inverse of the sum of connected two-point functions) times i:

$$\frac{\delta\Gamma}{\delta\phi^i(x)\delta\phi^j(y)} = i\langle\phi^i(x)\phi^j(y)\rangle_{\text{connected}} = iD^{-1}(x,y).$$

Assuming constant fields and Fourier transforming,

$$D(x,y) = \int \frac{d^4k}{(2\pi)^4}e^{-ip(x-y)}\tilde{D}(p),$$

where the momentum-space propagator $\tilde{D}(p)$ is given by [12]

$$\tilde{D}(p) = \frac{i}{p^2 - m_0^2 - M^2(p^2)},$$

that is, a geometric series of one-particle irreducible[2] two-point diagrams. The pole, at the physical mass, is shifted away from the bare mass m_0 by the self-energy $M^2(p^2)$. The poles of the propagator are the zeroes of its inverse, and so the physical masses squared m^2 are obtained by solving

$$0 = i\tilde{D}^{-1}(p^2) = \int \frac{d^4k}{(2\pi)^4} e^{-ip(x-y)} \frac{\delta\Gamma}{\delta\phi(x)\delta\phi(y)} \tag{1.17}$$

with respect to p^2. We seek massless particles, so let us study (1.17) with $p^2 = 0$. Since, this means that Γ is differentiated w.r.t. constant fields. But, from Eqn. (1.2) we see that this in turn implies

$$\Gamma[\phi] = \int d^4x(-V_{\text{eff}}(\phi)).$$

Thus, as we wished to show, the matrix

$$\frac{\partial^2 V_{\text{eff}}}{\partial\phi^i\partial\phi^j}$$

does indeed have a 0 eigenvalue for every zero-mass scalar in the spectrum; Goldstone's theorem holds also in the quantum theory.

1.4 The identification of Goldstone bosons in a scalar spectrum

We will now develop the machinery later used for identifying the Goldstone bosons in a theory of the type of those considered in this text. This work has been done in collaboration with J.E.C. Molina[3] and J. Wessén[3].

It is instructive to first explore the Standard Model. The SM electroweak symmetry $G = SU(2)_L \times U(1)_Y$ is broken into the EM symmetry group $H = U(1)_Q$. The Higgs field is fundamental under G,

$$\Phi = \begin{pmatrix} \phi^+ \\ v + \phi^0 \end{pmatrix} = \begin{pmatrix} \phi_1 + i\phi_2 \\ v + \phi_3 + i\phi_4 \end{pmatrix} \tag{1.18}$$

We first seek to find the generator of the unbroken symmetry (EM), i.e. Q. To do this, we note that the vacuum, which we chose to be

$$\langle\Phi\rangle = \begin{pmatrix} 0 \\ v \end{pmatrix},$$

[2]One-particle irreducible (1PI) diagrams are those which cannot be split into two diagrams by the removal of any one line.

[3] Theoretical High Energy Physics (THEP) at Lund University, Lund, Sweden

should be invariant under $U(1)_Q$. Since Φ is fundamentally represented under $SU(2)_L$, it transforms under the gauge transformation of G as

$$\Phi \to e^{\frac{i}{2}\omega_2^a \sigma^a} e^{i\omega_1 Y_\Phi} \Phi \approx \left(1 + \frac{i}{2}\omega_2^a \sigma^a + i\omega_1 Y_\Phi\right) \Phi \equiv \Phi + \delta\Phi$$

where σ^a are the Pauli matrices and $Y_\Phi = +1/2$ is the hypercharge assignment. In the second step we assume infinitesimal transformations, without loss of generality. We have also suppressed the spacetime dependence in the parameters ω_L^a, ω_Y. Acting on the vacuum and requiring no change,

$$\delta\langle\Phi\rangle = \left(\frac{i}{2}\omega_L^a \sigma^a + i\omega_Y Y_\Phi\right) \begin{pmatrix} 0 \\ v \end{pmatrix} = \frac{i}{2} \begin{pmatrix} (\omega_L^1 - i\omega_L^2)v \\ (-\omega_L^3 + \omega_Y)v \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

So, the vacuum is invariant under the gauge transformation with $\omega_L^1 = \omega_L^2 = 0$ and $\omega_Y = \omega_L^3 \equiv \omega_Q$. Thus, $Q = T^3 + Y$ as we expected.

The gauge transformation of $U(1)_Q$ on the Higgs doublet (which is in the fundamental representation of $U(1)_Q$) is then

$$\begin{aligned} e^{iQ\omega_Q} \begin{pmatrix} \phi_1 + i\phi_2 \\ v + \phi_3 + i\phi_4 \end{pmatrix} &= e^{\frac{i}{2}\sigma^3 \omega_Q} e^{iY_\Phi \omega_Q} \begin{pmatrix} \phi_1 + i\phi_2 \\ v + \phi_3 + i\phi_4 \end{pmatrix} \\ &= \begin{pmatrix} e^{i\omega_Q} & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \phi_1 + i\phi_2 \\ v + \phi_3 + i\phi_4 \end{pmatrix} \end{aligned}$$

This tells us that, indeed, $\phi^+ = \phi_1 + i\phi_2$ transforms as an object with $Q = +1$ under the EM group. Similarly, $v + \phi^0 = v + \phi_3 + i\phi_4$ is also in the fundamental representation and has EM charge $= 0$. In equations, assuming infinitesimal transformations,

$$\begin{aligned} \delta(\phi_1 + i\phi_2) &= i\omega_Q(\phi_1 + i\phi_2), \\ \delta(v + \phi_3 + i\phi_4) &= 0. \end{aligned}$$

The conjugate fields transform like

$$\begin{aligned} \delta(\phi_1 - i\phi_2) &= -i\omega_Q(\phi_1 - i\phi_2), \\ \delta(v + \phi_3 - i\phi_4) &= 0. \end{aligned}$$

From linearity we can find the individual transformation properties; subtracting and adding the above equations we find

$$\begin{aligned} \delta\phi_1 &= -\omega_Q \phi_2, \\ \delta\phi_2 &= \omega_Q \phi_1, \\ \delta\phi_3 &= \delta\phi_4 = 0. \end{aligned}$$

From analysis of the gauge boson spectrum we know that there are three mass and EM charge eigenstates; Z^0 and $W^\pm \equiv (W_1 \mp iW_2)/\sqrt{2}$, which must eat a Goldstone each in order to become massive. The scalar mass matrix reveals that ϕ_1, ϕ_2, ϕ_4 are massless (ϕ_3 becomes the physical Higgs). Thus, the three massless scalars mix in some particular way into three Goldstone states, with definite EM charges $0, \pm 1$, in order to take the roles of longitudinal polarization states of the gauge bosons. As we will see, there is only one possibility for each.

Let's start by finding the state $K_i \phi_i$ (summed over i; K_i are constants and $i \in \{1, 2, 4\}$) which has $Q = 0$. In other words,

$$\delta(K_i \phi_i) = K_i \delta\phi_i = K_1(-\omega_Q \phi_2) + K_2(\omega_Q \phi_1) + K_4 \cdot 0 \stackrel{!}{=} 0,$$

where, in the last step, the equality is enforced. Clearly, there are no nonzero constants $K_{1,2}$ that satisfy this. Then the uncharged Goldstone contains only ϕ_4. K_4 is arbitrary here but fixed to unity by requiring normalized kinetic terms. So, Z^0 eats the Goldstone $G^0 = \phi_4$.

The positively charged Goldstone is found in the same way, by requiring

$$\delta(K_i \phi_i) = K_i \delta\phi_i = K_1(-\omega_Q \phi_2) + K_2(\omega_Q \phi 1) + K_4 \cdot 0 \stackrel{!}{=} +i\omega_Q (K_i \phi_i)$$

Identifying coefficients we learn that $K_4 = 0$, $iK_1 = K_2$ and $iK_2 = -K_1$ (noting that the last two conditions are equivalent). So, the $Q = +1$ state G^+, eaten by W^+, is $K_1(\phi_1 + i\phi_2)$. $K_1 = 1/\sqrt{2}$ can be seen from requiring normalization.

Finally, G^-, which becomes the transverse mode of W^-, is found by setting

$$\delta(K_i \phi_i) = K_i \delta\phi_i = K_1(-\omega_Q \phi_2) + K_2(\omega_Q \phi 1) + K_4 \cdot 0 \stackrel{!}{=} -i\omega_Q (K_i \phi_i)$$

We find $K_4 = 0$ again, and $iK_2 = K_1 \iff iK_1 = -K_2$. So, $G^- = K_1(\phi_1 - i\phi_2) = \frac{1}{\sqrt{2}}(\phi_1 - i\phi_2)$.

To recapitulate, the three Goldstone modes engendered by the spontaneous breaking $SU(2)_L \times U(1)_Y \to U(1)_Q$ are, in terms of the EW gauge eigenstate Higgs fields,

$$\begin{aligned} G^0 &= \phi^4, \\ G^\pm &= \frac{1}{\sqrt{2}}(\phi_1 \pm i\phi_2). \end{aligned}$$

Since the Lagriangian is invariant under global phase transformations, we are free to redefine

$$\begin{aligned} G^+ &\to e^{i3\pi/2} G^+ = -iG^+ = \phi_2 - i\phi_1, \\ G^- &\to e^{-i3\pi/2} G^- = iG^- = \phi_2 + i\phi_1. \end{aligned} \tag{1.19}$$

We can also find the Goldstones by first finding the generators of the spontaneously broken part of G. The gauge-covariant derivative of the SM is

$$D_\mu = \partial_\mu - igW^i_\mu T^i - ig' Y B_\mu,$$

where the $SU(2)$ generators are half the Pauli matrices, $T^i = \sigma^i/2$, and the hypercharge Y generates the $U(1)$ transformations. Let us write this in terms of the mass eigenstate fields. These are obtained from diagonalization of the gauge boson mass matrices: $W_\mu^{1,2}$ are already mass eigenstates; the photon and Z are

$$\begin{aligned} A_\mu &= \frac{1}{\sqrt{g^2+g'^2}}(g'W_\mu^3 + gB_\mu), \\ Z_\mu^0 &= \frac{1}{\sqrt{g^2+g'^2}}(gW_\mu^3 - g'B_\mu). \end{aligned}$$

We get

$$\begin{aligned} D_\mu = \partial_\mu &- igT^1W_\mu^1 - igT^2W_\mu^2 \\ &- i\frac{1}{\sqrt{g^2+g'^2}}(g^2T^3 - g'^2Y)Z_\mu^0 - i\frac{gg'}{\sqrt{g^2+g'^2}}(T^3+Y)A_\mu. \end{aligned} \tag{1.20}$$

Each term in Eqn. (1.20) is a product of $-i$ times a charge, a physical gauge field, and a thereto associated generator. We can immediately read off $Q = T^3 + Y$ as the generator of the EM symmetry, again, and $e = gg'/\sqrt{g^2+g'^2}$ as the elementary electric charge.

Furthermore, the broken generators (i.e. those corresponding to the degrees of symmetry which do not live on as $U(1)_Q$ after spontaneous symmetry breaking) are T^1, T^2 and $(g^2T^3 - g'^2Y)$. As we found in Section 1.3.2, complex unity times a broken generator transforms the vacuum in the corresponding Goldstone directions. This allows us to find the Goldstone bosons corresponding to each generator and gauge boson. We get, for the first broken generator,

$$iT^1\langle\Phi\rangle = \frac{i}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}0\\ v\end{pmatrix} = \begin{pmatrix}i\frac{v}{2}\\ 0\end{pmatrix}.$$

This tells us that the Goldstone mode corresponding to the generator T^1, and thus eaten by W^1, is the complex part of the top component of Φ, defined in Eqn. (1.18), i.e. ϕ_2. Similarly,

$$iT^2\langle\Phi\rangle = \frac{i}{2}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\begin{pmatrix}0\\ v\end{pmatrix} = \begin{pmatrix}\frac{v}{2}\\ 0\end{pmatrix}$$

and

$$i(g^2T^3 - g'^2Y)\langle\Phi\rangle = \frac{i}{2}\left(g^2\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}0\\ v\end{pmatrix} - g'^2\begin{pmatrix}0\\ v\end{pmatrix}\right) = \begin{pmatrix}0\\ -i\frac{v}{2}(g^2+g'^2)\end{pmatrix}$$

tells us that the Goldstones eaten by W^2 and Z^0 are ϕ_1 and ϕ_4 respectively.

Now, of course, we prefer to express the mass eigenstates $W^{1,2}$ as the normalized, complex charge eigenstate fields $W^\pm = (W^1 \mp iW^2)/\sqrt{2}$. This implies that the Goldstone eaten by $W^\pm$ is

$$G^\pm = (\phi_2 \mp i\phi_1)/\sqrt{2}.$$

Consulting Eqn. (1.19), we see that the results agree.

Finally, we will make one observation which will speed up finding the generators corresponding to the physical gauge bosons, as per Eqn. (1.20). The covariant derivative contains terms of the form complex unity times a gauge coupling, a generator and a gauge field:

$$D_\mu \supset iT^a W^a, \tag{1.21}$$

where $T^a = g^a t^a$ (no sum); g^a and t^a are the corresponding gauge coupling and generator (in some representation), respectively, to the (gauge eigenstate) gauge boson W^a. We wish to express this in terms of physical gauge fields $\widehat{W}$. Let the rotation into physical eigenstates be achieved by

$$\widehat{W}^a = R^{ab} W^b$$

where R is an orthogonal[4] matrix. Then, the gauge eigenstates are

$$W^b = (R^T)^{ba} \widehat{W}^a = R^{ab} \widehat{W}^a$$

which implies (renaming a, b)

$$W^a = R^{ba} \widehat{W}^b.$$

Plugging this into Eqn. (1.21), we find that

$$D_\mu \supset i\widehat{T}^b \widehat{W}^b,$$

where now $\widehat{T}^b = R^{ba} T^a$ (recalling that $T^a = g^a t^a$ without summing). Thus, the generators mix exactly in the same way as the gauge bosons, only multiplied by the gauge coupling. In other words, to find out how the generators mix, we need only look at how the gauge bosons mix into physical states and replace the gauge eigenstate gauge fields by the gauge coupling times the corresponding generator.

As an example, consider the Standard Model Z boson. We know from the gauge boson mass spectrum that

$$Z^0_\mu = \frac{1}{\sqrt{g^2 + g'^2}} (g W^3_\mu - g' B_\mu).$$

Then, according to our prescription, the generator corresponding to this physical gauge boson should be

$$T_Z = \frac{1}{\sqrt{g^2 + g'^2}} (g(gT^3) - g'(g'Y)).$$

We can extract the normalization factor $1/\sqrt{g^2 + g'^2}$ and call it a coupling, leaving $(g^2 T^3 - g'^2 Y)$ as the generator, which is exactly what we found in Eqn. (1.20).

[4]It can be shown that any real, symmetric matrix can be diagonalized by an orthogonal matrix. The gauge boson mass matrix is real since its elements come from the square of the gauged scalar kinetic terms and obviously symmetric.

1.5 SARAH

SARAH [10, 11] is a Mathematica [17] package which calculates masses, mixings and vertices for gauge theories (supersymmetric and non-supersymmetric). It can also give tadpole equations and the renormalization group equations (RGEs) for all parameters up to the two-loop level. The model of choice is input by writing and loading so-called *model, particle* and *parameter* files. These contain the necessary information about the model structure: The model file encodes gauge symmetries; particle content and representations in the different eigenbases; superpotential (for SUSY theories) or Lagrangian (for non-SUSY models). The particle and parameter files, meanwhile, contain less crucial information regarding the particles and parameters of the model, such as output names and descriptions.

SARAH currently comes with a selection of around 50 models already constructed, most of which are extensions of the SM. If one wishes to use SARAH to calculate features of a model not included by default, constructing a model file manually is mostly straightforward. We will give a brief overview of the required steps in the following section.

1.5.1 The anatomy of a SARAH model file

In order to make this thesis somewhat self-contained, we will give a short outline of the required sections of a non-supersymmetric model file. For detailed instructions, we obviously refer to the SARAH manual [11].

Gauge groups. First, the gauge group of the theory is typically specified as a product of $U(1)$ and $SU(N)$ groups. For each such group, quantum numbers (such as hypercharge or colour) and dimension should be given. We can also choose which group indices should be expanded over in calculations. The gauge bosons for each group are added automatically.

Matter fields. Next, we must write the (Weyl spinor) fermions and scalars of the theory, including their charges under each gauge group.

Scalar potential and Yukawa Lagrangian. These two parts of the model's Lagrangian must be manually supplied. SARAH understands how to build invariants out of the scalar and fermion fields, since their representations are known, and as such can contract the indices without them having to be written out explicitly. For example, SARAH would understand a term such as $HH^\dagger$ for the SM Higgs, since there is only one way of contracting the (implicit) indices; as an $SU(2)$ product. However, in the event that there are several ways of constructing a gauge invariant, the desired contraction must be written out explicitly in the model file.

Vacuum structure. The vacuum structure must be supplied for all scalars which obtain a nonzero VEV.

Gauge and matter sector mixing. While SARAH can calculate the mixing matrices, we must tell the program which particles actually mix. This is done by supplying the gauge eigenstates that mix, and defining mass eigenstate and mixing matrix variables. SARAH then computes the latter. This must be done for fermions and scalars as well as for the gauge bosons.

Dirac/Weyl spinor structure. Lastly, we must input the way in which Dirac spinors should be constructed from the Weyl fields previously defined.

Examples of model files are included, as mentioned, in the SARAH package download [11]. Our model file for the MLRM is included, for reference, in Appendix A.

2 The minimal left-right-symmetric model

Left-right-symmetric models containing the gauge group $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ have been studied extensively since the 1970's [6–8, 18]. The extended symmetry in the electroweak sector leads to new phenomenology and several attractive theoretical features, not least as an intermediate effective theory between the SM and some higher-scale unified theory. We will consider the so-called Minimal Left-Right-Symmetric Model (MLRM), which is an gauge-group extension of the Standard Model to $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$. We take the transformation between L and R fields to be parity[5], and impose parity invariance before spontaneous breaking down to $U(1)_Q$. The latter is achieved by assigning VEVs to selected components of the triplet and bi-doublet Higgs fields.

We will begin by introducing the gauge group, fermion and gauge boson content in Section 2.1. In Section 2.2 we verify that the model is gauge anomaly-free. Section 2.3 treats the Higgs sector and symmetry breaking. We introduce the Higgs fields, calculate the gauge boson mass spectrum, and identify the Goldstones associated with breaking the MLRM gauge group to the SM. In Section 2.4 we write the scalar potential and, having solved the tadpole equations, obtain the scalar mass spectrum. Section 2.5 contains an analysis of the Yukawa sector. Furthermore, the entire Lagrangian is then put into the physical basis and presented in 2.6. Finally, we give a brief phenomenological overview and a summary in Sections 2.7 and 2.8, respectively.

All calculations have been performed in Mathematica. An MLRM model file was created for use with SARAH, and the results of the manual calculations were used to verify the correct construction of the model file. All results agree between the methods. There is also complete agreement with the literature in all results unless noted otherwise.

2.1 Fermion and vector particle content

2.1.1 Fermions

The fermion fields are assigned to the doublets

$$L_L^i = \begin{pmatrix} \nu \\ e \end{pmatrix}_L^i, \quad L_R^i = \begin{pmatrix} \nu \\ e \end{pmatrix}_R^i, \quad Q_L^i = \begin{pmatrix} u \\ d \end{pmatrix}_L^i, \quad Q_R^i = \begin{pmatrix} u \\ d \end{pmatrix}_R^i.$$

where i is a family index. These represent, respectively, $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ as

$$(\mathbf{2}, \mathbf{1}, -\frac{1}{2}), \quad (\mathbf{1}, \mathbf{2}, -\frac{1}{2}), \quad (\mathbf{1}, \mathbf{2}, \frac{1}{6}), \quad (\mathbf{2}, \mathbf{1}, \frac{1}{6}). \tag{2.1}$$

Thus, left- (right-) handed fermions occupy left-handed doublets (singlets) and right-handed singlets (doublets). The third component of "left-handed weak isospin" T_L^3, acting as the charge of $SU(2)_L$, is straightforwardly given a "right-handed" analogue in T_R^3.

[5]It is also possible to define charge conjugation as the transformation that takes fields from L to R.

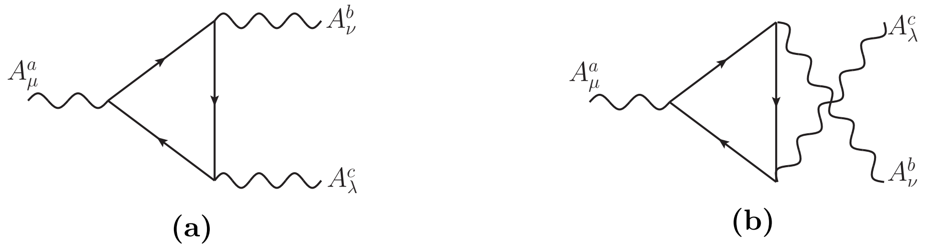

Figure 2.1: *The triangle diagrams which spoil gauge invariance for chiral theories [12].*

The generator of $U(1)_{B-L}$ is simply the half the difference in baryon and lepton number, $(B - L)/2$. The factor $1/2$ is a matter of convention. When this gauge group is supplemented by (unbroken) $SU(3)_C$, colour charge is assigned as per the SM.

2.1.2 Gauge bosons

The gauge boson spectrum (before breaking) is also a simple extension of the SM: We have two triplets,

$$W_L^{i\mu} = \begin{pmatrix} W^{1\mu} \\ W^{2\mu} \\ W^{3\mu} \end{pmatrix}_L, \quad W_R^{i\mu} = \begin{pmatrix} W^{1\mu} \\ W^{2\mu} \\ W^{3\mu} \end{pmatrix}_R,$$

the first transforming as the adjoint (singlet) under $SU(2)_{L(R)}$, and vice versa for the second. Additionally, a $U(1)_{B-L}$ gauge boson exists, called B^μ in analogy with the SM.

Then, the gauge-covariant derivative is

$$D_\mu = \partial_\mu - ig_L W^i_{L\mu} T^i_L - ig_R W^i_{R\mu} T^i_R - ig_{B-L} \frac{(B - L)}{2} B_\mu \tag{2.2}$$

where $T_{L,R}$ are just the Pauli matrices in the fundamental representation.

Furthermore, $g_L = g_R$ is commonly enforced if a parity-symmetric theory is desired. We will indeed make this identification in the following.

2.2 Gauge anomaly cancellation

It can be shown that diagrams of the type shown in Fig. 2.1, acting as corrections to the three-gauge boson couplings, may spoil gauge invariance in chiral theories [12]. In addition, there is similar, gravitational, anomalies may occur when gravitons replace the gauge bosons at the vertices [12]. Thus, we must ensure that, as in the SM, all contributions from such diagrams cancel. We will see how this happens below.

The diagram in Fig. 2.1 is proportional to the group-theoretic expression

$$\mathrm{tr}[(-1)^\chi T^a \{T^b, T^c\}], \tag{2.3}$$

where the T's are the generators of the corresponding gauge currents (see Fig. 2.1) [12]. The factor $(-1)^\chi$ (denoted γ^5 in [12]) is equal to -1 ($+1$) for left- (right-) handed fermions,

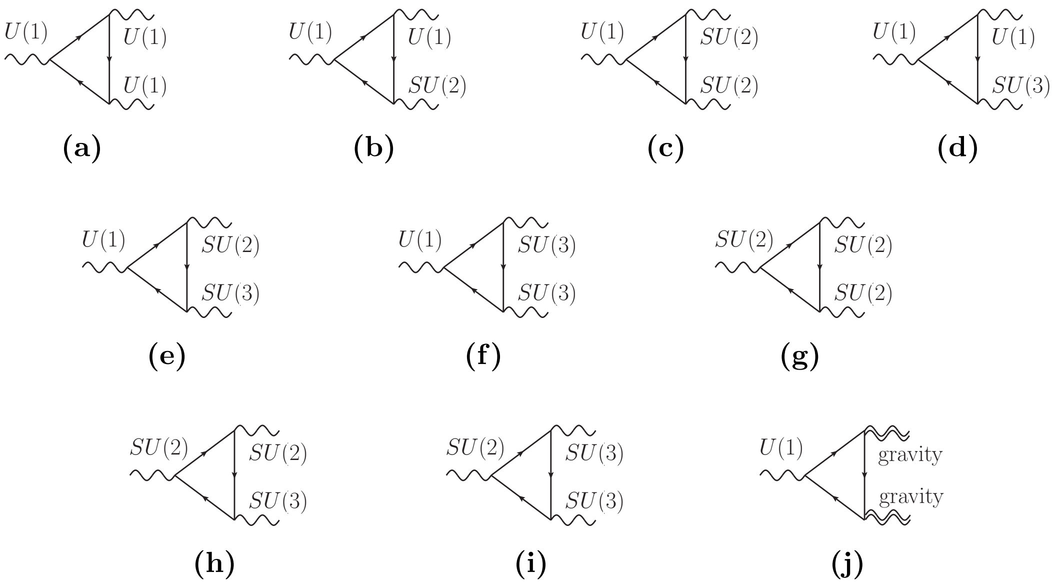

Figure 2.2: *The complete set of potentially anomalous triangle diagrams.*

while the commutator simply expresses the need to count diagrams with fermions running in both directions in the loops.

Diagrams that couple three bosons of non-chiral (left-right-symmetric) interactions, i.e. gravitons or gluons, do not contribute. So, for our theory of $SU(3)_C \otimes SU(2)_R \otimes SU(2)_L \otimes U(1)_{B-L}$, the potentially troublesome diagrams for consideration are those in Fig. 2.2. We will address them separately, essentially following Section 20.2 in Ref. [12].

Any diagram coupling exactly one $SU(2)$ or $SU(3)$ current to any others are proportional to the trace of one Pauli or Gell-Mann matrix (since the trace in Eqn. (2.3) is over a tensor product it separates: $\operatorname{tr} A \otimes B = \operatorname{tr} A \operatorname{tr} B$). These are Lie algebra generators and thus traceless, from which it follows that no such diagram (Fig. 2.2 (b), (d), (e), (h), (i)) can contribute.

Meanwhile, the diagram in Fig. 2.2(g) is also group-theoretically trivial, since the $SU(2)$ generators satisfy $\{\sigma^a, \sigma^b\} \propto \delta^{ab}$, implying again that $A_g \propto \operatorname{tr}[\sigma^c] = 0$, where, in the notation we now adopt, A_x is the amplitude of the diagram in Fig. 2.2(x).

The remaining, nontrivial amplitudes are thus A_a, A_c, A_f and A_j. In the case of A_c, the two $SU(2)$ currents may be either both $SU(2)_L$ or both $SU(2)_R$. The diagram connecting $SU(2)_L$ and $SU(2)_R$ to $U(1)$ is not a physical process since no fermion couples to both currents $SU(2)_L$ and $SU(2)_R$ currents.

The $(U(1)_{B-L})^3$ diagram A_a is simply proportional to $\operatorname{tr}[(\frac{B-L}{2})^3]$, where we interpret the trace as a sum over the quantum number $(B-L)/2$ cubed, for all fermions. We must also keep in mind that left-handed fermions enter with a minus sign due to the $(-1)^\chi$ factor. Then we have

$$A_a \propto \operatorname{tr}\left[\left(\frac{B-L}{2}\right)^3\right] = -2\left(-\frac{1}{2}\right)^3 + 2\left(-\frac{1}{2}\right)^3 - 3\cdot 2\left(\frac{1}{6}\right)^3 + 3\cdot 2\left(\frac{1}{6}\right)^3 = 0$$

according to the assignments in Eqn. (2.1).

The anomaly A_c, with both $SU(2)$ currents either L or R, is proportional to

$$\mathrm{tr}[\sigma^a \sigma^b (B-L)/2] \propto \delta^{ab} \sum_{f_{L,R}} (B-L)_{f_{L,R}}$$

by properties of the Pauli matrices. The sum runs over only L (R) fermions for the $(SU(2)_L)^2$ $((SU(2)_R)^2)$ diagram. We also count each quark three times for colour, and, as usual, L-fermion contributions are subtracted. Thus,

$$A_c^L \propto \sum_{f_L} \frac{(B-L)_{f_L}}{2} = -2\left(-\frac{1}{2}\right) - 3 \cdot 2\left(\frac{1}{3}\right) = 0$$

and

$$A_c^R \propto \sum_{f_R} \frac{(B-L)_{f_R}}{2} = 2\left(-\frac{1}{2}\right) + 3 \cdot 2\left(\frac{1}{6}\right) = 0.$$

For A_f, which couples two $SU(3)_C$ currents to $U(1)_{B-L}$, we have

$$\mathrm{tr}[\lambda^a \lambda^b (B-L)/2] = \frac{1}{4}\delta^{ab} \sum_q (B-L)_q \propto -3 \cdot 2\left(\frac{1}{6}\right) + 3 \cdot 2\left(\frac{1}{6}\right) = 0,$$

where, naturally, only quarks run in the loops.

Finally, the A_j diagram is proportional to the trace over the $(B-L)/2$ matrix,

$$A_j \propto \mathrm{tr}[(B-L)/2] = -2\left(\frac{1}{2}\right) + 2\left(\frac{1}{2}\right) - 3 \cdot 2\left(\frac{1}{6}\right) + 3 \cdot 2\left(\frac{1}{6}\right) = 0.$$

Thus, we conclude that there exist no chiral gauge anomalies or gravitational anomalies in the theory. The analysis done in SARAH confirms this result.

2.3 Higgs sector and symmetry breaking

2.3.1 Higgs fields and vacuum structure

We wish, as in the SM, to spontaneously break $SU(2)_L \otimes U(1)_Y \to U(1)_Q$. Thus, we must first break $SU(2)_R \otimes U(1)_{B-L} \to U(1)_Y$, requiring an extended Higgs sector. For phenomenological reasons we require that this happens at a higher scale. This is to ensure a high mass for the new (apparently hidden) W_R, Z_R bosons; we discuss this further in Section 2.7.

The breaking of the right-handed symmetry can, in principle, be performed by introducing two new Higgs doublets [19]. However, we instead introduce two triplet Higgs fields, conveniently represented by

$$T_L^i \delta_L^i \equiv \Delta_L = \begin{pmatrix} \delta_L^+/\sqrt{2} & \delta_L^{++} \\ \delta_L^0 & -\delta_L^+/\sqrt{2} \end{pmatrix}, \; T_R^i \delta_R^i \equiv \Delta_R = \begin{pmatrix} \delta_R^+/\sqrt{2} & \delta_R^{++} \\ \delta_R^0 & -\delta_R^+/\sqrt{2} \end{pmatrix}. \tag{2.4}$$

This is done to open the possibility of Majorana mass terms for the right-handed neutrinos via the see-saw mechanism [20] (see Section 2.7). These fields transform as the representations $(\mathbf{3}, \mathbf{1}, 1)$ and $(\mathbf{1}, \mathbf{3}, 1)$, respectively, of the $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ gauge group.

In order to spontaneously break $SU(2)_L \otimes U(1)_Y$ down to $U(1)_Q$, we introduce the bi-doublet

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix},$$

belonging to the $(\mathbf{2}, \mathbf{2^*}, 0)$ representation[6].

These representational assignments mean that the Higgs fields transform under $U_{L,R} \in SU(2)_{L,R}$ as

$$\begin{aligned} \Phi &\to U_L \Phi U_R^\dagger \\ \Delta_L &\to U_L \Delta_L U_L^\dagger \\ \Delta_R &\to U_R \Delta_R U_R^\dagger \end{aligned} \tag{2.5}$$

In order to spontaneously break the symmetries, as we will see in the next section, we must chose appropriate Φ and Δ vacuum structure. Since we wish to end up with $U(1)_Q$ unbroken, the neutral Higgs field components gain VEVs[7]. We define them by

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} k e^{i\alpha_k} & 0 \\ 0 & k' e^{i\alpha_{k'}} \end{pmatrix},$$
$$\langle \Delta_L \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_L e^{i\beta_L} & 0 \end{pmatrix}, \quad \langle \Delta_R \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_R e^{i\beta_R} & 0 \end{pmatrix}.$$

Since they are in general complex, there are four phases. However, we may gauge-transform away two of them. This owes to the transformation properties in Eqn. (2.5): The vacua are invariant (by construction) under the $U(1)_Q$ transformation. The EM generator is $Q = T_R^3 + T_L^3 + (B-L)/2$, as we will find in Section 2.3.4 (Eqn. (2.11)). Let us then consider the three operators that commute with Q; $T_{L,R}^3$ and $(B-L)/2$. Under these gauge transformations, the vacua transform as

$$\langle \Phi \rangle \to e^{iT_L^3 \omega_L} \langle \Phi \rangle e^{-iT_R^3 \omega_R},$$
$$\langle \Delta_L \rangle \to e^{i\omega_{B-L}} e^{iT_L^3 \omega_L} \langle \Delta_L \rangle e^{-iT_L^3 \omega_L},$$
$$\langle \Delta_R \rangle \to e^{i\omega_{B-L}} e^{iT_R^3 \omega_L} \langle \Delta_R \rangle e^{-iT_R^3 \omega_R},$$

where we have plugged in $(B-L)/2 = 1$ for the triplet fields. This implies that the phases transform as

$$\alpha_k \to \alpha_k - \frac{1}{2}\omega_L + \frac{1}{2}\omega_R,$$

[6]Note that for $SU(2)$, $\mathbf{2^*}$=$\mathbf{2}$.

[7]If the vacuum is not invariant under $U(1)_Q$, we do not end up with EM charge conservation or massless photons.

$$\alpha_{k'} \to \alpha_{k'} + \frac{1}{2}\omega_L - \frac{1}{2}\omega_R,$$
$$\beta_L \to \beta_L + \omega_{B-L} - \omega_L,$$
$$\beta_R \to \beta_R + \omega_{B-L} - \omega_R.$$

Since two independent combinations of the parameters appear in the above transformations, two phases can be rotated away [21] . Thus, we remove two phases, conventionally leaving

$$\begin{aligned} \langle\Phi\rangle &= \frac{1}{\sqrt{2}}\begin{pmatrix} k & 0 \\ 0 & k' e^{i\alpha_{k'}} \end{pmatrix}, \\ \langle\Delta_L\rangle &= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 \\ v_L e^{i\beta_L} & 0 \end{pmatrix}, \quad \langle\Delta_R\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}. \end{aligned} \tag{2.6}$$

There must exist a distinct hierarchy between the VEVs of the Higgs fields, both in order to hide the experimentally unobserved gauge bosons associated with $SU(2)_R$, and to match other phenomenology (see Section 2.7); we require $v_R \gg k, k' \gg v_L$, with $\sqrt{k^2 + k'^2}$ lying around the EW scale. The R-breaking VEV v_R is usually taken to be at least 10^{10} GeV [20].

The gauged Higgs sector Lagrangian, without the scalar potential (treated in Section 2.4), is simply

$$\begin{aligned} \mathscr{L}_{\text{Higgs}} = \mathrm{Tr}[(D_\mu\Delta_L)^\dagger(D^\mu\Delta_L)] + \mathrm{Tr}[(D_\mu\Delta_R)^\dagger(D^\mu\Delta_R)] + \\ + \mathrm{Tr}[(D_\mu\Phi)^\dagger(D^\mu\Phi)] \end{aligned} \tag{2.7}$$

with D_μ is given by Eqn. (2.2).

2.3.2 Gauge boson masses

As in the SM, the gauge boson mass eigenstates do not coincide with the weak eigenstates. Acting on the Higgs multiplets with the covariant derivative we find

$$D_\mu\Delta_{L,R} = \partial_\mu\Delta_{L,R} - \frac{ig}{2}\left[W^i_{L,R\mu}\sigma^i, \Delta_{L,R}\right] - ig_{B-L}B_\mu\Delta_{L,R},$$

where we have set $g_L = g_R \equiv g$, and plugged in $B - L = 2$ for the Δ fields, and

$$D_\mu\Phi = \partial_\mu\Phi - \frac{ig}{2}\left(W^i_{L\mu}\sigma^i\Phi - \Phi W^i_{R\mu}\sigma^i\right).$$

Using the above in the gauged Lagrangian (2.7), and evaluating at the vacuum with the Higgs VEVs (2.6), we obtain, after some algebra, the W mass term

$$\mathscr{L}^{W^\pm}_{\text{mass}} = \begin{pmatrix} W^-_L & W^-_R \end{pmatrix} \begin{pmatrix} \frac{1}{4}g^2(k^2 + k'^2 + 2v_L^2) & -\frac{1}{2}g^2 kk' e^{-i\alpha_{k'}} \\ -\frac{1}{2}g^2 kk' e^{i\alpha_{k'}} & \frac{1}{4}g^2(k^2 + k'^2 + 2v_R^2) \end{pmatrix} \begin{pmatrix} W^+_L \\ W^+_R \end{pmatrix}.$$

We use the ubiquitous notation $W^{\pm}_{L,R} = \frac{1}{\sqrt{2}}(W^1_{L,R} \mp iW^2_{L,R})$, suppressing spacetime indices from now on. We also rename g_{B-L} to g' and introduce $k^2_{\pm} = k^2 \pm k'^2$ for notational brevity.

Clearly, since the mass matrix is not diagonal, the $W^{\pm}_{L,R}$ states are not mass eigenstates. Diagonalizing and letting $W^{\pm}_{1,2}$ denote the mass eigenstates, we find

$$M^2_{W_{1,2}} = \frac{g^2}{4}\left[k^2 + k'^2 + v_L^2 + v_R^2 \mp \sqrt{(v_L^2 - v_R^2)^2 + 4k^2k'^2}\right].$$

Invoking the hierarchy $v_R \gg k, k' \gg v_L$, mentioned above, we find

$$\begin{aligned} M^2_{W_1} &\approx \frac{g^2}{4}k^2_+\left(1 - \frac{2k^2k'^2}{k^2_+v_R^2}\right), \\ M^2_{W_2} &\approx \frac{g^2}{2}v_R^2. \end{aligned}$$

Note that we have expressed the W_1 mass-squared in the form of the correct SM expression plus as small correction. In other words, the gauge eigenstates are also approximate mass eigenstates, and since the expressions for currents, etc., are simpler in the $W^{\pm}_{L,R}$ basis, it is often used. The mixing is given by

$$\begin{pmatrix} W^{\pm}_L \\ W^{\pm}_R \end{pmatrix} = \begin{pmatrix} \cos\zeta & -\sin\zeta e^{i\lambda} \\ \sin\zeta e^{-i\lambda} & \cos\zeta \end{pmatrix}\begin{pmatrix} W^{\pm}_1 \\ W^{\pm}_2 \end{pmatrix}.$$

The phase is $\lambda = -\alpha_{k'}$ (which is often set to 0, see the following sections) while the mixing angle is given by [21]

$$\tan\zeta = -\frac{kk'}{v_R^2},$$

which is clearly suppressed.

As mentioned above, the W^3's mix with the B field to produce $Z_{L,R}$ and γ, in analogy with the SM. The mass part of the Lagrangian is,

$$\mathscr{L}^{W^0_{L,R},B}_{\text{mass}} = \frac{1}{2}\begin{pmatrix} W^3_{L\mu} & W^3_{R\mu} & B_\mu \end{pmatrix} M_0^2 \begin{pmatrix} W^{3\mu}_L \\ W^{3\mu}_R \\ B^\mu \end{pmatrix}$$

where the mass matrix is

$$M_0^2 = \begin{pmatrix} \frac{g^2}{4}(k^2_+ + 4v_L^2) & -\frac{g^2}{4}k^2_+ & -gg'v_L^2 \\ -\frac{g^2}{4}k^2_+ & \frac{g^2}{4}(k^2_+ + 4v_R^2) & -gg'v_R^2 \\ -gg'v_L^2 & -gg'v_R^2 & g'^2(v_L^2 + v_R^2) \end{pmatrix}.$$

Again, diagonalizing in the $v_L \to 0$ limit, we obtain the mass eigenvalues

$$\begin{aligned} M_A &= 0, \\ M^2_{Z_{1,2}} &= \frac{1}{4}\left[g^2(k^2_+ + 2v_R^2) + 2g'^2v_R^2 \mp \sqrt{g^4k^4_+ + 4v_R^4(g^2 + g'^2)^2 - 4g^2g'^2v_R^2k^2_+}\right]. \end{aligned}$$

Again in the limit of v_R large, the latter two are

$$
\begin{aligned}
M_{Z_1}^2 &\approx \frac{k_+^2 g^2}{4\cos^2\theta_W}\left(1-\frac{k_+^2}{4v_R^2\cos^4\theta_Y}\right),\\
M_{Z_2}^2 &\approx g^2 v_R^2.
\end{aligned}
$$

where we have pre-emptively plugged in the expressions introduced below for the mixing angles. Once again, the Z_1 mass is equal to the SM value plus a small correction. The mass matrix is diagonalized by the transformation (in the $v_R \gg k_+$ limit)

$$
\begin{pmatrix} A \\ Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} s_W & c_W s_Y & c_W c_Y \\ -c_W & s_W s_Y & s_W c_Y \\ 0 & -c_Y & s_Y \end{pmatrix} \begin{pmatrix} W_L^3 \\ W_R^3 \\ B \end{pmatrix}
$$

where the mixing angles are defined via

$$
\begin{aligned}
s_W \equiv \sin\theta_W &= \frac{g'}{\sqrt{g^2+2g'^2}},\\
c_W \equiv \cos\theta_W &= \sqrt{\frac{g^2+g'^2}{g^2+2g'^2}},\\
s_Y \equiv \sin\theta_Y &= \frac{g'}{\sqrt{g^2+g'^2}},\\
c_Y \equiv \cos\theta_Y &= \frac{g}{\sqrt{g^2+g'^2}}.
\end{aligned} \tag{2.8}
$$

Thus, the weak eigenstates expressed in physical fields are

$$
\begin{aligned}
W_L^0 &= s_W A - c_W Z_1,\\
W_R^0 &= c_W s_Y A + s_W s_Y Z_1 - c_Y Z_2,\\
B &= c_W c_Y A + s_W c_Y Z_1 + s_Y Z_2.
\end{aligned} \tag{2.9}
$$

The mixing can be thought of as occurring in two steps: first, the neutral components of the W field mix with B to produce the two weak eigenstate fields $Z_{L,R}$ and the photon A. Then, the weak eigenstate $Z_{L,R}$ mix to produce the mass eigenstates $Z_{1,2}$

Furthermore, the hierarchy $v_R \gg k_+$ means that the latter mixing is small, and, as with the charged bosons, the electroweak eigenstates $Z_{L,R}$ (where $Z_L = Z$ is identified with the Standard Model gauge boson) are almost mass eigenstates.

2.3.3 $SU(2)_R \otimes U(1)_{B-L} \to U(1)_Y$ Goldstone bosons

We now wish to explicitly find the Goldstone modes associated with the breaking of the MLRM gauge group down to the Standard Model. The strategy is to find the generators which are spontaneously broken and apply them to the vacuum to find the Goldstone directions, as demonstrated in Section 1.4.

We will first consider the breaking of $SU(2)_R \otimes U(1)_{B-L}$ to $U(1)_Y$ only, taking $k = k' = v_L = 0, v_R \neq 0$. This will allow us to find the three Goldstones eaten by the new, heavy $W_R^\pm, Z_2$ gauge bosons. While the groups involved in this breaking look the same as those involved in the SM symmetry breaking, an important difference is that the vacuum Δ_R now transforms as the adjoint under $SU(2)_R$. The vacuum triplet $(\delta_1, \delta_2, \delta_3)$ is usually put in the form (2.4):

$$\Delta_R = \delta_R^i \sigma^i/2 = \begin{pmatrix} \delta_R^+/\sqrt{2} & \delta_R^{++} \\ \delta_R^0 & -\delta_R^+/\sqrt{2} \end{pmatrix}.$$

Thus, the components are

$$\begin{aligned} \delta_R^1 &= \delta_R^{++} + \delta_R^0, \\ \delta_R^2 &= i(\delta_R^{++} - \delta_R^0), \\ \delta_R^3 &= \sqrt{2}\delta_R^+. \end{aligned} \tag{2.10}$$

The vacuum expectation values are $\langle \delta_R^0 \rangle = v_R/\sqrt{2}$ and $\langle \delta_R^+ \rangle = \langle \delta_R^{++} \rangle = 0$.

We begin by finding the broken generator. Requiring the vacuum $\langle \Delta_R \rangle$ does not change under the transformation,

$$\delta \langle \delta_R^a \rangle = -\epsilon^{abc} \omega_R^b \delta_R^c + i \frac{(B-L)_{\delta_R}}{2} \omega_{B-L} \delta_R^a \stackrel{!}{=} 0.$$

The first and second terms encode the behaviour of the δ_R components under infinitesimal $SU(2)_R$ and $U(1)_{B-L}$ transformations respectively. The $B-L$ charge is 2 for all the δ_R's. Solving these equations for each a and real ω's, we obtain

$$\begin{aligned} \omega_R^1 &= \omega_R^2 = 0, \\ \omega_R^3 &= \omega_{B-L} \equiv \omega. \end{aligned}$$

In other words, the gauge transformation under which the vacuum is invariant is generated by $Y \equiv T_R^3 + \frac{B-L}{2}$, where T_R^3 is the third generator of $SU(2)_R$, represented in the adjoint by $-i\epsilon^{3bc}$.

Using the prescription developed in Section 1.4, we can find the broken generators by considering the gauge boson mixing. Considering only the breaking $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \to SU(2)_L \otimes U(1)_Y$, the only massive gauge bosons come from the mixing of the $SU(2)_R$ fields W_R with the $U(1)_{B-L}$ field B as

$$\begin{pmatrix} W_R^1 \\ W_R^2 \\ A_R \\ Z_R \end{pmatrix} = \frac{1}{\sqrt{g^2+g'^2}} \begin{pmatrix} \sqrt{g^2+g'^2} & 0 & 0 & 0 \\ 0 & \sqrt{g^2+g'^2} & 0 & 0 \\ 0 & 0 & g' & g \\ 0 & 0 & -g & g' \end{pmatrix} \begin{pmatrix} W_R^1 \\ W_R^2 \\ W_R^3 \\ B_{B-L} \end{pmatrix}$$

where g (g') is the $SU(2)$ ($U(1)$) gauge coupling. The field A_R, which is massless, corresponds, according to our prescription, to the generator

$$\frac{gg'}{\sqrt{g^2+g'^2}} \left(T_R^3 + \frac{B-L}{2} \right).$$

This is just proportional to the hypercharge, an unbroken symmetry (since the gauge field remains massless), as we have already seen. As a sanity check, let us confirm this result by computing the result of this operator acting on the vacuum:

$$i\left(T_R^3+\frac{B-L}{2}\right)\frac{v_R}{\sqrt{2}}\begin{pmatrix}1\\-i\\0\end{pmatrix}\propto\left(\begin{pmatrix}0&1&0\\-1&0&0\\0&0&0\end{pmatrix}+i\right)\begin{pmatrix}1\\-i\\0\end{pmatrix}=0,$$

as we suspected.

The three broken generators are then T_R^1, T_R^2 and $(-g^2T_R^3+g'^2\frac{B-L}{2})$. To find the Goldstone modes, we apply these generators to the vacuum. From (2.10), we see that the vacuum is

$$\left\langle\begin{pmatrix}\delta_R^1\\\delta_R^2\\\delta_R^3\end{pmatrix}\right\rangle=\frac{v_R}{\sqrt{2}}\begin{pmatrix}1\\-i\\0\end{pmatrix}.$$

As mentioned, T_R^3 is represented by $-i\epsilon^{3bc}$ in the adjoint. Applying i times this generator to the vacuum, we find

$$\begin{aligned}i\left(-g^2T_R^3+g'^2\frac{B-L}{2}\right)\frac{v_R}{\sqrt{2}}\begin{pmatrix}1\\-i\\0\end{pmatrix}&=\frac{v_R}{\sqrt{2}}\left(-g^2\begin{pmatrix}0&1&0\\-1&0&0\\0&0&0\end{pmatrix}+ig'^2\right)\begin{pmatrix}1\\-i\\0\end{pmatrix}\\&=\frac{v_R(g^2+g'^2)}{\sqrt{2}}\begin{pmatrix}i\\1\\0\end{pmatrix}.\end{aligned}$$

The Goldstone mode corresponding to this broken generator, and eaten by Z_R, is then proportional to $(\operatorname{Im}\delta_R^1+\operatorname{Re}\delta_R^2)$, or, using Eqn. (2.10) and normalizing, $\operatorname{Im}\delta_R^0$.

The two Goldstones corresponding to $W_R^{1,2}$ are found from

$$ig\begin{pmatrix}0&0&0\\0&0&-i\\0&i&0\end{pmatrix}\frac{v_R}{\sqrt{2}}\begin{pmatrix}1\\-i\\0\end{pmatrix}=\frac{gv_R}{\sqrt{2}}\begin{pmatrix}0\\0\\i\end{pmatrix}$$

and

$$ig\begin{pmatrix}0&0&i\\0&0&0\\-i&0&0\end{pmatrix}\frac{v_R}{\sqrt{2}}\begin{pmatrix}1\\-i\\0\end{pmatrix}=\frac{gv_R}{\sqrt{2}}\begin{pmatrix}0\\0\\1\end{pmatrix}$$

to be $\operatorname{Im}\delta_R^3=\sqrt{2}\operatorname{Im}\delta_R^+$ and $\operatorname{Re}\delta_R^3=\sqrt{2}\operatorname{Re}\delta_R^+$, respectively. In complete analogy to the SM, the $W_R^{1,2}$ mass eigenstates are put into charge eigenstates $W_R^\pm=\frac{1}{\sqrt{2}}(W_R^1\mp W_R^2)$ (which are approximate mass eigenstates; see Section 2.3.2). Thus, calling the Goldstone eaten by $W_R^\pm$ $G_R^\pm$, we have

$$\begin{aligned}G_R^+&=\operatorname{Im}\delta_R^+-i\operatorname{Re}\delta_R^+,\\G_R^-&=\operatorname{Im}\delta_R^++i\operatorname{Re}\delta_R^+.\end{aligned}$$

As in the SM case discussed in Section 1.4, we are also free to rotate the Goldstone fields by a phase; the Lagrangian is invariant. A prudent choice is $e^{\pm i\pi/2}$, yielding the redefined Goldstones

$$\begin{aligned} G_R^+ &= \mathrm{Re}\,\delta_R^+ + i\,\mathrm{Im}\,\delta_R^+ = \delta_R^+, \\ G_R^- &= \mathrm{Re}\,\delta_R^+ - i\,\mathrm{Im}\,\delta_R^+ = \delta_R^-. \end{aligned}$$

2.3.4 $SU(2)_L \otimes U(1)_Y \to U(1)_Q$ Goldstone bosons

Next, let us consider the breaking of $SU(2)_L \otimes U(1)_Y \to U(1)_Q$ by means of assigning the Higgs bi-doublet

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}$$

the VEV

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} k & 0 \\ 0 & k' \end{pmatrix}.$$

As we saw, the hypercharge is given by $Y = T_R^3 + \frac{B-L}{2}$. Let us compute how Φ transforms under this symmetry. We know that, since Φ is in the anti-fundamental representation of $SU(2)_R$, $\Phi \to \Phi U_R^\dagger$ for an $SU(2)_R$ gauge transformation U_R. Since $(B-L)_\Phi = 0$, Φ transforms as

$$\Phi \to \Phi e^{-i\omega_Y T_R^3}$$

under a hypercharge transformation. This is, plugging in the generator T_R^3,

$$\Phi \to \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix} \begin{pmatrix} e^{-i\omega_Y/2} & 0 \\ 0 & e^{+i\omega_Y/2} \end{pmatrix}$$

or

$$\begin{aligned} \phi_1^0 &\to e^{-i\omega_Y \frac{1}{2}} \phi_1^0, \\ \phi_2^- &\to e^{-i\omega_Y \frac{1}{2}} \phi_2^-, \\ \phi_1^p &\to e^{+i\omega_Y \frac{1}{2}} \phi_1^p, \\ \phi_2^0 &\to e^{+i\omega_Y \frac{1}{2}} \phi_2^0. \end{aligned}$$

So, we see that Φ seems to split into two doublets,

$$\Phi_1 = \begin{pmatrix} \phi_1^0 \\ \phi_2^- \end{pmatrix}, \qquad Y = -\frac{1}{2}$$

and

$$\Phi_2 = \begin{pmatrix} \phi_1^+ \\ \phi_2^0 \end{pmatrix}, \qquad Y = +\frac{1}{2}.$$

To identify the broken generators, we need the gauge boson mixings. These come from the Lagrangian terms

$$\mathscr{L} \supset \mathrm{Tr}\,|D_\mu \Phi_1|^2 + \mathrm{Tr}\,|D_\mu \Phi_2|^2,$$

evaluated at the vacuum, where

$$D_\mu \Phi_{1,2} = \left(-\frac{ig}{2} W_L^a \sigma^a \mp \frac{ig'}{2} B_Y \right) \Phi_{1,2}.$$

The mixing is identical to what occurs in the SM:

$$\begin{pmatrix} W_L^1 \\ W_L^2 \\ A \\ Z \end{pmatrix} = \frac{1}{\sqrt{g^2+g'^2}} \begin{pmatrix} \sqrt{g^2+g'^2} & 0 & 0 & 0 \\ 0 & \sqrt{g^2+g'^2} & 0 & 0 \\ 0 & 0 & g' & g \\ 0 & 0 & -g & g' \end{pmatrix} \begin{pmatrix} W_L^1 \\ W_L^2 \\ W_L^3 \\ B_Y \end{pmatrix}$$

Then, the broken generators corresponding to $W_L^{1,2}$ and Z are gT_L^1, gT_L^2 and $(-g^2 T_L^3 + g'^2 Y)$, respectively. The prescription is to apply i times these generators to the vacua

$$\langle \Phi_1 \rangle = \begin{pmatrix} k/\sqrt{2} \\ 0 \end{pmatrix}$$

and

$$\langle \Phi_2 \rangle = \begin{pmatrix} 0 \\ k'/\sqrt{2} \end{pmatrix}.$$

The resultant of these directions in field space are parallel to the Goldstone modes.

Let us first find the Goldstone eaten by Z. We have

$$\begin{aligned} i(-g^2 T_L^3 + g'^2 Y) \begin{pmatrix} k/\sqrt{2} \\ 0 \end{pmatrix} &= i \left(-\frac{g^2}{2\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} - \frac{g'^2}{2\sqrt{2}} \right) \begin{pmatrix} k \\ 0 \end{pmatrix} \\ &= -\frac{g^2+g'^2}{2\sqrt{2}} \begin{pmatrix} ki \\ 0 \end{pmatrix} \end{aligned}$$

and

$$\begin{aligned} i(-g^2 T_L^3 + g'^2 Y) \begin{pmatrix} 0 \\ k'/\sqrt{2} \end{pmatrix} &= i \left(-\frac{g^2}{2\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \frac{g'^2}{2\sqrt{2}} \right) \begin{pmatrix} 0 \\ k' \end{pmatrix} \\ &= -\frac{g^2+g'^2}{2\sqrt{2}} \begin{pmatrix} 0 \\ -k'i \end{pmatrix}. \end{aligned}$$

Adding these directions and normalizing, we find the Goldstone boson

$$\begin{aligned} G_1^0 &= \frac{1}{\sqrt{k^2+k'^2}} (k \operatorname{Im} \phi_1^0 - k' \operatorname{Im} \phi_2^0) \\ &= \operatorname{Im} \left[\frac{1}{\sqrt{k^2+k'^2}} (k\phi_1^0 + k' \phi_2^{0*}) \right] \equiv \operatorname{Im} \phi_-^0. \end{aligned}$$

It remains to find the Goldstones eaten by $W_L^{1,2}$, and, in turn, the EM charge eigenstates $W_L^\pm$. The Goldstone corresponding to W^1, G_L^1 is in the direction of

$$igT_L^1 \langle \Phi \rangle = \frac{ig}{2\sqrt{2}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} k & 0 \\ 0 & k' \end{pmatrix} = \frac{g}{2\sqrt{2}} \begin{pmatrix} 0 & k'i \\ ki & 0 \end{pmatrix},$$

i.e. proportional to $k' \operatorname{Im} \phi_1^+ + k \operatorname{Im} \phi_2^-$. The W^2 Goldstone G_L^2, similarly goes as

$$igT_L^2 \langle \Phi \rangle = \frac{ig}{2\sqrt{2}} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} k & 0 \\ 0 & k' \end{pmatrix} = \frac{g}{2\sqrt{2}} \begin{pmatrix} 0 & k' \\ -k & 0 \end{pmatrix},$$

or is proportional to $k' \operatorname{Re} \phi_1^+ - k \operatorname{Re} \phi_2^-$. Constructing the charge eigenstates $W^\pm = (W^1 \mp iW^2)/\sqrt{2}$, we obtain

$$\begin{aligned} G_L^+ &= \frac{1}{\sqrt{2}}(G_L^1 - iG_L^2) \propto (k' \operatorname{Im} \phi_1^+ + k \operatorname{Im} \phi_2^-) - i(k' \operatorname{Re} \phi_1^+ - k \operatorname{Re} \phi_2^-) \\ &= e^{i\pi/2}(\phi_1^+ - i\phi_2^+) \end{aligned}$$

and

$$\begin{aligned} G_L^- &= \frac{1}{\sqrt{2}}(G_L^1 + iG_L^2) \propto (k' \operatorname{Im} \phi_1^+ + k \operatorname{Im} \phi_2^-) + i(k' \operatorname{Re} \phi_1^+ - k \operatorname{Re} \phi_2^-) \\ &= e^{-i\pi/2}(\phi_1^- - i\phi_2^-). \end{aligned}$$

We have rotated the fields by $e^{\pm i\pi/2}$ in order to agree with the expressions given in Ref. [18]. Thus, the Goldstones eaten by $W_L^\pm$, rotated and properly normalized, are (recalling $k_+ \equiv \sqrt{k^2 + k'^2}$)

$$G_L^\pm = \frac{1}{k_+}(\phi_1^\pm - i\phi_2^\pm).$$

Since we found in the previous section that $Y = T_R^3 + \frac{B-L}{2}$, we now see that the EM charge generator is

$$Q = T_L^3 + Y = T_L^3 + T_R^3 + \frac{B-L}{2}. \tag{2.11}$$

As a final consistency check, let us verify that this operator indeed keeps the vacuum invariant. We find the changes in $\langle \Phi_{1,2} \rangle$ under this transformation to be

$$i(T_L^3 + Y)\langle \Phi_1 \rangle = \left(\frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} - \frac{1}{2} \right) \begin{pmatrix} k/\sqrt{2} \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

and

$$i(T_L^3 + Y)\langle \Phi_2 \rangle = \left(\frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \frac{1}{2} \right) \begin{pmatrix} 0 \\ k'/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

just as we expected.

2.4 Scalar potential and Higgs mass spectrum

In construction of the Higgs potential we will only assume parity invariance (and, of course, renormalizability). In its most general form under these restrictions, the potential is [18]

$$
\begin{aligned}
V(\Delta_L, \Delta_R, \Phi) = & -\mu_1^2 \operatorname{Tr} \Phi^\dagger \Phi - \mu_2^2 \left[\operatorname{Tr} \tilde{\Phi}\Phi^\dagger + \operatorname{Tr} \tilde{\Phi}^\dagger \Phi\right] \\
& - \mu_3^2 \left[\operatorname{Tr} \Delta_L \Delta_L^\dagger + \operatorname{Tr} \Delta_R \Delta_R^\dagger\right] \\
& + \lambda_1 (\operatorname{Tr} \Phi^\dagger \Phi)^2 + \lambda_2 \left[(\operatorname{Tr} \tilde{\Phi}\Phi^\dagger)^2 + (\operatorname{Tr} \tilde{\Phi}^\dagger \Phi)^2\right] \\
& + \lambda_3 \operatorname{Tr} \tilde{\Phi}\Phi^\dagger \operatorname{Tr} \tilde{\Phi}^\dagger \Phi + \lambda_4 \operatorname{Tr} \Phi^\dagger \Phi \left[\operatorname{Tr} \tilde{\Phi}\Phi^\dagger + \operatorname{Tr} \tilde{\Phi}^\dagger \Phi\right] \\
& + \rho_1 \left[(\operatorname{Tr} \Delta_L \Delta_L^\dagger)^2 + (\operatorname{Tr} \Delta_R \Delta_R^\dagger)^2\right] \\
& + \rho_2 \left[\operatorname{Tr} \Delta_L \Delta_L \operatorname{Tr} \Delta_L^\dagger \Delta_L^\dagger + \operatorname{Tr} \Delta_R \Delta_R \operatorname{Tr} \Delta_R^\dagger \Delta_R^\dagger\right] \\
& + \rho_3 \operatorname{Tr} \Delta_L \Delta_L^\dagger \operatorname{Tr} \Delta_R \Delta_R^\dagger \\
& + \rho_4 \left[\operatorname{Tr} \Delta_L \Delta_L \operatorname{Tr} \Delta_R^\dagger \Delta_R^\dagger + \operatorname{Tr} \Delta_L^\dagger \Delta_L^\dagger \operatorname{Tr} \Delta_R \Delta_R\right] \\
& + \alpha_1 \operatorname{Tr} \Phi^\dagger \Phi \left[\operatorname{Tr} \Delta_L^\dagger \Delta_L + \operatorname{Tr} \Delta_R^\dagger \Delta_R\right] \\
& + \left[\alpha_2 e^{i\delta} \left[\operatorname{Tr} \tilde{\Phi}\Phi^\dagger \operatorname{Tr} \Delta_L \Delta_L^\dagger + \operatorname{Tr} \tilde{\Phi}^\dagger \Phi \operatorname{Tr} \Delta_R \Delta_R^\dagger\right] + \text{h.c.}\right] \\
& + \alpha_3 (\operatorname{Tr} \Phi \Phi^\dagger \Delta_L \Delta_L^\dagger + \operatorname{Tr} \Phi^\dagger \Phi \Delta_R \Delta_R^\dagger) \\
& + \beta_1 (\operatorname{Tr} \Phi \Delta_R \Phi^\dagger \Delta_L^\dagger + \operatorname{Tr} \Phi^\dagger \Delta_L \Phi \Delta_R^\dagger) \\
& + \beta_2 (\operatorname{Tr} \tilde{\Phi} \Delta_R \Phi^\dagger \Delta_L^\dagger + \operatorname{Tr} \tilde{\Phi}^\dagger \Delta_L \Phi \Delta_R^\dagger) \\
& + \beta_3 (\operatorname{Tr} \Phi \Delta_R \tilde{\Phi}^\dagger \Delta_L^\dagger + \operatorname{Tr} \Phi^\dagger \Delta_L \tilde{\Phi} \Delta_R^\dagger),
\end{aligned}
\tag{2.12}
$$

where $\tilde{\Phi} = \sigma_2 \Phi^* \sigma_2$ is the charge conjugated field. The parameters $\mu^2_{1,2,3}$, $\lambda_{1,2,3,4}$, $\rho_{1,2,3,4}$, $\alpha_{1,2,3}$ and $\beta_{1,2,3}$ are all real, except α_2 which may be complex, indicated explicitly above by the inclusion of the (CP-violating) phase δ. We will treat the case where there is no explicit or spontaneous CP violation (real scalar potential, $\delta = 0$, and $\alpha_{k'} = 0$); referred to in the literature as the *manifest left-right-symmetric* limit. We define normalized real and imaginary parts of the neutral fields,

$$\phi^0_{1,2} = \frac{1}{\sqrt{2}} (\phi^{0r}_{1,2} + i\phi^{0i}_{1,2})$$

and analogously for $\delta^0_{L,R}$.

The potential is, after spontaneous symmetry breaking, extremal at the VEVs (2.6). This yields six tadpole equations,

$$\frac{\partial V}{\partial \phi_1^{0r}} = \frac{\partial V}{\partial \phi_2^{0r}} = \frac{\partial V}{\partial \delta_R^{0r}} = \frac{\partial V}{\partial \delta_L^{0r}} = \frac{\partial V}{\partial \phi_2^{0i}} = \frac{\partial V}{\partial \delta_L^{0i}} = 0.$$

The first four equalities, evaluated at the VEVs, together imply

$$\begin{aligned}
\mu_1^2 &= [2v_L v_R(\beta_2 k^2 - \beta_3 k'^2) + (v_L^2 + v_R^2)(\alpha_1 k_-^2 - \alpha_3 k'^2)]/(2k_-^2) + k_+^2\lambda_1 + 2kk'\lambda_4, \\
\mu_2^2 &= [v_L v_R(\beta_1 k_-^2 - 2kk'(\beta_2 - \beta_3)) + (v_L^2 + v_R^2)(2\alpha_2 k_-^2 + \alpha_3 kk')]/(4k_-^2) \\
&\quad + kk'(2\lambda_2 + \lambda_3) + \lambda_4 k_+^2/2, \\
\mu_3^2 &= (\alpha_1 k_+^2 + 4\alpha_2 kk' + \alpha_3 k'^2 + 2\rho_1 v_R^2 + 2\rho_1(v_L^2 + v_R^2))/2, \\
\beta_2 &= (-\beta_1 kk' - \beta_3 k'^2 + v_L v_R(2\rho_1 - \rho_3))/k^2. \qquad (2.13)
\end{aligned}$$

Note the last equation: In the scenario $\beta_1 = \beta_2 = \beta_3 = 0$, which we shall justify and adopt later, it reads

$$0 = v_L v_R(\rho_3 - 2\rho_1).$$

This is known as the *VEV see-saw* relation. Clearly either v_L, v_R or $(\rho_3 - 2\rho_1)$ must vanish. We know that v_R must be nonzero to break $SU(2)_R$ and give large mass to W_R, Z_R. The factor $(\rho_3 - 2\rho_1)$ is known to be nonzero due to phenomenology: As we will find, several of the new Higgs bosons have masses proportional (to first order) to $(\rho_3 - 2\rho_1)$. If they are massless, they would open up new Z decay channels with widths comparable to $Z \to \nu\bar{\nu}$ channels [18]. Even with small mass contributions from loop corrections, such extra decays would be easily detectable, and we thus conclude that the only possibility in the $\beta_i = 0$ case is $v_L = 0$.

We will now derive the scalar mass spectrum. First, the mass matrices M_{R}^2, M_{I}^2, M_+^2 and M_{++}^2, in the bases

$$\{\phi_1^{0r}, \phi_2^{0r}, \delta_R^{0r}, \delta_L^{0r}\},$$

$$\{\phi_1^{0i}, \phi_2^{0i}, \delta_R^{0i}, \delta_L^{0i}\},$$

$$\{\phi_1^+, \phi_2^+, \delta_R^+, \delta_L^+\}$$

and

$$\{\delta_R^{++}, \delta_L^{++}\},$$

respectively, are constructed from the bilinear terms resulting from expanding the potential (2.12) around the VEVs (2.6). Then, these matrices are rotated into the flavour-diagonal bases

$$\{\phi_-^{0r}, \phi_+^{0r}, \delta_R^{0r}, \delta_L^{0r}\},$$

$$\{\phi_-^{0i}, \phi_+^{0i}, \delta_R^{0i}, \delta_L^{0i}\},$$

$$\{(k\phi_1^+ + k'\phi_2^+)/k_+, (k\phi_2^+ - k'\phi_1^+)/k_+, \delta_R^+, \delta_L^+\}$$

and

$$\{\delta_L^{++}, \delta_R^{++}\},$$

where

$$\phi_+^0 \equiv \frac{1}{k_+}(-k'\phi_1^0 + k\phi_2^{0*}), \quad \phi_-^0 \equiv \frac{1}{k_+}(k\phi_1^0 + k'\phi_2^{0*})$$

and, in particular,

$$\phi_+^{0r} = \frac{1}{k_+}(-k'\phi_1^{0r} + k\phi_2^{0r}), \quad \phi_-^{0r} = \frac{1}{k_+}(k\phi_1^{0r} + k'\phi_2^{0r}).$$

Plugging in the relations (2.13) allow us to eliminate the LHS parameters. We also take $\beta_i = 0$, following several previous studies [18, 22], in order to avoid fine-tuning among them. This is discussed further in Section 2.7.

Thus, the mass matrices, in the

$$\{\phi_-^{0r}, \phi_+^{0r}, \delta_R^{0r}, \delta_L^{0r}\},$$

$$\{\phi_-^{0i}, \phi_+^{0i}, \delta_R^{0i}, \delta_L^{0i}\},$$
$$\{(k\phi_1^+ + k'\phi_2^+)/k_+, (k\phi_2^+ - k'\phi_1^+)/k_+, \delta_R^+, \delta_L^+\},$$
$$\{\delta_L^{++}, \delta_R^{++}\},$$

bases respectively, are

$$\begin{aligned}
M_{r11}^2 &= 2\lambda_1 k_+^2 + 8k_1^2 k_2^2(2\lambda_2 + \lambda_3)/k_+^2 + 8k_1 k_2 \lambda_4,\\
M_{r12}^2 &= M_{r21}^2 = 4k_1 k_2 k_-^2(2\lambda_2 + \lambda_3)/k_+^2 + 2\lambda_4 k_-^2,\\
M_{r13}^2 &= M_{r31}^2 = \alpha_1 v_R k_+ + k_2 v_R(4\alpha_2 k_1 + \alpha_3 k_2)/k_+,\\
M_{r23}^2 &= M_{r32}^2 = v_R(2\alpha_2 k_-^2 + \alpha_3 k_1 k_2)/k_+,\\
M_{r33}^2 &= 2\rho_1 v_R^2,\\
M_{r14}^2 &= M_{r41}^2 = M_{r42}^2 = M_{r24}^2 = M_{r43}^2 = M_{r34}^2 = 0,\\
M_{r44}^2 &= \frac{v_R^2}{2}(\rho_3 - 2\rho_1),
\end{aligned}$$

$$M_i^2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -2k_+^2(2\lambda_2 - \lambda_3) + \frac{\alpha_3 v_R^2 k_+^2}{2k_-^2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{v_R^2}{2}(\rho_3 - 2\rho_1) \end{pmatrix},$$

$$M_+^2 = \begin{pmatrix} \frac{\alpha_3 k_+^2 v_R^2}{2k_-^2} & 0 & \frac{\alpha_3 v_R k_+}{\sqrt{8}} & 0 \\ 0 & 0 & 0 & 0 \\ \frac{\alpha_3 v_R k_+}{\sqrt{8}} & 0 & \frac{\alpha_3 k_-^2}{4} & 0 \\ 0 & 0 & 0 & \frac{\alpha_3 k_-^2}{2} + \frac{v_R^2}{2}(\rho_3 - 2\rho_1) \end{pmatrix}$$

and

$$M_{++}^2 = \begin{pmatrix} 2\rho_2 v_R^2 + \frac{\alpha_3 k_-^2}{2} & 0 \\ 0 & \frac{\alpha_3 k_-^2}{2} + \frac{v_R^2}{2}(\rho_3 - 2\rho_1) \end{pmatrix}.$$

Following [18], we present these matrices below in a shorthand notation: replacing any parameter $\alpha_{1,2,3}$ by the generic α, $\lambda_{1,2,3,4}$ by λ, and so on. Expanding in the hierarchy $v_R \gg k, k'$ and keeping only the most important terms we find

$$M_r^2 = \begin{pmatrix} \lambda\kappa^2 & \lambda\kappa^2 & \alpha v_R \kappa & 0 \\ \lambda\kappa^2 & \alpha v_R^2 & \alpha v_R \kappa & 0 \\ \alpha v_R \kappa & \alpha v_R \kappa & 2\rho_1 v_R^2 & 0 \\ 0 & 0 & 0 & \frac{v_R^2}{2}(\rho_3 - 2\rho_1) \end{pmatrix},$$

$$M_i^2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \alpha v_R^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{v_R^2}{2}(\rho_3 - 2\rho_1) \end{pmatrix},$$

$$M_+^2 = \begin{pmatrix} \alpha v_R^2 & 0 & \alpha v_R \kappa & 0 \\ 0 & 0 & 0 & 0 \\ \alpha v_R \kappa & 0 & \alpha\kappa^2 & 0 \\ 0 & 0 & 0 & \frac{v_R^2}{2}(\rho_3 - 2\rho_1) \end{pmatrix}$$

and

$$M_{++}^2 = \begin{pmatrix} 2\rho_2 v_R^2 & 0 \\ 0 & \frac{v_R^2}{2}(\rho_3 - 2\rho_1) \end{pmatrix}.$$

Diagonalizing these matrices allows us to find the physical Higgs masses. Identifying the six Goldstone modes which are eaten by the gauge bosons $Z_{L,R}$, $W_{L,R}^{\pm}$ is done in detail in Sections 2.3.3 and 2.3.4. We will recount the results of that analysis here. The results agree fully with Ref. [18].

The mass matrix M_r^2, containing real parts of the fields, diagonalizes in the $v_R \gg \kappa$ limit with four nonzero eigenvalues. The first, which we interpret as the SM Higgs boson, is

$$M_{H_0^0}^2 = \frac{k^2}{2}\left(\frac{\alpha^2}{\rho_1} - 2\lambda\right). \tag{2.14}$$

This is the only Higgs boson which is not at the heavy $SU(2)_R$-breaking scale v_R. The three remaining masses belong to three heavy, neutral, scalar Higgs bosons $H_{1,2,3}^0$:

$$\begin{aligned} M_{H_1^0}^2 &\approx \alpha v_R^2, \\ M_{H_2^0}^2 &\approx \rho_1 v_R, \\ M_{H_3^0}^2 &= \frac{v_R^2}{2}(\rho_3 - 2\rho_1). \end{aligned}$$

Clearly, M_i^2 contains the two zero-mass Goldstone modes $G_1^0 = \phi_-^{0i} \equiv \operatorname{Im}\phi_-^0$ and $G_2^0 = \delta_R^{0i}$; the two neutral degrees of freedom which become the longitudinal polarization modes of, respectively, the Z_1 and Z_2 physical gauge bosons. There are also two neutral pseudoscalar Higgs bosons, $A^0_{1,2}$ with masses

$$M^2_{A^0_1} \approx \alpha v_R^2,$$
$$M^2_{A^0_2} = \frac{v_R^2}{2}(\rho_3 - 2\rho_1).$$

The third matrix, M^2_+, written in the basis

$$\left\{ \frac{1}{k_+}(k\phi_1^+ + k'\phi_2^+),\ \frac{1}{k_+}(k\phi_2^+ - k'\phi_1^+),\ \delta_R^+,\ \delta_L^+ \right\},$$

in the form above already betrays one (complex) Goldstone degree of freedom $G_L^\pm$, proportional to the mixture $(k\phi_2^+ - k'\phi_1^+)/k_+$, which is absorbed by W_L. The second, $G_R^\pm$, is, in the $v_R \gg k_+$ limit, almost exclusively proportional to δ_R^+, which becomes the longitudinal state of W_R after symmetry breaking. Two degrees of freedom remain, realized in physical, singly-charged Higgs states $H^\pm_{1,2}$ with masses

$$M^2_{H_1^\pm} \approx \alpha v_R^2$$

and

$$M^2_{H_2^\pm} = \frac{v_R^2}{2}(\rho_3 - 2\rho_1).$$

Finally, the doubly charged matrix M^2_{++} is already in diagonal form. We find two doubly-charged Higgs bosons $\delta^{\pm\pm}_{1,2}$ with masses

$$M^2_{\delta_1^{\pm\pm}} \approx \rho_2 v_R^2,$$
$$M^2_{\delta_2^{\pm\pm}} = \frac{v_R^2}{2}(\rho_3 - 2\rho_1).$$

The mass matrices above are all diagonal in the $v_R \gg k_+$ limit. Thus, in this limit, the gauge eigenstates (in the basis given above) and physical eigenstates coincide. The form of our results for the Higgs masses agrees with [21] but disagrees with [22] for the SM Higgs mass. We discuss this discrepancy in 2.8.

Below we present the gauge eigenstate Higgs fields in terms of the physical Higgs fields and Goldstone modes for $k_+/v_R \to 0$.

$$
\begin{aligned}
\phi_1^0 &\approx \frac{1}{\sqrt{2}k_+}[kH_0^0 - k'H_1^0 + i(kG_1^0 - k'A_1^0)] \\
\phi_2^0 &\approx \frac{1}{\sqrt{2}k_+}[kH_0^0 + k'H_1^0 - i(kG_1^0 + k'A_1^0)] \\
\phi_1^+ &\approx \frac{1}{\sqrt{2}k_+}[kH_2^+ - k'G_L^+] \\
\phi_2^+ &\approx \frac{1}{\sqrt{2}k_+}[k'H_2^+ + kG_L^+] \\
\delta_L^0 &\approx \frac{1}{\sqrt{2}}[H_3^0 + iA_2^0] \\
\delta_R^0 &\approx \frac{1}{\sqrt{2}}[H_2^0 + iG_2^0] \\
\delta_L^+ &\approx H_1^+ \\
\delta_R^+ &\approx G_R^+
\end{aligned} \tag{2.15}
$$

Note that $\delta_{L,R}^{\pm\pm}$ are both gauge and mass eigenstates.

To summarize, there are 20 degrees of freedom stored in the ten complex numbers in the Higgs fields; four in the bi-doublet, three times two in the triplets. We saw above that SSB produces six massive bosons ($W_{L,R}^{\pm}, Z_{L,R}$); six Goldstone modes have been eaten, leaving 14 degrees of freedom for the physical Higgs bosons: Four real scalars $H_{0,1,2,3}^0$; two real pseudoscalars $A_{1,2}^0$; four complex scalars $H_{1,2}^{\pm}$ and $\delta_{1,2}^{\pm\pm}$. All of the physical Higgs states but H_0^0 lie hidden at the high scale v_R.

2.4.1 SARAH implementation of the scalar potential

A short section on the implementation of the scalar potential in the SARAH model file (found in full in Appendix A) is warranted here. While most parts of the implementation of the MLRM into SARAH were straightforward, there are several things to keep in mind here.

Consider the terms of Eqn. (2.12) containing the field $\tilde{\Phi} = \sigma_2 \Phi^* \sigma_2$. This is the charge conjugate field to Φ. Our first approach was to introduce this in the model file as a separate field in the scalar field definitions, with all gauge group charges conjugated, as a kind of auxiliary field. This did produce the correct potential; however, we discovered[8] that SARAH was including kinetic terms for this field which contributed to the gauge boson masses, et cetera. This is obviously wrong. SARAH does not support including "external" objects like σ_2 in the scalar potential, or, indeed, complex conjugation (without also transposition). Furthermore, it was not possible to expand the contractions component-wise by hand (for example, writing $\mathrm{Tr}\, \tilde{\Phi}\Phi^\dagger = \phi_2^{0*}\phi_1^{0*} + \dots$); SARAH does not seem to support this

[8]Partly due to correspondence with the F. Staub, the creator of SARAH.

feature either. Every term in the model file Lagrangian must be written in terms of, for our case, the full bi-doublet or triplet objects.

Our workaround was to calculate exactly what the correct contractions were, and then constructing these terms in the model file potential using only the non-charge conjugated fields. That is, we have written contractions like $\tilde{\Phi}\Phi^\dagger$ using $\Phi^\dagger\Phi^\dagger$, contracting the indices manually, so that it is, despite its appearance, a gauge-invariant. SARAH does support such manual contraction, achieved by writing out the indices and tensors (Kronecker, Levi-Civita etc.) explicitly.

Let us provide an example. The contraction discussed above, part of the μ_2^2 term in Eqn. (2.12), is

$$(\tilde{\Phi})^l_r(\Phi^\dagger)^r_l = 2(\phi_1^{0*}\phi_2^{0*} - \phi_1^-\phi_2^+)$$

which we see is equal to

$$(\Phi^\dagger)^r_l(\Phi^\dagger)^{r'}_{l'}\epsilon^{ll'}\epsilon_{rr'} = 2(\phi_1^{0*}\phi_2^{0*} - \phi_1^-\phi_2^+).$$

Consulting Appendix A, we see the term discussed written as

```
-mu22 epsTensor[lef2,lef1] epsTensor[rig2,rig1] conj[phi].conj[phi]
```

So, by replacing all terms in the potential containing $\tilde{\Phi}$ by contractions containing only Φ and $\Phi^\dagger$, the correct potential could be input without having to resort to separately defined fields.

Finally, it should be noted that SARAH does output warnings when checking the Lagrangian, due to it understandably misinterpreting terms like $\Phi\Phi$ and $\Phi^\dagger\Phi^\dagger$ as non-invariants. However, as we have illustrated, the explicit contractions ensure that they are proper gauge invariants, so these errors may be safely ignored.

2.5 Yukawa sector

We write, with generality, the Yukawa Lagrangian for (gauge eigenstate) quarks Q as

$$\mathscr{L}_{\text{Yukawa}} = \overline{Q}_{Li}(h_{ij}\Phi + \tilde{h}_{ij}\tilde{\Phi})Q_{Rj} + \text{h.c.} \tag{2.16}$$

where, again, $\tilde{\Phi} \equiv \sigma_2\Phi^*\sigma_2$ and $h, \tilde{h}$ are hermitian three-by-three matrices with generation indices i, j. This hermicity is required by invariance under the parity transformation

$$Q_L \leftrightarrow Q_R, \quad \Phi \leftrightarrow \Phi^\dagger.$$

Evaluating at the VEVs (2.6), we obtain the mass matrices for up- and down-type quarks:

$$M_U = kh + k'e^{-i\alpha}\tilde{h},$$
$$M_D = k'e^{i\alpha}h + k\tilde{h}.$$

The hierarchy that exists between the top and bottom masses implies that $h, \tilde{h}$ and k, k' should not lie at the same scales [21]. Thus, we assume $k \gg k'$ and $h_{ij} \gg \tilde{h}_{ij}$. This allows us to approximate

$$\begin{aligned} M_U &\approx kh, \\ M_D &= k' e^{i\alpha} h + k\tilde{h}. \end{aligned}$$

Since our gauge couplings are flavour-independent, and h is hermitian, we can work in a basis such that M_U is diagonal. We write

$$M_U = S_U \widehat{M}_U$$

where $\widehat{M}_U = \mathrm{diag}\{m_u, m_c, m_t\}$ is the diagonal up-type mass matrix and $S_U = \mathrm{diag}\{s_u, s_c, s_t\}$, where $s_q = \pm 1$ depending on the sign of the corresponding eigenvalue of M_U. Thus, we have defined the masses as strictly positive and extracted the sign. M_D is not necessarily diagonal in this basis, since it also contains a term $\propto \tilde{h}$, and $\tilde{h}$ is not, in general, diagonalized in the same basis as h. With this in mind, we define the Cabibbo–Kobayashi–Maskawa [23, 24] (CKM) matrices, which bring M_D into diagonal form, via

$$M_D = V_R^{\mathrm{CKM}} \widehat{M}_D V_L^{\mathrm{CKM}\dagger} S_U$$

where, in general, $V_R \equiv V_R^{\mathrm{CKM}} \neq V_L^{\mathrm{CKM}} \equiv V_L$, where we also introduced some shorthand notation. Using the above we find

$$k\tilde{h} = V_L \widehat{M}_L V_R^\dagger S_U - \frac{k'}{k} S_U \widehat{M}_U e^{i\alpha}.$$

Subtracting the hermitian conjugate from the above, and recalling the hermicity of $\tilde{h}$, we find the equation

$$M_D \widehat{V}_R^\dagger - \widehat{V}_R \widehat{M}_D = \frac{k}{k'} 2i \sin\alpha V_L^\dagger \widehat{M}_U S_U V_L, \tag{2.17}$$

where $\widehat{V}_R = V_L^\dagger S_U V_R$ has been introduced, relating the left and right CKM matrices. Note that, in the case of manifest left-right symmetry ($\alpha = 0$) discussed above in the context of the scalar potential, $V_R = S_U V_L S_D$; that is, the CKM matrices are equal up to the signs of the eigenvalues.

Being a three-by-three unitary matrix, V_R has nine independent parameters. Eqn. (2.17) contains nine relations which can be used to find V_R. We will outline how this may be done, following [21]. Noting that $m_b \ll m_s \ll M_D$ and keeping only terms proportional to the heaviest quark mass, the RHS of (2.17) can, from its antihermicity, be written

$$\begin{pmatrix} -2iM_D \,\mathrm{Im}\, \widehat{V}_{R11} & -m_s \widehat{V}_{R12} & -m_b \widehat{V}_{R13} \\ m_s \widehat{V}_{R12}^* & -2im_s \,\mathrm{Im}\, \widehat{V}_{R22} & -m_b \widehat{V}_{R23} \\ m_b \widehat{V}_{R13}^* & m_b \widehat{V}_{R23}^* & -2im_b \,\mathrm{Im}\, \widehat{V}_{R33} \end{pmatrix}.$$

Meanwhile, the RHS of (2.17) depends on the usual SM CKM matrix, the quark masses $\widehat{M}_U$ and the CP-violating factor $\xi \sin\alpha$, where we introduced $\xi \equiv k/k'$. We recall the well-known Wolfenstein parametrization [25] of the CKM matrix:

$$V_L = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4), \tag{2.18}$$

written as an expansion in the parameter $\lambda \equiv \sin\theta_{\mathrm{Cabibbo}}$, with η as the CP-violating parameter. Using this, and solving for the elements of $\widehat{V}_R$ to $\mathcal{O}(\lambda^3)$, Ref. [21] finds ($r \equiv \xi m_t/m_b$)

$$\begin{aligned}
\mathrm{Im}\,\widehat{V}_{R11} &= -r\sin\alpha \frac{m_b m_c}{M_D m_t}\lambda^2 \left(s_c + s_t \frac{m_t}{m_c} A^2\lambda^4((1-\rho)^2+\eta^2)\right), \\
\mathrm{Im}\,\widehat{V}_{R22} &= -r\sin\alpha \frac{m_b m_c}{m_s m_t} \left(s_c + s_t \frac{m_t}{m_c} A^2\lambda^4\right), \\
\mathrm{Im}\,\widehat{V}_{R11} &= -r\sin\alpha s_t, \\
\widehat{V}_{R12} &= 2ir\sin\alpha \frac{m_b m_c}{m_s m_t}\lambda \left(s_c + s_t \frac{m_t}{m_c} A^2\lambda^4(1-\rho+i\eta)\right), \\
\widehat{V}_{R13} &= -2ir\sin\alpha A\lambda^3 s_t(1-\rho+i\eta), \\
\widehat{V}_{R23} &= 2ir\sin\alpha A\lambda^2 s_t.
\end{aligned}$$

Again to order λ^3, and defining three new phases $\theta_i \equiv S_{Dii}\,\mathrm{Im}\,V_{Rii}$ for $i = 1, 2, 3$, the remaining elements can be found from unitarity:

$$\begin{aligned}
\widehat{V}_{Rii} &= S_{Dii} e^{i\theta_i} \qquad \text{(no sum over } i\text{)}, \\
\widehat{V}_{R21} &= -s_d s_s \widehat{V}^*_{R12} e^{i(\theta_1+\theta_2)}, \\
\widehat{V}_{R31} &= -s_d s_b \widehat{V}^*_{R13} e^{i(\theta_1+\theta_3)}, \\
\widehat{V}_{R32} &= -s_b s_s \widehat{V}^*_{R32} e^{i(\theta_3+\theta_2)}.
\end{aligned}$$

Then the right-handed CKM matrix can be written

$$\widehat{V}_R = P_U \widetilde{V}_L P_D$$

with $P_U = \mathrm{diag}(s_u, s_c e^{2i\theta_2}, s_t e^{2i\theta_3})$, $P_D = \mathrm{diag}(s_d e^{i\theta_1}, s_s e^{-i\theta_2}, s_b e^{-i\theta_3})$ and

$$\widetilde{V}_L = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 e^{-2i\theta_2} \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 e^{2i\theta_2} & 1 \end{pmatrix},$$

which is just the Wolfenstein CKM matrix with an additional phase on the (3,2) and (2,3) elements. Performing this matrix product, the final result is

$$\widehat{V}_R = \begin{pmatrix} s_d s_u \left(1-\frac{1}{2}\lambda^2\right) e^{i\theta_1} & s_s s_u \lambda e^{-i\theta_2} & s_b s_u A\lambda^3(\rho - i\eta)e^{-i\theta_3} \\ -s_d s_c \lambda e^{i(\theta_1+2\theta_2)} & s_s s_c (1-\frac{1}{2}\lambda^2)e^{i\theta_2} & s_b s_c A\lambda^2 e^{-i\theta_3} \\ s_d s_t A\lambda^3(1-\rho-i\eta)e^{i(\theta_1+2\theta_3)} & -s_s s_t A\lambda^2 e^{i(\theta_2+2\theta_3)} & s_b s_t e^{i\theta_3} \end{pmatrix}$$

In orders of λ, $\widehat{V}_R$ has the same structure as the SM CKM matrix (cf. Eqn. (2.18)): Mixing between generations one and two is $\sim \lambda$, between generations one and three $\sim \lambda^3$ and between generations two and three $\sim \lambda^2$. However, every term now contains a CP-violating phase which leads to enriched phenomenology in CP-related areas [21].

Let us here also add that entering the Yukawa-sector Lagrangian (2.16) into SARAH posed the same difficulties as described in Section 2.4.1; they were, however, easily resolved using the same methods as described there.

2.6 Tree level Lagrangian in physical basis

Here we collect the full MLRM tree-level Lagrangian, developed in the preceding sections, in the mass basis.

2.6.1 Yukawa sector

Quarks. The quark mass terms are simply

$$\mathscr{L}^q_{\text{mass}} = \bar{U}'_{Li}(M_U)_{ij}U'_{Rj} + \bar{D}'_{Li}(M_D)_{ij}D'_{Rj} + \text{h.c.}$$

where U' (D') is the 3-dimensional vector of up (down)-type quarks in the weak basis.

The quark-scalar Lagrangian, with quark mass matrices (see Section 2.5) $h^Q = (kM_U - k'M_D)/k_-^2$ and $\tilde{h}^Q = (kM_D - k'M_U)/k_-^2$ (denoted $h, \tilde{h}$ here for brevity) and Higgs bi-doublet with its charge-conjugate

$$\phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}, \qquad \tilde{\phi} = \begin{pmatrix} \phi_2^{0*} & -\phi_2^+ \\ -\phi_1^- & \phi_1^{0*} \end{pmatrix}$$

is

$$\mathscr{L}^Q_{\text{Yukawa}} = \overline{Q}'_{\text{L}i}(h_{ij}\phi + \tilde{h}_{ij}\tilde{\phi})Q'_{\text{R}j} + \text{h.c.} \equiv \mathscr{L}_{\text{N}} + \mathscr{L}_{\text{C}}$$

where, in the last step, we have divided the Lagrangian into neutral and charged scalar boson parts. These are, respectively,

$$\mathscr{L}_{\text{N}} = \bar{d}'_{\text{L}i}(h_{ij}\phi_2^0 + \tilde{h}_{ij}\phi_1^{0*})d'_{\text{R}j} + \bar{u}'_{\text{L}i}(h_{ij}\phi_1^0 + \tilde{h}_{ij}\phi_2^{0*})u'_{\text{R}j} + \text{h.c.}$$

and

$$\mathscr{L}_{\text{C}} = \bar{d}'_{\text{L}i}(h_{ij}\phi_2^- - \tilde{h}_{ij}\phi_1^-)u'_{\text{R}j} + \bar{u}'_{\text{L}i}(h_{ij}\phi_1^+ - \tilde{h}_{ij}\phi_2^+)d'_{\text{R}j} + \text{h.c.}$$

Plugging in the physical Higgs states (2.15), and rotating the weak quark fields q' into the mass eigenbasis states q using the CKM matrices described in Section 2.3.1, we have

$$\begin{aligned}
\mathscr{L}_N = \bar{d}_{\mathrm{L}i}\bigg(& \frac{k(\widehat{M}_U)_{ij} - k'(\widehat{M}_D)_{ij}}{k_-^2}\frac{1}{\sqrt{2}k_+}[kH_0^0 + k'H_1^0 - i(kG_1^0 + k'A_1^0)] \\
& + \frac{k(\widehat{M}_D)_{ij} - k'(\widehat{M}_U)_{ij}}{k_-^2}\frac{1}{\sqrt{2}k_+}[kH_0^0 - k'H_1^0 - i(kG_1^0 - k'A_1^0)]\bigg) d_{\mathrm{R}j} \\
& + \bar{u}_{\mathrm{L}i}\bigg(\frac{k(\widehat{M}_U)_{ij} - k'(\widehat{M}_D)_{ij}}{k_-^2}\frac{1}{\sqrt{2}k_+}[kH_0^0 - k'H_1^0 + i(kG_1^0 - k'A_1^0)] \\
& + \frac{k(\widehat{M}_D)_{ij} - k'(\widehat{M}_U)_{ij}}{k_-^2}\frac{1}{\sqrt{2}k_+}[kH_0^0 + k'H_1^0 + i(kG_1^0 + k'A_1^0)]\bigg) u_{\mathrm{R}j} + \text{h.c.}
\end{aligned}$$

and

$$\begin{aligned}
\mathscr{L}_C = \bar{d}_{\mathrm{L}i}\bigg(& \frac{k(\widehat{M}_U)_{ij} - k'(\widehat{M}_D)_{ij}}{k_-^2}\frac{1}{k_+}[k'H_2^- + kG_L^-] \\
& - \frac{k(\widehat{M}_D)_{ij} - k'(\widehat{M}_U)_{ij}}{k_-^2}\frac{1}{k_+}[kH_2^+ - k'G_L^+]\bigg) u_{\mathrm{R}j} \\
& + \bar{u}_{\mathrm{L}i}\bigg(\frac{k(\widehat{M}_U)_{ij} - k'(\widehat{M}_D)_{ij}}{k_-^2}\frac{1}{k_+}[kH_2^+ - k'G_L^+] \\
& - \frac{k(\widehat{M}_D)_{ij} - k'(\widehat{M}_U)_{ij}}{k_-^2}\frac{1}{k_+}[k'H_2^+ + kG_L^+]\bigg) d_{\mathrm{R}j} + \text{h.c.}
\end{aligned}$$

Note that the mass matrices $M_{u,d}$ have been diagonalized by

$$M_D = V_R^{\mathrm{CKM}}\widehat{M}_D V_L^{\mathrm{CKM}\dagger} S_U$$

as discussed in Section 2.3.1.

Leptons. The lepton-Higgs interaction Lagrangian contains, in general, both Dirac and Majorana pieces. Again denoting the (Dirac) lepton mass matrices $h^L = h, \tilde{h}^L = \tilde{h}$ and the Majorana mass matrix h_M, the most general such Lagrangian invariant under $SU(2)_L \otimes SU(2)_R$ and parity is [18]

$$\begin{aligned}
\mathscr{L}^L_{\text{Yukawa}} = & \{\overline{L}_{Li}(h_{ij}\phi + \tilde{h}_{ij}\tilde{\phi})L_{Rj} + \text{h.c.}\} \\
& + \{\overline{L}^c_{Ri}(h_M)_{ij}\Sigma_L L_{Lj} + \overline{L}^c_{Li}(h_M)_{ij}\Sigma_R L_{Rj} + \text{h.c.}\}
\end{aligned}$$

where the first term yields, upon SSB, Dirac masses, and the second part Majorana masses. The field $\Sigma_{L,R} = i\sigma_2\Delta_{L,R}$. In the gauge basis, we have

$$
\begin{aligned}
\mathscr{L}_{\text{lepton-scalar}} = {}& \bar{\nu}'_{Li}(h_{ij}\phi_1^0 + \tilde{h}_{ij}\phi_2^{0*})\nu'_{Rj} + \bar{\nu}'_{Li}(h_{ij}\phi_1^+ - \tilde{h}_{ij}\phi_2^+)l'_{Rj} \\
&+ \bar{l}'_{Li}(h_{ij}\phi_2^- - \tilde{h}_{ij}\phi_1^-)\nu'_{Rj} + \bar{l}'_{Li}(h_{ij}\phi_2^0 - \tilde{h}_{ij}\phi_1^{0*})l'_{Rj} \\
&+ \bar{\nu}'^c_{Ri}(h_M)_{ij}\delta_L^0\nu'_{Lj} - \frac{\delta_L^+}{\sqrt{2}}(\bar{\nu}'^c_{Ri}(h_M)_{ij}l'_{Lj} + \bar{l}'^c_{Ri}(h_M)_{ij}\nu'_{Lj}) \\
&+ \bar{\nu}'^c_{Li}(h_M)_{ij}\delta_R^0\nu'_{Rj} - \frac{\delta_R^+}{\sqrt{2}}(\bar{\nu}'^c_{Li}(h_M)_{ij}l'_{Rj} + \bar{l}'^c_{Li}(h_M)_{ij}\nu'_{Rj}) \\
&- \bar{l}'^c_{Ri}(h_M)_{ij}\delta_L^{++}l'_{Lj} - \bar{l}'^c_{Li}(h_M)_{ij}\delta_R^{++}l'_{Rj} + \text{h.c.}
\end{aligned}
$$

where the primes indicate mass eigenstate fields and L^c is the charge conjugate of L. Inserting the physical fields in (2.15) we obtain the lepton mass Lagrangian

$$
\mathscr{L}^l_{\text{mass}} = \bar{l}'_{Li}(M_l)_{ij}l'_R + \bar{l}'_{Ri}(M_l^\dagger)_{ij}l'_L,
$$

where the lepton mass matrix is $M_l = M_l^\dagger = \frac{1}{\sqrt{2}}(k\tilde{h}_l + k'h_l)$. The neutrino mass Lagrangian is

$$
\mathscr{L}^\nu_{\text{mass}} = \frac{1}{2}(\bar{n}'^c_L M_\nu n'_R + \bar{n}'^c_R M_\nu^* n'_L), \tag{2.19}
$$

with fields $n'_R = \begin{pmatrix} \nu'^c_R \\ \nu'_R \end{pmatrix}$ and $n'_L = \begin{pmatrix} \nu'_L \\ \nu'^c_L \end{pmatrix}$. The neutrino mass matrix is

$$
M_\nu = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix},
$$

where $M_D = \frac{1}{\sqrt{2}}(kh_l + k'\tilde{h}_l)$, $M_R = \sqrt{2}h_M v_R$, and the (1,1) element follows from $v_L = 0$.

Meanwhile,

$$
\begin{aligned}
\mathscr{L}_{\text{lepton}-\text{scalar}} = {} & \bar{\nu}'_{Li}\Bigg(h_{ij}\frac{1}{\sqrt{2}k_+}[kH_0^0 - k'H_1^0 + i(kG_1^0 - k'A_1^0)] \\
& + \tilde{h}_{ij}\frac{1}{\sqrt{2}k_+}[kH_0^0 + k'H_1^0 + i(kG_1^0 + k'A_1^0)]\Bigg)\nu'_{Rj} \\
& + \bar{\nu}'_{Li}\Bigg(h_{ij}\frac{1}{k_+}[kH_2^+ - k'G_L^+] - \tilde{h}_{ij}\frac{1}{k_+}[k'H_2^+ + kG_L^+]\Bigg) l'_{Rj} \\
& + \bar{l}'_{Li}\Bigg(h_{ij}\frac{1}{k_+}[k'H_2^- + kG_L^-] - \tilde{h}_{ij}\frac{1}{k_+}[kH_2^- - k'G_L^-]\Bigg)\nu'_{Rj} \\
& + \bar{l}'_{Li}\Bigg(h_{ij}\frac{1}{\sqrt{2}k_+}[kH_0^0 + k'H_1^0 - i(kG_1^0 + k'A_1^0)] \\
& - \tilde{h}_{ij}\frac{1}{\sqrt{2}k_+}[kH_0^0 - k'H_1^0 - i(kG_1^0 - k'A_1^0)]\Bigg) l'_{Rj} \\
& + \bar{\nu}'^c_{Li}(h_M)_{ij}\frac{1}{\sqrt{2}}[H_2^0 + iG_2^0]\nu'_{Rj} \\
& - \frac{H_1^+}{\sqrt{2}}(\bar{\nu}'^c_{Ri}(h_M)_{ij}l'_{Lj} + \bar{l}'^c_{Ri}(h_M)_{ij}\nu'_{Lj}) \\
& + \bar{\nu}'^c_{Li}(h_M)_{ij}\frac{1}{\sqrt{2}}[H_2^0 + iG_2^0]\nu'_{Rj} \\
& - \frac{G_R^+}{\sqrt{2}}(\bar{\nu}'^c_{Li}(h_M)_{ij}l'_{Rj} + \bar{l}'^c_{Li}(h_M)_{ij}\nu'_{Rj}) \\
& - \bar{l}'^c_{Ri}(h_M)_{ij}\delta_L^{++}l'_{Lj} - \bar{l}'^c_{Li}(h_M)_{ij}\delta_R^{++}l'_{Rj} + \text{h.c.},
\end{aligned}
$$

once again recalling that $\delta^{++}_{L,R}$ are already physical.

2.6.2 Gauge boson-fermion interactions

The gauge boson-fermion interactions are obtained from the Lagrangian

$$\mathscr{L}_{\text{gauge}-\text{fermion}} = \bar{Q}_{Li}i\not{D}Q_{Li} + \bar{L}_{Li}i\not{D}L_{Li} + (L \to R)$$

with the covariant derivative (2.2).

Quarks. Considering first quark fields, we have charged and neutral currents of the forms

$$\mathscr{L}^{\text{charged current}}_{\text{gauge}-\text{quark}} = \bar{U}'_L\gamma^\mu\frac{g}{\sqrt{2}}W^+_{L\mu}D'_L + \bar{D}'_L\gamma^\mu\frac{g}{\sqrt{2}}W^-_{L\mu}U'_L + (L \to R)$$

and

$$
\begin{aligned}
\mathscr{L}^{\text{neutral current}}_{\text{gauge}-\text{quark}} = {} & \bar{U}'_L\gamma^\mu\left[\frac{g}{2}W^3_{L\mu} + \frac{g'}{6}B_\mu\right]U'_L + \bar{D}'_L\gamma^\mu\left[-\frac{g}{2}W^3_{L\mu} + \frac{g'}{6}B_\mu\right]D'_L \\
& + (L \to R)
\end{aligned}
$$

in the gauge eigenbasis (where, once again, U', D' are 3-dimensional vectors of up- and down-type quarks, respectively). Rotating into physical fields, we obtain

$$\mathscr{L}^{\rm cc}_{\rm g-q} = \bar{U}_L \gamma^\mu V_L^{\rm CKM} \frac{g}{\sqrt{2}} W^+_{L\mu} D_L + \bar{D}_L \gamma^\mu V_L^{\rm CKM\dagger} \frac{g}{\sqrt{2}} W^-_{L\mu} U_L + (L \to R)$$

and

$$\begin{aligned}
\mathscr{L}^{\rm nc}_{\rm g-q} = {} & \bar{U}_L \gamma^\mu \left[\frac{g}{2}(s_W A_\mu - c_W Z_\mu) + \frac{g'}{6}(c_W c_Y A_\mu + s_W c_Y Z_1 + s_Y Z_{2\mu}) \right] U_L \\
& + \bar{D}_L \gamma^\mu \left[\frac{g}{2}(c_W Z_\mu - s_W A_\mu) + \frac{g'}{6}(c_W c_Y A_\mu + s_W c_Y Z_1 + s_Y Z_{2\mu}) \right] D_L \\
& + \bar{U}_L \gamma^\mu \bigg[\frac{g}{2}(c_W s_Y A_\mu + s_W s_Y Z_\mu - c_Y Z_{2\mu}) \\
& \qquad\qquad + \frac{g'}{6}(c_W c_Y A_\mu + s_W c_Y Z_1 + s_Y Z_{2\mu}) \bigg] U_L \\
& + \bar{D}_L \gamma^\mu \bigg[\frac{g}{2}(c_W s_Y A_\mu + s_W s_Y Z_\mu - c_Y Z_{2\mu}) \\
& \qquad\qquad + \frac{g'}{6}(c_W c_Y A_\mu + s_W c_Y Z_1 + s_Y Z_{2\mu}) \bigg] D_L .
\end{aligned}$$

Leptons. The charged and neutral current gauge boson-lepton Lagrangians are, in terms of weak eigenstate fields,

$$\mathscr{L}^{\rm charged\ current}_{\rm gauge-lepton} = \bar{\nu}'_L \gamma^\mu \frac{g}{\sqrt{2}} W^+_{L\mu} l'_L + \bar{l}'_L \gamma^\mu \frac{g}{\sqrt{2}} W^-_{L\mu} \nu'_L + (L \to R)$$

and

$$\mathscr{L}^{\rm neutral\ current}_{\rm gauge-lepton} = \bar{\nu}'_L \gamma^\mu \left[\frac{g}{2} W^3_{L\mu} - \frac{g'}{2} B_\mu \right] \nu'_L - \bar{l}'_L \gamma^\mu \left[\frac{g}{2} W^3_{L\mu} + \frac{g'}{2} B_\mu \right] l'_L + (L \to R)$$

Inserting the physical fields, and introducing the Pontecorvo–Maki–Nakagawa–Sakata [26, 27] (PMNS) matrices, which simply mix leptons to physical states in exact analogy to how the CKM matrices mix quarks, we have

$$\mathscr{L}^{\rm cc}_{\rm gauge-lepton} = \bar{\nu}_L V_L^{\rm PMNS} \gamma^\mu \frac{g}{\sqrt{2}} W^+_{L\mu} l_L + \bar{l}_L V_L^{\rm PMNS\dagger} \gamma^\mu \frac{g}{\sqrt{2}} W^-_{L\mu} \nu_L + (L \to R)$$

and

$$\begin{aligned}
\mathscr{L}^{\text{nc}}_{\text{gauge-lepton}} = {} & \bar{\nu}_L \gamma^\mu \left[\frac{g}{2}(s_W A_\mu - c_W Z_{1\mu}) \right. \\
& \qquad \left. - \frac{g'}{2}(c_W c_Y A_\mu + s_W c_Y Z_1 + s_Y Z_{2\mu}) \right] \nu_L \\
& - \bar{l}_L \gamma^\mu \left[\frac{g}{2}(s_W A_\mu - c_W Z_{1\mu}) \right. \\
& \qquad \left. + \frac{g'}{2}(c_W c_Y A_\mu + s_W c_Y Z_1 + s_Y Z_{2\mu}) \right] l_L \\
& + \bar{\nu}_R \gamma^\mu \left[\frac{g}{2}(c_W s_Y A_\mu + s_W s_Y Z_{1\mu} - c_Y Z_{2\mu}) \right. \\
& \qquad \left. - \frac{g'}{2}(c_W c_Y A_\mu + s_W c_Y Z_1 + s_Y Z_{2\mu}) \right] \nu_R \\
& - \bar{l}_R \gamma^\mu \left[\frac{g}{2}(c_W s_Y A_\mu + s_W s_Y Z_{1\mu} - c_Y Z_{2\mu}) \right. \\
& \qquad \left. + \frac{g'}{2}(c_W c_Y A_\mu + s_W c_Y Z_1 + s_Y Z_{2\mu}) \right] l_R
\end{aligned}$$

Simplifying using the expressions (2.8) for the coefficients, we find that $\nu_{L,R}$ do not interact with the photon field, as we require.

2.6.3 Gauge boson-scalar interactions

Starting from the kinetic Higgs Lagrangian

$$\begin{aligned}
\mathscr{L}^{\text{kinetic}}_{\text{H}} = {} & \text{Tr}[(D_\mu \Delta_L)^\dagger (D^\mu \Delta_L)] + \text{Tr}[(D_\mu \Delta_R)^\dagger (D^\mu \Delta_R)] \\
& + \text{Tr}[(D_\mu \Phi)^\dagger (D^\mu \Phi)]
\end{aligned}$$

and expanding around the vacuum, we obtain the following pieces:

$$\mathscr{L}^{\text{kinetic}}_{\text{Higgs}} + \mathscr{L}^{\text{kinetic}}_{\text{Goldstone}} + \mathscr{L}^{\text{mass}}_{\text{gauge}} + \mathscr{L}^{\text{bilinear}} + \mathscr{L}_{\text{gauge-scalar}}$$

The first two terms are kinetic terms for the physical Higgs and Goldstone states:

$$\begin{aligned}
\mathscr{L}^{\text{kinetic}}_{\text{Higgs}} = {} & \frac{1}{2}(\partial_\mu H^0_i)(\partial^\mu H^0_i) + \frac{1}{2}(\partial_\mu A^0_j)(\partial^\mu A^0_j) \\
& + (\partial_\mu H^+_j)(\partial^\mu H^-_j) + (\partial_\mu \delta^{++}_L)(\partial^\mu \delta^{--}_L) + (\partial_\mu \delta^{++}_R)(\partial^\mu \delta^{--}_R),
\end{aligned}$$

where i runs from 0 to 3 and j from over 1,2, and

$$\mathscr{L}^{\text{kinetic}}_{\text{Goldstone}} = \frac{1}{2}(\partial_\mu G^0_j)(\partial^\mu G^0_j) + (\partial_\mu G^+_L)(\partial^\mu G^-_L) + (\partial_\mu G^+_R)(\partial^\mu G^-_R).$$

The gauge boson mass Lagrangian, treated in detail in Section 2.3.1, is

$$\mathscr{L}^{\text{mass}}_{\text{gauge}} = \frac{1}{2}\begin{pmatrix} A_\mu & Z_{1\mu} & Z_{2\mu}\end{pmatrix} M_Z^2 \begin{pmatrix} A^\mu \\ Z_1^\mu \\ Z_2^\mu \end{pmatrix} + (W^-_{L\mu} \quad W^-_{R\mu}) M_W^2 \begin{pmatrix} W_L^{+\mu} \\ W_R^{+\mu}\end{pmatrix},$$

where the mass matrices are diagonal with the eigenvalues found in the aforementioned Section. We recall that $W^{\pm}_{L,R}$ are mass eigenstates to $\mathcal{O}(kk'/v_R^2)$.

$\mathscr{L}^{\text{bilinear}}$ contains bilinear terms which are made to cancel when gauge-fixing terms added; this is beyond the scope of this study, but is done in [22].

The remaining part of the Lagrangian, $\mathscr{L}_{\text{gauge-scalar}}$, contains interactions between the gauge and scalar bosons. We first present the terms containing three fields. We do not, in the interest of compactness, print the insertion of the physical states (2.15).

Couplings between two gauge bosons and one scalar.

$$\begin{aligned}
\mathscr{L}_{\text{gauge-scalar}} \supset{}& \frac{g^2}{2}\left[(k'\phi_1^- - k\phi_2^-)W_R^{3\mu}W^+_{L\mu} + (k'\phi_2^- - k\phi_1^-)W_L^{3\mu}W^+_{R\mu}\right] \\
&+ \sqrt{2} g v_R W^+_{R\mu}\delta_R^-\left(g'B^\mu - \frac{g}{2}W_R^{0\mu}\right) - \frac{g^2 v_R}{\sqrt{2}} W^+_{R\mu} W_R^{+\mu}\delta_R^{--} \\
&+ \frac{g^2}{4\sqrt{2}}(k\phi_1^0 + k'\phi_2^0)(W^3_{L\mu} - W^3_{R\mu})(W_L^{0\mu} - W_R^{0\mu}) \\
&- \frac{g^2}{\sqrt{2}}\delta_R^0(gW^3_{R\mu} - g'B_\mu)(gW_R^{0\mu} - g'B^\mu) \\
&+ \frac{g^2}{\sqrt{8}}(k\phi_1^0 + k'\phi_2^0)(W^+_{L\mu}W_L^{-\mu} + W^+_{R\mu}W_R^{-\mu}) \\
&- \frac{g^2}{\sqrt{2}}(k'\phi_1^0 + k\phi_2^{0*})W^-_{L\mu}W_R^{+\mu} + \frac{g^2 v_R}{\sqrt{2}}\delta_R^0 W^+_{R\mu}W_R^{-\mu} \\
&+ \text{h.c.}
\end{aligned}$$

Couplings between one gauge boson and two scalars.

$$
\begin{aligned}
\mathscr{L}_{\text{gauge-scalar}} \supset \frac{ig}{2} & \left[(\partial^\mu \phi_1^+)\phi_1^- - (\partial^\mu \phi_2^-)\phi_2^+\right](W^3_{L\mu} + W^3_{R\mu}) \\
& - ig' \left[(\partial^\mu \delta_R^-)\delta_R^+ + (\partial^\mu \delta_L^-)\delta_L^+\right] B_\mu \\
& - i(\partial^\mu \delta_R^{--})\delta_R^{++}(gW^0_{R\mu} + g' B_\mu) \\
& - i(\partial^\mu \delta_L^{--})\delta_L^{++}(gW^0_{L\mu} + g' B_\mu) \\
& - ig \left[(\partial^\mu \delta_R^+)\delta_R^{--} - (\partial^\mu \delta_R^{--})\delta_R^+\right] W^+_{R\mu} \\
& - ig \left[(\partial^\mu \delta_L^+)\delta_L^{--} - (\partial^\mu \delta_L^{--})\delta_L^+\right] W^+_{L\mu} \\
& + \frac{ig}{\sqrt{2}}\Big\{(\partial^\mu \phi_2^-)\phi_1^{0*} - (\partial^\mu \phi_1^-)\phi_2^0 + (\partial^\mu \phi_2^0)\phi_1^- \\
& \qquad\qquad - (\partial^\mu \phi_1^{0*})\phi_2^- - \sqrt{2}\left[(\partial^\mu \delta_L^-)\delta_L^0 - (\partial^\mu \delta_L^0)\delta_L^-\right]\Big\} W^+_{L\mu} \\
& + \frac{ig}{\sqrt{2}}\Big\{(\partial^\mu \phi_2^-)\phi_1^{0*} - (\partial^\mu \phi_1^-)\phi_2^0 + (\partial^\mu \phi_2^0)\phi_1^- \\
& \qquad\qquad - (\partial^\mu \phi_1^{0*})\phi_2^- - \sqrt{2}\left[(\partial^\mu \delta_L^-)\delta_L^0 - (\partial^\mu \delta_L^0)\delta_L^-\right]\Big\} W^+_{L\mu} \\
& + \frac{ig}{2}\left[(\partial^\mu \phi_1^0)\phi_1^{0*} - (\partial^\mu \phi_2^0)\phi_2^{0*}\right](W^0_{L\mu} - W^0_{R\mu}) \\
& + i(\partial \delta_R^{0*})\delta_R^0(gW^0_{R\mu} - g' B_\mu) + i(\partial \delta_L^{0*})\delta_L^0(gW^0_{L\mu} - g' B_\mu) \\
& + \text{h.c.}
\end{aligned}
$$

Couplings between two gauge bosons and two scalars.

$$\begin{aligned}
\mathscr{L}_{\text{gauge-scalar}} \supset & \frac{g^2}{4}(\phi_1^+\phi_1^- + \phi_2^+\phi_2^-)(W^0_{L\mu} + W^0_{R\mu})(W_L^{0\mu} + W_R^{0\mu}) \\
& + \frac{g^2}{2}(\phi_1^+\phi_1^- + \phi_2^+\phi_2^-)(W^-_{L\mu}W_L^{+\mu} + W^-_{R\mu}W_R^{+\mu}) \\
& + g'^2(\delta_L^+\delta_L^- + \delta_R^+\delta_R^-)B_\mu B^\mu \\
& + 2g^2\delta_R^+\delta_R^- W^+_{R\mu}W_R^{-\mu} + 2g^2\delta_L^+\delta_L^- W^+_{L\mu}W_L^{-\mu} \\
& + \delta_R^{++}\delta_R^{--}\left[(gW^0_{R\mu} + g'B_\mu)(gW_R^{0\mu} + g'B^\mu) + g^2W^-_{R\mu}W_R^{+\mu}\right] \\
& + \delta_L^{++}\delta_L^{--}\left[(gW^0_{L\mu} + g'B_\mu)(gW_L^{0\mu} + g'B^\mu) + g^2W^-_{L\mu}W_L^{+\mu}\right] \\
& + \frac{g^2}{4}(\phi_1^0\phi_1^{0*} + \phi_2^0\phi_2^{0*})(W^0_{L\mu} - W^0_{R\mu})(W_L^{0\mu} - W_R^{0\mu}) \\
& + \delta_R^0\delta_R^{0*}(gW^0_{R\mu} - g'B_\mu)(gW_R^{0\mu} - g'B^\mu) \\
& + \delta_L^0\delta_L^{0*}(gW^0_{L\mu} - g'B_\mu)(gW_L^{0\mu} - g'B^\mu) \\
& + g^2\delta_L^0\delta_L^{0*}W^+_{L\mu}W_L^{-\mu} + g^2\delta_R^0\delta_R^{0*}W^+_{R\mu}W_R^{-\mu} \\
& - g^2\phi_1^0\phi_2^{0*}W^-_{L\mu}W_R^{+\mu} - g^2\phi_1^{0*}\phi_2^0W^+_{L\mu}W_R^{-\mu} \\
& - \Bigg\{ g^2\phi_2^+\phi_1^+W^-_{L\mu}W_L^{-\mu} \\
& \qquad + g\delta_R^+\delta_R^{--}(gW^0_{R\mu} + 2g'B_\mu)W_R^{+\mu} \\
& \qquad + g\delta_L^+\delta_L^{--}(gW^0_{L\mu} + 2g'B_\mu)W_L^{+\mu} \\
& \qquad + \frac{g^2}{2}(\phi_1^-\phi_2^0 - \phi_2^-\phi_1^{0*})W^0_{R\mu}W_L^{+\mu} \\
& \qquad + \frac{g^2}{2}(\phi_2^-\phi_2^{0*} - \phi_1^-\phi_1^0)W^0_{L\mu}W_R^{+\mu} \\
& \qquad + g\delta_R^-\delta_R^0(-gW^0_{R\mu} + 2g'B_\mu)W_R^{+\mu} \\
& \qquad + g\delta_L^-\delta_L^0(-gW^0_{L\mu} + 2g'B_\mu)W_L^{+\mu} \\
& \qquad - g^2\delta_R^{--}\delta_R^0W^+_{R\mu}W_R^{+\mu} - g^2\delta_L^{--}\delta_L^0W^+_{L\mu}W_L^{+\mu} + \text{h.c.}\Bigg\}
\end{aligned}$$

2.6.4 Gauge boson interactions

The gauge-gauge Lagrangian is, in the gauge eigenbasis,

$$\mathscr{L}_{\text{gauge-gauge}} = -\frac{1}{4}W_L^{i\mu\nu}W^i_{L\mu\nu} - \frac{1}{4}W_R^{i\mu\nu}W^i_{R\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu},$$

where we have introduced the field strength tensors

$$\begin{aligned}
W^{1\mu\nu}_{L,R} &= \partial^\mu W^{1\nu}_{L,R} - \partial^\nu W^{1\mu}_{L,R} + g(W^{2\mu}_{L,R}W^{3\nu}_{L,R} - W^{3\mu}_{L,R}W^{2\nu}_{L,R}),\\
W^{2\mu\nu}_{L,R} &= \partial^\mu W^{2\nu}_{L,R} - \partial^\nu W^{2\mu}_{L,R} + g(W^{3\mu}_{L,R}W^{1\nu}_{L,R} - W^{1\mu}_{L,R}W^{3\nu}_{L,R}),\\
W^{3\mu\nu}_{L,R} &= \partial^\mu W^{3\nu}_{L,R} - \partial^\nu W^{3\mu}_{L,R} + g(W^{1\mu}_{L,R}W^{2\nu}_{L,R} - W^{2\mu}_{L,R}W^{1\nu}_{L,R}),\\
B^{\mu\nu} &= \partial^\mu B^\nu - \partial^\nu B^\mu.
\end{aligned}$$

Inserting the physical fields $A, W^\pm_{L,R}, Z_{L,R}$ (see Section 2.3.1; we use $v_R \gg k, k'$ throughout, which means that $W^\pm_{L,R}$ are already physical fields and the neutral fields are given by Eqn. (2.9)) we have

$$\begin{aligned}
\sqrt{2}W^{1\mu\nu}_L &= \partial^\mu(W^{+\nu}_L + W^{-\nu}_L) - \partial^\nu(W^{+\mu}_L + W^{-\mu}_L)\\
&\quad + ig\Big[(W^{+\mu}_L - W^{-\mu}_L)(s_W A^\nu - c_W Z^\nu_1)\\
&\quad - (s_W A^\mu - c_W Z^\mu_1)(W^{+\nu}_L - W^{-\nu}_L)\Big],
\end{aligned}$$

$$\begin{aligned}
\sqrt{2}W^{1\mu\nu}_R &= \partial^\mu(W^{+\nu}_R + W^{-\nu}_R) - \partial^\nu(W^{+\mu}_R + W^{-\mu}_R)\\
&\quad + ig\Big[(W^{+\mu}_R - W^{-\mu}_R)(c_W s_Y A^\nu - s_W s_Y Z^\nu_1 - c_Y Z^\nu_2)\\
&\quad - (c_W s_Y A^\mu - s_W s_Y Z^\mu_1 - c_Y Z^\mu_2)(W^{+\nu}_R - W^{-\nu}_R)\Big],
\end{aligned}$$

$$\begin{aligned}
\sqrt{2}W^{2\mu\nu}_L &= i\partial^\mu(W^{+\nu}_L - W^{+\nu}_L) - i\partial^\nu(W^{+\mu}_L - W^{+\mu}_L)\\
&\quad + g\Big[(s_W A^\nu - c_W Z^\nu_1)(W^{+\mu}_L + W^{-\mu}_L)\\
&\quad - (W^{+\nu}_L + W^{-\nu}_L)(s_W A^\mu - c_W Z^\mu_1)\Big],
\end{aligned}$$

$$\begin{aligned}
\sqrt{2}W^{2\mu\nu}_R &= i\partial^\mu(W^{+\nu}_R - W^{+\nu}_R) - i\partial^\nu(W^{+\mu}_R - W^{+\mu}_R)\\
&\quad + g\Big[(c_W s_Y A^\nu - s_W s_Y Z^\nu_1 - c_Y Z^\nu_2)(W^{+\mu}_R + W^{-\mu}_R)\\
&\quad - (W^{+\nu}_R + W^{-\nu}_R)(c_W s_Y A^\mu - s_W s_Y Z^\mu_1 - c_Y Z^\mu_2)\Big],
\end{aligned}$$

$$\begin{aligned}
W^{3\nu\mu}_L &= \partial^\mu(s_W A^\nu - c_W Z^\nu_1) - \partial^\nu(s_W A^\mu - c_W Z^\mu_1)\\
&\quad + \frac{i}{2}g\Big[(W^{+\mu}_L + W^{-\mu}_L)(W^{+\nu}_L - W^{-\nu}_L) - (W^{+\mu}_L - W^{-\mu}_L)(W^{+\nu}_L + W^{-\nu}_L)\Big],
\end{aligned}$$

$$
\begin{aligned}
W_R^{3\nu\mu} &= \partial^\mu(c_W s_Y A^\nu - s_W s_Y Z_1^\nu - c_Y Z_2^\nu) \\
&- \partial^\nu(c_W s_Y A^\mu - s_W s_Y Z_1^\mu - c_Y Z_2^\mu) \\
&+ \frac{i}{2} g \left[(W_R^{+\mu} + W_R^{-\mu})(W_R^{+\nu} - W_R^{-\nu}) - (W_R^{+\mu} - W_R^{-\mu})(W_R^{+\nu} + W_R^{-\nu}) \right],
\end{aligned}
$$

$$
\begin{aligned}
B^{\mu\nu} &= \partial^\mu(c_W c_Y A^\nu + s_W c_Y Z_1^\nu + s_Y Z_2^\nu) \\
&- \partial^\nu(c_W c_Y A^\mu + s_W c_Y Z_1^\mu + s_Y Z_2^\mu).
\end{aligned}
$$

We will not explicitly insert the above into the Lagrangian; the operation is straightforward. For the full expanded expressions we refer to, among others, [22].

2.7 Phenomenological overview

While we do not focus on experimental aspects in this paper, we will give here a brief review of the current phenomenological state of the MLRM.

Since the model introduces right-handed neutrinos, the possibility of making the left-handed neutrinos naturally very light arises via Majorana masses and the see-saw mechanism. Let us review this process.

Having arrived at the neutrino mass Lagrangian (2.19) in Section 2.6,

$$
\mathscr{L}^\nu_{\text{mass}} = \frac{1}{2} \bar{n}_L'^c M_\nu n_R' + \text{h.c.},
$$

with fields $n_R' = \begin{pmatrix} \nu_R'^c \\ \nu_R' \end{pmatrix}$ and $n_L' = \begin{pmatrix} \nu_L' \\ \nu_L'^c \end{pmatrix}$. We found the neutrino mass matrix to have the form

$$
M_\nu = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix}, \tag{2.20}
$$

where M_D are Dirac mass matrices, dependent on the SM-scale VEVs. M_R, meanwhile, depends on the large VEV v_R.

The see-saw mechanism [5] is essentially realized when diagonalizing a matrix of the type (2.20) above, where the $(2,2)$ element is much larger than the off-diagonal elements. Let us, for simplicity, consider a case where the dimensionality is reduced, so that M_R and M_D are numbers and not matrices. We find the eigenvalues

$$
\lambda_{1,2} = \frac{1}{2}\left(M_R \pm \sqrt{M_R^2 + 4M_D^2} \right).
$$

Using $M_R \gg M_D$, the eigenvalues are, to leading order,

$$
\begin{aligned}
\lambda_1 &\sim M_R, \\
\lambda_2 &\sim \frac{M_D^2}{M_R}.
\end{aligned}
$$

Thus, we find one neutrino mass proportional to the large scale introduced in v_R, and the other suppressed by it. This *see-sawing*, where the light mass is pushed down as the scale v_R is raised, supplies a natural explanation for the light left-handed neutrinos, from only the hierarchy $v_R \gg k, k'$, where the SM cannot.

Measured upper limits of the SM neutrinos can in this way place lower limits on the scale v_R; [20] writes at least 10^{10} GeV.

In Section 2.4, we mention that the β_i parameters in the scalar potential may be set to zero. This choice appears in the literature [18, 22], motivated by the elimination of the need to fine-tune the β_i's: Consider the last equation in (2.13), the so-called VEV see-saw relation. The authors of [18] argue that if reasonable constraints (from neutrino and gauge boson masses) are placed on v_R and v_L, the β_i's must be fine-tuned to six or seven orders of magnitude. After failing to find a symmetry constraining $\beta_i = 0$ within the model, they conclude that this can reasonably be achieved by assuming the MLRM is embedded in some GUT, a hidden symmetry of which forces the β_i's to vanish exactly.

Since the model predicts several new particles, searches for such supply limits on the free parameters of the model. Two important classes of such are searches for new gauge bosons and for charged Higgses.

Currently, the best mass limits on new neutral or charged gauge bosons (so-called Z' and W') come from the LHC. For left-right-symmetric models CMS has excluded a heavy, charged W_R boson, assuming a heavy right-handed muon neutrino N_μ also exists [28]. The excluded area in the plane spanned by (M_{N_μ}, M_{W_R}) reaches a lower bound on the W_R mass of 2.5 TeV.

For singly-charged Higgs bosons, ATLAS reports [29] a model-dependent lower limit on the mass of approximately 100 GeV. This limit depends heavily on the $t \to bH^+$ branching ratio, and assumes $\mathrm{Br}[H^+ \to \nu\tau] = 100\%$: Please consult [29] for the full exclusion zones in parameter space.

Possible charged Higgs signals at the LHC are reviewed in [30]. They find a current limit on MLRM doubly-charged Higgs boson mass of $M_{H^{\pm\pm}} \gtrsim 400 - 500$ GeV, and that this limit should be improved by around 100 GeV during the coming $\sqrt{s} = 14$ TeV LHC run.

Thus, the MLRM is very much alive; while no signal has been detected, due to the large scale of v_R required anyway for light neutrino mass generation, the model is not immediately threatened by this fact.

2.8 Conclusions and outlook

We have introduced the particle content, symmetries and vacuum structure of the MLRM, and verified that it does not suffer from any chiral gauge anomalies. We have broken the MLRM group down to $U(1)_Q \otimes SU(3)_C$ and found the Goldstone modes associated with each broken generator, using a general method. We have not seen this calculation done explicitly in the previous literature. We have also rederived the Lagrangian in the gauge and physical bases, using purpose-written Mathematica code as well as SARAH. The methods agree fully.

The above results have been cross-referenced with the literature and agree, with the exception of the SM Higgs mass (2.14), which disagrees with Ref. [22]. It does, however, agree with Ref. [18]; a seminal paper to the field, with calculations presented, which Ref. [22] also cites. In the light of this, we are confident in our results, and assume there is a mistake or typographical error in Ref. [22].

Since SARAH is a powerful tool for non-SUSY models (it is generally harder to calculate RGEs for non-SUSY than SUSY models), having a complete MLRM SARAH framework established opens several interesting research avenues. We have short-term plans for, among other things, calculating the RGEs in SARAH and studying the vacuum stability at the 1-loop level.

We have seen that left-right-symmetric models, like the one studied here, have a number of very attractive features. Most notably, the seemingly arbitrary way in which the SM treats chirality by favouring left-handedness is completely cured. While the hierarchy $v_R \gg v_L$ breaks parity at low scale, like the SM, it is restored at high scale. Furthermore, right-handed neutrinos with large Majorana masses explain the observed small but finite masses of the left-handed SM neutrinos, through the see-saw mechanism, in an appealing way.

While no direct signals of predictions made by this model have been measured, no phenomenological evidence against it has been found.

However, the MLRM is significantly more complex than the SM in certain sectors; the scalar potential, for example, has 18 free parameters just by itself (as compared to two in the SM). In order to remedy this, we must look to theories with unification and higher symmetries.

3 Non-SUSY Trinification

Trinification, first suggested in 1984 by de Rújula, Georgi and Glashow [9], is traditionally a GUT with gauge group

$$SU(3)_L \otimes SU(3)_R \otimes SU(3)_C$$

where the gauge interactions are unified (or indeed *trinified*) by a discrete $\mathbb{Z}_3$ symmetry. Such models have been studied in a variety of contexts over the last 30 years [31–37], and have been found to have several attractive features: All fermions and scalars are symmetrically and elegantly represented as bi-fundamentals of the above groups, and Trinfication-class models, since they do not require adjoint-representation Higgs fields to break their gauge group down to the SM, may be embedded in heterotic string theories [34]. Furthermore, trinified theories have been shown to have suppressed proton decay rates [31, 34], a problem which otherwise plagues GUTs.

However, there are issues with this type of model. Trinified models have, in general, large flexibility in the fermion couplings it can accommodate [34]. While this tends to gives a model phenomenological survivability, its predictive power is diminished by the large amount of free parameters (here, Yukawa couplings) it must introduce.

We propose a novel trinified model, symmetric under

$$[SU(3)_L \otimes SU(3)_R \otimes SU(3)_C] \otimes SU(3)_f \otimes \mathbb{Z}_3.$$

$SU(3)_f$ is a global family symmetry. We will see that this addition vastly reduces the amount of free parameters (ending up with fewer than ten; only one Yukawa and one gauge coupling) and further unifies the theory. Additionally, we show the existence of a potential minimum where the potential takes a remarkably simple form, parametrized by only one quadratic and one quartic coupling.

The particle content and group representations are introduced in Section 3.1. We then study the SSB of global and gauge symmetries in Section 3.2. In Section 3.3 we write the tree-level Lagrangian of the theory, before deriving the charges and masses of the scalars, fermions and gauge bosons of the theory, as well as identifying the Goldstone bosons associated with the SSB of the gauge symmetries, in Section 3.4. Finally, we give some concluding remarks and an outlook regarding future research in Section 3.5.

This section was developed in collaboration with R. Pasechnik[9], J.E.C. Molina[9], A.P. Morais[9,10], J. Wessén[9] and M.O.P. Sampaio[10].

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3.1 Particle content

The fermions are grouped into the 27-plets (three-index objects)

$$L,\ Q_L,\ Q_R,$$

transforming under the gauge and global symmetry group $[SU(3)_1 \otimes SU(3)_2 \otimes SU(3)_3] \otimes SU(3)_{\mathrm{f}}$ as

$$(\mathbf{3}, \mathbf{3}^*, \mathbf{1}, \mathbf{3}), (\mathbf{3}^*, \mathbf{1}, \mathbf{3}, \mathbf{3}), (\mathbf{1}, \mathbf{3}, \mathbf{3}^*, \mathbf{3}),$$

respectively. The scalars, meanwhile, are placed in the object H whose representation of the above group is

$$(\mathbf{3}, \mathbf{3}^*, \mathbf{1}, \mathbf{3}).$$

The three $SU(3)_{1,2,3}$ symmetries are local, and the remaining is a global family symmetry. We will be using $i, j, k, \ldots$ to denote $SU(3)_1$ indices; $a, b, c, \ldots$ for $SU(3)_2$; $s, t, u, \ldots$ for $SU(3)_3$ and $\alpha, \beta, \gamma, \ldots$ as the family index. Upper indices indicate the fundamental representation and lower the anti-fundamental: Thus, for example, the object $L_a^{i\alpha}$ belongs to the fundamental representations of $SU(3)_1$ and family space, and the anti-fundamental representation of $SU(3)_2$.

In everything that follows, we will identify the groups $SU(3)_{1,2,3}$ as $SU(3)_{L,R,C}$ for left, right and colour, but this is in principle only decided by the vacuum structure presented in the next section. The particular choice of vacuum we make below results in the aforementioned group identification.

Then, as the notation anticipates, we interpret L as a lepton multiplet and $Q_{L,R}$ as left- and right-handed quarks, respectively, while the H components are Higgs fields.

3.2 Symmetry breaking

In order to break the gauge symmetry $SU(3)_L \otimes SU(3)_R \otimes SU(3)_C$ down to $SU(3)_C \otimes U(1)_Q$, we assign the Higgs fields VEVs as follows.

$$\langle H_a^{i1} \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{p}{\sqrt{2}} \end{pmatrix}; \ \langle H_a^{i2} \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{q}{\sqrt{2}} & 0 & 0 \end{pmatrix}; \ \langle H_a^{i3} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} u & 0 & 0 \\ 0 & v & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

This is, as we will see, sufficient for breaking Trinification fully. We could, however, imagine a more complicated vacuum structure with more Higgs components getting VEVs. For simplicity, we will consider only this, simplest, case. The VEVs p and q break $SU(3)_L \otimes SU(3)_R \to SU(2)_L \otimes SU(2)_R \otimes U(1)$ and $SU(2)_R \otimes U(1) \to U(1)$, where the aim is to move from the full symmetry to a left-right-symmetric model like the one studied in the previous chapter. The VEVs in the third generation ensure SM breaking.

The Higgs multiplet VEV transforms under $SU(3)_{L,R}$ as

$$\langle H_a^{i\alpha} \rangle \to \langle H_a^{i\alpha} \rangle + \delta \langle H_a^{i\alpha} \rangle$$

where

$$\delta\langle H_a^{i\alpha}\rangle = (i\omega_L^\zeta (T_L^\zeta)^i_j \delta_a^b - i\omega_R^\zeta (T_R^\zeta)^b_a \delta^i_j)\langle H_b^{j\alpha}\rangle. \tag{3.1}$$

The $\omega_{L,R}$'s are 8+8 real infinitesimals which parametrize the gauge transformation. We wish to find the transformation which leaves the vacuum invariant, or for which $\delta\langle H\rangle = 0$. Since we break the SM fully, this last unbroken symmetry should be $U(1)_Q$. Solving the system of equations obtained when (3.1) = 0, we find

$$\omega_L^3 = \omega_R^3 = -\sqrt{3}\omega_L^8 = -\sqrt{3}\omega_R^8$$

and the rest 0. We define

$$\omega_L^8 = \omega_R^8 \equiv \omega \implies \omega_L^3 = \omega_R^3 = -\sqrt{3}\omega,$$

which means that the EM charge is

$$Q = T_L^8 + T_R^8 - \sqrt{3}(T_L^3 + T_R^3). \tag{3.2}$$

We now turn to the breaking of the global $SU(3)_f$ family symmetry. Since a global transformation is just a special case of a gauge transformation, the most general (infinitesimal) global transformation of the vacuum we can write produces the change

$$\delta\langle H_a^{i\alpha}\rangle = (i\omega_f^\zeta (T_f^\zeta)^\alpha_\beta \delta^i_j \delta^b_a + i\omega_L^\zeta (T_L^\zeta)^i_j \delta_a^b \delta^\alpha_\beta - i\omega_R^\zeta (T_R^\zeta)^b_a \delta^i_j \delta^\alpha_\beta)\langle H_b^{j\beta}\rangle,$$

since H transforms in the fundamental representation under $SU(3)_f$ and $SU(3)_L$, and in the anti-fundamental under $SU(3)_R$. Solving the system of equations resulting from the demand of no change of the vacuum under the above transformation, we obtain the following solutions.

$$\begin{gathered}\omega_{L,R}^{1,2,4,5,6,7} = 0,\\ \omega_L^3 = \omega_R^3 = -\sqrt{3}\omega_L^8 = -\sqrt{3}\omega_R^8,\\ \omega_f^{1,2,3,4,5,6,7,8} = 0.\end{gathered}$$

It should not come as a surprise that we have found the $U(1)_Q$ transformation again here, since, as we mentioned, any global transformation is also a gauge transformation. From the last line, we learn that $SU(3)_f$ is, in fact, fully broken by our vacuum; there are no parameters available with which to parametrize a transformation.

3.3 Tree-level Lagrangian

3.3.1 Kinetic terms

The fermion kinetic Lagrangian is

$$\mathscr{L} \supset L_{i\alpha}^{\dagger a} i\not{D} L_a^{i\alpha} + Q_{Ls\alpha}^{\dagger i} i\not{D} Q_{Li}^{s\alpha} + Q_{Ra\alpha}^{\dagger s} i\not{D} Q_{Rs}^{a\alpha}.$$

Note that the first field in each term is conjugated with respect to all indices. The covariant derivatives are

$$(D_\mu L)^i_a = \left[\delta^i_j \delta^b_a \partial_\mu - i\frac{g}{2}(A^\zeta_\mu T_\zeta)^i_j \delta^b_a + i\frac{g}{2}(B^\zeta_\mu T_\zeta)^b_a \delta^i_j\right] L^j_b$$
$$(D_\mu Q_L)^s_i = \left[\delta^j_i \delta^s_t \partial_\mu + i\frac{g}{2}(A^\zeta_\mu T_\zeta)^j_i \delta^s_t - i\frac{g}{2}(C^\zeta_\mu T_\zeta)^s_t \delta^j_i\right] Q^t_{Lj}$$
$$(D_\mu Q_R)^a_s = \left[\delta^a_b \delta^t_s \partial_\mu - i\frac{g}{2}(B^\zeta_\mu T_\zeta)^a_b \delta^t_s + i\frac{g}{2}(C^\zeta_\mu T_\zeta)^t_s \delta^a_b\right] Q^b_{Rt},$$

where family index has been left implicit. The groups $SU(3)_{L,R,C}$ have corresponding gauge fields $A^\zeta_\mu, B^\zeta_\mu, C^\zeta_\mu$, $\zeta \in \{1, \ldots, 8.\}$. Each field, lying in the adjoint representation, has 8 components, matching the eight $SU(3)$ generators T^ζ. The couplings to the gauge fields are $g_1 = g_2 = g_3 = g$, respecting the $\mathbb{Z}_3$ symmetry.

Meanwhile, the kinetic terms for the scalars are

$$\mathscr{L} \supset \frac{1}{2}[(D_\mu H)^\dagger]^a_{i\alpha}[(D^\mu H)]^{i\alpha}_a$$

where

$$(D_\mu H)^i_a = \left[\delta^i_j \delta^b_a \partial_\mu - i\frac{g}{2}(A^\zeta_\mu T_\zeta)^i_j \delta^b_a + i\frac{g}{2}(B^\zeta_\mu T_\zeta)^b_a \delta^i_j\right] H^j_b,$$

identical in structure to the term for L, since H transforms the same way as L.

3.3.2 Yang-Mills sector

The Yang-Mills contribution to the Lagrangian is written, as usual,

$$\mathscr{L} \supset -\frac{1}{4}A^{\mu\nu}_\zeta A^\zeta_{\mu\nu} - \frac{1}{4}B^{\mu\nu}_\zeta B^\zeta_{R\mu\nu} - \frac{1}{4}C^{\mu\nu}_\zeta C^\zeta_{\mu\nu}.$$

The field strength tensors are constructed in the normal way:

$$A^{\mu\nu}_\zeta \equiv \partial^\mu A^\nu_\zeta - \partial^\nu A^\mu_\zeta + g f^{\zeta\eta\theta} A^\mu_\eta A^\nu_\theta$$

where g and f are the corresponding couplings and structure functions respectively.

3.3.3 Yukawa sector

In our highly unified theory, the only Yukawa term is

$$\mathscr{L} \supset y\epsilon_{\alpha\beta\gamma} H^{i\alpha}_a Q^{s\beta}_{Li} Q^{a\gamma}_{Rs}$$

where y is the coupling. Note that there is only one coupling since the interaction must be a scalar under all gauge groups *and* $SU(3)_f$, and this is the only term possible to construct.

3.3.4 Scalar potential

The quartic terms in the scalar potential are naïvely given by all possible ways of fully contracting the product of two pairs of the object $(HH^\dagger)$. Defining the tensor

$$\mathcal{H}^{ib\alpha}_{ja\beta} \equiv H^{i\alpha}_a H^{\dagger b}_{j\beta},$$

these are

$$\begin{aligned} V \supset\ &\lambda_1(\mathcal{H}^{ia\alpha}_{ia\alpha})^2 + \lambda_2(\mathcal{H}^{ia\beta}_{ia\alpha})(\mathcal{H}^{jb\alpha}_{jb\beta}) + \lambda_3(\mathcal{H}^{ia\alpha}_{ib\alpha})(\mathcal{H}^{jb\beta}_{ja\beta}) + \lambda_4(\mathcal{H}^{ia\alpha}_{ja\alpha})(\mathcal{H}^{jb\beta}_{ib\beta}) \\ &+ \lambda_5(\mathcal{H}^{ia\alpha}_{jb\alpha})(\mathcal{H}^{jb\beta}_{ia\beta}) + \lambda_6(\mathcal{H}^{ia\beta}_{ib\alpha})(\mathcal{H}^{jb\alpha}_{ja\beta}) + \lambda_7(\mathcal{H}^{ia\beta}_{ja\alpha})(\mathcal{H}^{ib\alpha}_{ib\beta}) + \lambda_8(\mathcal{H}^{ia\beta}_{jb\alpha})(\mathcal{H}^{jb\alpha}_{ia\beta}). \end{aligned}$$

Expanding these terms (and adding the quadratic μ^2 term), we have

$$\begin{aligned} V \supset & -\mu^2 H^{i\alpha}_a H^{\dagger a}_{i\alpha} + \lambda_1(H^{i\alpha}_a H^{\dagger a}_{i\alpha})^2 + \lambda_2(H^{i\beta}_a H^{\dagger a}_{i\alpha})(H^{j\alpha}_b H^{\dagger b}_{j\beta}) \\ &+ \lambda_3(H^{i\alpha}_b H^{\dagger a}_{i\alpha})(H^{j\beta}_a H^{\dagger b}_{j\beta}) + \lambda_4(H^{i\alpha}_a H^{\dagger a}_{j\alpha})(H^{j\beta}_b H^{\dagger b}_{i\beta}) \\ &+ \lambda_5(H^{i\alpha}_b H^{\dagger a}_{j\alpha})(H^{j\beta}_a H^{\dagger b}_{i\beta}) + \lambda_6(H^{i\beta}_b H^{\dagger a}_{i\alpha})(H^{j\alpha}_a H^{\dagger b}_{j\beta}) \\ &+ \lambda_7(H^{i\beta}_a H^{\dagger a}_{j\alpha})(H^{j\alpha}_b H^{\dagger b}_{i\beta}) + \lambda_8(H^{i\beta}_b H^{\dagger a}_{j\alpha})(H^{j\alpha}_a H^{\dagger b}_{i\beta}). \end{aligned}$$

However, when they are expanded, the $\lambda_{1,2,3,4}$ terms are simply reproduced by the $\lambda_{8,5,7,6}$ terms, respectively, so we are left with only four independent quartic terms. The scalar potential is then

$$\begin{aligned} V = & -\mu^2 H^i_{a\alpha} H^{\dagger a\alpha}_i + \lambda_1(H^i_{a\alpha} H^{\dagger a\alpha}_i)^2 + \lambda_2(H^i_{a\beta} H^{\dagger a\alpha}_i)(H^j_{b\alpha} H^{\dagger b\beta}_j) \\ &+ \lambda_3(H^i_{b\alpha} H^{\dagger a\alpha}_i)(H^j_{a\beta} H^{\dagger b\beta}_j) + \lambda_4(H^i_{a\alpha} H^{\dagger a\alpha}_j)(H^j_{b\beta} H^{\dagger b\beta}_i) + \text{h.c.}. \end{aligned}$$

The requirement that the potential is extremal, i.e. that its first derivatives w.r.t. the fields vanish, allow us to simplify the above expression. The tadpole equations are

$$\left\langle \frac{\partial V}{\partial H^{i\alpha}_a} \right\rangle = 0$$

for all components $H^{i\alpha}_a$. These conditions imply several trivial relations, as well as and the equations relating the potential parameters and the VEVs

$$\begin{aligned} 0 &= -\frac{1}{2}\mu^2 + (p^2+q^2+u^2+v^2)\lambda_1 + p^2(\lambda_2+\lambda_3) + (p^2+q^2)\lambda_4, \\ 0 &= -\frac{1}{2}\mu^2 + (p^2+q^2+u^2+v^2)\lambda_1 + q^2\lambda_2 + (q^2+u^2)\lambda_3 + (p^2+q^2)\lambda_4, \\ 0 &= -\frac{1}{2}\mu^2 + (p^2+q^2+u^2+v^2)\lambda_1 + (u^2+v^2)\lambda_2 + (q^2+u^2)\lambda_3 + u^2\lambda_4, \\ 0 &= -\frac{1}{2}\mu^2 + (p^2+q^2+u^2+v^2)\lambda_1 + (u^2+v^2)\lambda_2 + v^2(\lambda_3+\lambda_4). \end{aligned}$$

Solving yields

$$\mu^2 = \lambda_1(p^2 + q^2 + u^2 + v^2),$$
$$\lambda_2 = \lambda_3 = \lambda_4 = 0.$$

The potential at this point is characterized by only two parameters! Note also that since $\mu^2 > 0$, we have $\lambda_1 > 0$. Using this, we can evaluate the potential at the vacuum,

$$V\Big|_{H_a^{i\alpha}=0} = -\frac{\lambda_1}{4}(p^2 + q^2 + u^2 + v^2)^2 < 0,$$

a point, which, as we shall see in Section 3.4.1, is a minimum.

3.4 Mass spectrum and charges

3.4.1 Scalars

The potential terms quadratic in the fields make up the mass matrix for the scalars of the model. Separating the fields into real and imaginary parts, $H_a^{i\alpha} = H_{ra}^{i\alpha} + iH_{ia}^{i\alpha}$, we write the mass matrix M_H^2 in the basis

$$\begin{aligned}
\{&H_{r1}^{11}, H_{r2}^{11}, H_{r3}^{11}, H_{r1}^{21}, H_{r2}^{21}, H_{r3}^{21}, H_{r1}^{31}, H_{r2}^{31}, H_{r3}^{31},\\
&H_{r1}^{12}, H_{r2}^{12}, H_{r3}^{12}, H_{r1}^{22}, H_{r2}^{22}, H_{r3}^{22}, H_{r1}^{32}, H_{r2}^{32}, H_{r3}^{32},\\
&H_{r1}^{13}, H_{r2}^{13}, H_{r3}^{13}, H_{r1}^{23}, H_{r2}^{23}, H_{r3}^{23}, H_{r1}^{33}, H_{r2}^{33}, H_{R3}^{33},\\
&H_{i1}^{11}, H_{i2}^{11}, \dots\}
\end{aligned}$$

and find

$$\begin{aligned}
(M_H^2)_{9,9} &= 4p^2\lambda_1,\\
(M_H^2)_{16,16} &= 4q^2\lambda_1,\\
(M_H^2)_{19,19} &= 4u^2\lambda_1,\\
(M_H^2)_{23,23} &= 4v^2\lambda_1,\\
(M_H^2)_{9,16} = (M_H^2)_{16,9} &= 4pq\lambda_1,\\
(M_H^2)_{9,19} = (M_H^2)_{19,9} &= 4pu\lambda_1,\\
(M_H^2)_{9,23} = (M_H^2)_{23,9} &= 4pv\lambda_1,\\
(M_H^2)_{16,19} = (M_H^2)_{19,16} &= 4qu\lambda_1,\\
(M_H^2)_{16,23} = (M_H^2)_{23,16} &= 4qv\lambda_1,\\
(M_H^2)_{19,23} = (M_H^2)_{23,19} &= 4uv\lambda_1,
\end{aligned}$$

with all other elements 0. This matrix is straightforwardly diagonalized, yielding only one nonzero eigenvalue:

$$M_\phi^2 = 4\lambda_1(p^2 + q^2 + u^2 + v^2)$$

The physical field ϕ is the linear combination

$$\phi = \frac{1}{\sqrt{u^2+v^2+p^2+q^2}}(uH^{13}_{r1} + vH^{23}_{r2} + pH^{31}_{r3} + qH^{32}_{r1}).$$

Since this, the only nonzero mass, is positive definite (again recalling that $\mu^2 > 0$ implies $\lambda_1 > 0$), the vacuum is indeed a true minimum. Clearly, it lies at the high Trinification-breaking scale p.

Thus, at tree level, there is only one massive scalar. The remaining 53 massless particles have among them 15 Goldstone modes, associated with the gauge symmetry SSB, identified below. There should also be further Goldstones associated with the breaking of the global SSB from $SU(3)_f$, which we have not identified.

To match with SM phenomenology, the massless Higgs bosons should acquire masses when loop effects are taken into account. This would occur by quantum effects generating spontaneous symmetry breaking, as described in Section 1.2.

Let us now find the charges of the scalars. From the discussion in Section 3.2, the unbroken gauge transformation is

$$H^{i\alpha}_a \to \left(e^{i\omega(-\sqrt{3}T^3_L + T^8_L)}\right)^i_j \left(e^{-i\omega(-\sqrt{3}T^3_R + T^8_R)}\right)^b_a H^{j\alpha}_b.$$

The $SU(3)$ generators are just the Gell-Mann matrices, for which

$$-\sqrt{3}T^3 + T^8 = \begin{pmatrix} 1/3 & 0 & 0 \\ 0 & -2/3 & 0 \\ 0 & 0 & 1/3 \end{pmatrix},$$

so, from the properties of the matrix exponential,

$$H^{i\alpha}_a \to \begin{pmatrix} e^{i\omega/3} & & \\ & e^{-i\omega 2/3} & \\ & & e^{i\omega/3} \end{pmatrix}^i_j \begin{pmatrix} e^{-i\omega/3} & & \\ & e^{i\omega 2/3} & \\ & & e^{-i\omega/3} \end{pmatrix}^b_a H^{j\alpha}_b.$$

Since the contraction over R-indices instructs us to multiply columns of the first matrix with the rows of the second, the RHS can be written as the matrix product

$$\begin{pmatrix} e^{i\omega/3} & & \\ & e^{-i\omega 2/3} & \\ & & e^{i\omega/3} \end{pmatrix} \begin{pmatrix} H^1_1 & H^1_2 & H^1_3 \\ H^2_1 & H^2_2 & H^2_3 \\ H^3_1 & H^3_2 & H^3_3 \end{pmatrix} \begin{pmatrix} e^{-i\omega/3} & & \\ & e^{i\omega 2/3} & \\ & & e^{-i\omega/3} \end{pmatrix}.$$

Performing this, we find the transformation properties of each element under the EM gauge group. For example, H^1_1 picks up $e^{i\omega/3}$ from the L transformation and $e^{-i\omega/3}$ from the R transformation, so

$$H^1_1 \to e^{i\omega(\frac{1}{3}-\frac{1}{3})} H^1_1.$$

Thus, H_1^1 transforms as an object with 0 charge under $U(1)_Q$. Similarly,

$$H_2^1 \to e^{i\omega(\frac{1}{3}+\frac{2}{3})} H_2^1,$$

so H_2^1 has $Q = +1$. We can repeat this for each component. The EM charges of the Higgs components are then

$$Q(H) = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}.$$

3.4.2 Gauge symmetry Goldstone bosons

To identify the Goldstones from the breaking of the gauge symmetries, we know, from the method developed in 1.4, to find the broken generators from the gauge boson mixing matrix. To leading order in $\frac{u,v}{p,q}$, the mixed gauge boson states are given in Table 2. To reiterate, we obtain the broken generator corresponding to each massive gauge boson by mixing the generators $T^a_{L,R}$ corresponding to the gauge eigenstates A^a, B^a in the same way as the latter mix into physical states, except that each $T_{L,R}$ must also be multiplied by the corresponding gauge coupling $g_{L,R}$. In this model, there is the additional simplification of $g_L = g_R = g$, so all the g's multiplying each $T_{L,R}$ can be pulled out. Thus, the *unbroken* generator corresponding to the photon $\frac{1}{2\sqrt{2}}(-\sqrt{3}(A_3 + B_3) + A_8 + B_8)$ is proportional to $-\sqrt{3}(T_L^3 + T_R^3) + T_L^8 + T_R^8$, as we have already determined.

Reading off Table 2 directly, we find the broken generators to be (up to normalization)

$$T_R^5;\ T_R^4;\ T_L^5;\ T_L^4;\ T_R^7;\ T_R^6;\ T_L^7;\ T_L^6;\ T_L^2;\ T_L^1;\ T_R^2;\ T_R^1;$$
$$T_R^8 - T_L^8;\ \sqrt{3}(T_L^8 + T_R^8) - 3T_R^3 + 5T_L^3;\ 2T_R^3 + \sqrt{3}(T_L^8 + T_R^8).$$

As per our prescription, we now apply i times these generators to the vacuum and see which direction the vacuum is pushed. Recalling the vacuum structure

$$\langle H_a^{i1}\rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{p}{\sqrt{2}} \end{pmatrix};\ \langle H_a^{i2}\rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{q}{\sqrt{2}} & 0 & 0 \end{pmatrix};\ \langle H_a^{i3}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} u & 0 & 0 \\ 0 & v & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

we will do the first two calculations explicitly. Let us start with gT_L^5, the generator corresponding to the mass eigenstate gauge boson A^5. We have

$$igT^5 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & p/\sqrt{2} \end{pmatrix} = \frac{ig}{2\sqrt{2}} \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & p \end{pmatrix} = \begin{pmatrix} 0 & 0 & \frac{gp}{2\sqrt{2}} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$igT^5 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ q/\sqrt{2} & 0 & 0 \end{pmatrix} = \frac{ig}{2\sqrt{2}} \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ q & 0 & 0 \end{pmatrix} = \begin{pmatrix} \frac{gq}{2\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$igT^5 \frac{1}{\sqrt{2}} \begin{pmatrix} u & 0 & 0 \\ 0 & v & 0 \\ 0 & 0 & 0 \end{pmatrix} = \frac{ig}{2\sqrt{2}} \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \begin{pmatrix} u & 0 & 0 \\ 0 & v & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{-gu}{2\sqrt{2}} & 0 & 0 \end{pmatrix},$$

acting on each generation of the vacuum in turn. Thus, the Goldstone direction, which is the resultant of these pushes in field space, is, normalized to unity,

$$G_L^5 = \frac{1}{\sqrt{p^2+q^2+u^2}}(pH_{r3}^{11} + qH_{r1}^{12} - uH_{r1}^{33}),$$

where we recall the notation $H_{ra}^{i\alpha}$ for the real part of the (i, a) component of the αth generation H field.

Turning to T_R^5, we must take care to multiply the generator and the vacuum correctly. The vacuum transforms in the anti-fundamental representation of $SU(3)_R$. Thus, we need the generator acting on it in the anti-fundamental representation as well. The generators in the two representations are related by

$$T_{\mathrm{A}} = -T_{\mathrm{F}}^* = -T_{\mathrm{F}}^{\mathrm{T}}, \tag{3.3}$$

where A stands for anti-fundamental and F for fundamental. The first equality in (3.3) follows from, assuming a field in the fundamental transforms as

$$\phi \rightarrow \exp(i\omega^a T_{\mathrm{F}}^a)\phi,$$

the transformation of the conjugate (anti-fundamentally represented) field

$$\phi^* \rightarrow \exp(i\omega^a(-T_{\mathrm{F}}^*)^a)\phi^* \equiv \exp(i\omega^a T_{\mathrm{A}}^a)\phi^*;$$

the second equality in (3.3) follows from the $SU(N)$ generator property $T^{\mathrm{T}} = T^*$.

Then, the correct contraction we seek is

$$ig(T_{R,\mathrm{A}}^5)_b^a \langle H_a^{i\alpha}\rangle = -ig((T_{R,\mathrm{F}}^5)^{\mathrm{T}})_b^a \langle H_a^{i\alpha}\rangle.$$

Performing this, we see that this generator pushes the vacuum in the direction

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -\frac{gp}{2\sqrt{2}} & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{gq}{2\sqrt{2}} \end{pmatrix} + \begin{pmatrix} 0 & 0 & \frac{gu}{2\sqrt{2}} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

and so find the Goldstone corresponding to T_R^5

$$G_R^5 = \frac{1}{\sqrt{q^2+p^2+u^2}}(-pH_{r1}^{31} + qH_{r3}^{32} + uH_{r3}^{13}).$$

Using the same methods and notation, we present the full number of Goldstone bosons associated with the breaking of $SU(3)_R \otimes SU(3)_L \rightarrow U(1)_Q$ in Table 1 below.

Table 1: *Broken generators associated with the spontaneous breaking $SU(3)_R \otimes SU(3)_L \to U(1)_Q$ and their corresponding Goldstone modes.*

Broken generator	Goldstone state
T_R^5	$\frac{1}{\sqrt{q^2+p^2+u^2}}(-pH_{r1}^{31} + qH_{r3}^{32} + uH_{r3}^{13})$
T_R^4	$\frac{1}{\sqrt{q^2+p^2+u^2}}(-pH_{i1}^{31} - qH_{i3}^{32} - uH_{i3}^{13})$
T_L^5	$\frac{1}{\sqrt{p^2+q^2+u^2}}(pH_{r3}^{11} + qH_{r1}^{12} - uH_{r1}^{33})$
T_L^4	$\frac{1}{\sqrt{q^2+p^2+u^2}}(pH_{i3}^{11} + qH_{i1}^{12} + uH_{i1}^{33})$
T_R^7	$\frac{1}{\sqrt{p^2+v^2}}(-pH_{r2}^{31} + vH_{r3}^{23})$
T_R^6	$\frac{1}{\sqrt{q^2+v^2}}(-qH_{i2}^{31} - vH_{i3}^{23})$
T_L^7	$\frac{1}{\sqrt{p^2+p^2+v^2}}(pH_{r3}^{21} + qH_{r1}^{22} - vH_{r2}^{33})$
T_L^6	$\frac{1}{\sqrt{p^2+p^2+v^2}}(pH_{i3}^{21} + qH_{i1}^{22} + vH_{i2}^{33})$
T_L^2	$\frac{1}{\sqrt{u^2+v^2}}(vH_{r2}^{13} - uH_{r1}^{23})$
T_L^1	$\frac{1}{\sqrt{u^2+v^2}}(vH_{i2}^{13} + uH_{i1}^{23})$
T_R^2	$\frac{1}{\sqrt{q^2+u^2+v^2}}(-qH_{i2}^{32} - uH_{i2}^{13} - vH_{i1}^{23})$
T_R^1	$\frac{1}{\sqrt{q^2+u^2+v^2}}(+qH_{r2}^{32} + uH_{r2}^{13} - vH_{r1}^{23})$
$T_R^8 - T_L^8$	$\frac{1}{\sqrt{4p^2+q^2/4+u^2+v^2}}(2pH_{i3}^{31} + \frac{q}{2}H_{i1}^{32} - uH_{i1}^{13} - vH_{i2}^{23})$
$\sqrt{3}(T_L^8 + T_R^8) - 3T_R^3 + 5T_L^3$	$\frac{1}{\sqrt{4(p^2+q^2)+9u^2+25v^2}}(2pH_{i3}^{31} + 2qH_{i1}^{32} + 3uH_{13}^{i1} - 5vH_{i2}^{23})$
$2T_R^3 + \sqrt{3}(T_L^8 + T_R^8)$	$\frac{1}{\sqrt{25q^2+4(u^2+v^2)}}(-5q^2H_{i1}^{32} - 2uH_{i1}^{13}) + 2vH_{i2}^{23}$

3.4.3 Leptons

Since L transforms in the same way as H, we know immediately that

$$Q(L) = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}.$$

The leptons do not acquire tree-level masses in this model, since the only Yukawa term is $\sim HQ_LQ_R$. If the model is to successfully recreate the SM spectrum, charged lepton masses must be generated by quantum corrections, just as for the massless scalar bosons discussed in the previous section.

3.4.4 Quarks

The changes in the quarks Q_L and Q_R under a full infinitesimal gauge transformation are

$$\delta Q^s_{Li} = (i\omega^\zeta_C (T^\zeta_C)^s_t \delta^j_i - i\omega^\zeta_L (T^\zeta_L)^j_i \delta^s_t) Q^t_{Lj}$$

and

$$\delta Q^a_{Rs} = (i\omega^\zeta_R (T^\zeta_R)^a_b \delta^t_s - i\omega^\zeta_C (T^\zeta_C)^t_s \delta^a_b) Q^b_{Rt},$$

so when we consider the electromagnetic gauge transformation (3.2) we find

$$Q_L \to \begin{pmatrix} Q^1_{L1} & Q^1_{L2} & Q^1_{L3} \\ Q^2_{L1} & Q^2_{L2} & Q^2_{L3} \\ Q^3_{L1} & Q^3_{L2} & Q^3_{L3} \end{pmatrix} \begin{pmatrix} e^{-i\omega/3} & 0 & 0 \\ 0 & e^{i\omega 2/3} & 0 \\ 0 & 0 & e^{-i\omega/3} \end{pmatrix}$$

implying the EM charges

$$Q(Q_L) = \begin{pmatrix} -1/3 & 2/3 & -1/3 \\ -1/3 & 2/3 & -1/3 \\ -1/3 & 2/3 & -1/3 \end{pmatrix},$$

and

$$Q_R \to \begin{pmatrix} e^{i\omega/3} & 0 & 0 \\ 0 & e^{-i\omega 2/3} & 0 \\ 0 & 0 & e^{i\omega/3} \end{pmatrix} \begin{pmatrix} Q^1_{R1} & Q^1_{R2} & Q^1_{R3} \\ Q^2_{R1} & Q^2_{R2} & Q^2_{R3} \\ Q^3_{R1} & Q^3_{R2} & Q^3_{R3} \end{pmatrix},$$

yielding

$$Q(Q_R) = \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ -2/3 & -2/3 & -2/3 \\ 1/3 & 1/3 & 1/3 \end{pmatrix}.$$

In order to obtain the quark mass spectrum, we collect the quark bilinear terms into a matrix

$$(M_Q)_{ij} = \left. \frac{\partial^2 \mathscr{L}_{\text{Yukawa}}}{\partial Q_j \partial Q_j} \right|_{\langle H \rangle},$$

where $Q_{i,j}$ is shorthand for any $(Q_L)^\alpha_i$ or $(Q_R)^{a\alpha}$. Note that colour indices have been suppressed; any physical state or mass found is simply three times degenerate in colour. Then, the quark masses can be easily found by diagonalizing the matrix

$$M_Q^2 = M_Q^\dagger M_Q.$$

We find nine Dirac mass eigenstates (27 when accounting for colour; in the following we will leave the colour degeneracy implicit). The masses are, to leading order in $\frac{u,v}{p,q}$,

$$\begin{pmatrix} 0 \\ 0 \\ 2p^2y^2 \\ 2q^2y^2 \\ 2u^2y^2 \\ 2u^2y^2 \\ 2v^2y^2 \\ 2v^2y^2 \\ 2(p^2+q^2)y^2 \end{pmatrix}.$$

Thus, there are two exactly massless quarks, for which masses should be generated when loops are taken into account in order to match any phenomenology. Among the remaining, three masses lie at the high scale $\sim p, q$ (note that we do not assume any hierarchy between p and q here), while four lie at the SM scale $\sim u, v$. Assuming the massless quarks obtain small loop masses (compared to the Trinification scale p), there are six light quarks; these might be interpreted as the SM quarks.

If we set $u = v = 0$, the mixing of the states becomes transparent. The Yukawa Lagrangian is then

$$\mathscr{L}_{\text{Yukawa}} = y\epsilon_{\alpha\beta\gamma} H^{i\alpha}_a Q^\beta_{Li} Q^{a\gamma}_R + \text{h.c.}$$

where the contraction over colour has been suppressed. Expanding and evaluating at the vacuum for this simplified case, we get

$$\begin{aligned}\mathscr{L}_{\text{Yukawa}} &= -y\sqrt{2}\left[Q^3_{L3}(qQ^1_{R1} - pQ^2_{R3}) - qQ^1_{L3}Q^3_{R1} + pQ^2_{L3}Q^3_{R3}\right] + \text{h.c.} \\ &= -y\sqrt{2}\sqrt{p^2+q^2}\left[Q^3_{L3}\frac{qQ^1_{R1} - pQ^2_{R3}}{\sqrt{p^2+q^2}} + \text{h.c.}\right] \\ &\quad + y\sqrt{2}q\left[Q^1_{L3}Q^3_{R1} + \text{h.c.}\right] - y\sqrt{2}p\left[Q^2_{L3}Q^3_{R3} + \text{h.c.}\right],\end{aligned}$$

which corresponds to three Dirac spinors

$$\begin{pmatrix} Q^3_{L3} \\ \frac{q(Q^1_{R1})^\dagger - p(Q^2_{R3})^\dagger}{\sqrt{p^2+q^2}} \end{pmatrix}, \quad \begin{pmatrix} Q^1_{L3} \\ -(Q^3_{R1})^\dagger \end{pmatrix}, \quad \begin{pmatrix} Q^2_{L3} \\ (Q^3_{R3})^\dagger \end{pmatrix}$$

with the masses-squared

$$2(p^2+q^2)y^2,\ 2q^2y^2,\ 2p^2y^2$$

respectively. Clearly, these masses lie at the large Trinification-breaking and LR-breaking scales (p and q, respectively).

3.4.5 Gauge bosons

In the basis $\{A^1, A^2, \ldots, A^8, B^1, B^2, \ldots, B^8\}$ the nonzero elements of the gauge boson mass matrix are

$$
\begin{aligned}
(M^2_{GB})_{1,1} &= g^2(u^2+v^2)/16, \\
(M^2_{GB})_{2,2} &= g^2(u^2+v^2)/16, \\
(M^2_{GB})_{3,3} &= g^2(u^2+v^2)/16, \\
(M^2_{GB})_{4,4} &= g^2(p^2+q^2+u^2)/16, \\
(M^2_{GB})_{5,5} &= g^2(p^2+q^2+u^2)/16, \\
(M^2_{GB})_{6,6} &= g^2(p^2+q^2+v^2)/16, \\
(M^2_{GB})_{7,7} &= g^2(p^2+q^2+v^2)/16, \\
(M^2_{GB})_{8,8} &= g^2(4p^2+q^2+u^2+v^2)/48, \\
(M^2_{GB})_{9,9} &= g^2(q^2+q^2+v^2)/16), \\
(M^2_{GB})_{10,10} &= g^2(q^2+q^2+v^2)/16, \\
(M^2_{GB})_{11,11} &= g^2(p^2+q^2+v^2)/16, \\
(M^2_{GB})_{12,12} &= g^2(p^2+q^2+u^2)/16, \\
(M^2_{GB})_{13,13} &= g^2(p^2+q^2+u^2)/16, \\
(M^2_{GB})_{14,14} &= g^2(p^2+v^2)/16, \\
(M^2_{GB})_{15,15} &= g^2(p^2+v^2)/16, \\
(M^2_{GB})_{16,16} &= g^2(4p^2+q^2+u^2+v^2)/48, \\
(M^2_{GB})_{1,9} &= (M^2_{GB})_{9,1} = -g^2uv/8, \\
(M^2_{GB})_{2,10} &= (M^2_{GB})_{10,2} = -g^2uv/8, \\
(M^2_{GB})_{3,8} &= (M^2_{GB})_{8,3} = g^2(u^2-v^2)/(16\sqrt{3}), \\
(M^2_{GB})_{3,11} &= (M^2_{GB})_{11,3} = -g^2(u^2+v^2)/16, \\
(M^2_{GB})_{3,16} &= (M^2_{GB})_{16,3} = g^2(v^2-u^2)/(16\sqrt{3}), \\
(M^2_{GB})_{8,11} &= (M^2_{GB})_{11,8} = g^2(2q^2-u^2+v^2)/(16\sqrt{3}), \\
(M^2_{GB})_{8,16} &= (M^2_{GB})_{16,8} = -g^2(4p^2-2q^2+u^2+v^2)/48, \\
(M^2_{GB})_{11,16} &= (M^2_{GB})_{16,11} = g^2(q^2+u^2-v^2)/(16\sqrt{3}),
\end{aligned}
$$

This matrix is diagonalized to give the physical gauge boson spectrum. The phenomenologically justified VEV hierarchy $p \gg q \gg u \sim v$ is imposed, and results, both masses and states, are given to leading order in $\frac{p,q}{uv}$ in Table 2. The first state in Table 2, which is massless, is identified as the photon. There are three further states that are massless in the limit of no SM breaking $u = v = 0$; these are the three electroweak gauge bosons. Let us now find the EM charge eigenstates. A^a lies in the adjoint representation of $SU(3)_L$ and is a singlet under $SU(3)_R$, and vice versa for B^a. Thus, the transformation properties

Table 2: *Physical $SU(3)_{L,R}$ gauge bosons given in terms of the gauge eigenstates A, B, and their respecitve masses squared.*

Physical eigenstate	Mass-squared
$\frac{1}{2\sqrt{2}}(-\sqrt{3}(A_3+B_3)+A_8+B_8)$	0
B_5	$\frac{g^2}{16}(p^2+q^2)$
B_4	$\frac{g^2}{16}(p^2+q^2)$
A_5	$\frac{g^2}{16}(p^2+q^2)$
B_4	$\frac{g^2}{16}(p^2+q^2)$
B_7	$\frac{g^2}{16}p^2$
B_6	$\frac{g^2}{16}p^2$
A_7	$\frac{g^2}{16}(p^2+q^2)$
A_6	$\frac{g^2}{16}(p^2+q^2)$
A_2	$\frac{g^2}{16}(v^2+u^2)$
A_1	$\frac{g^2}{16}(v^2+u^2)$
B_2	$\frac{g^2}{16}q^2$
B_1	$\frac{g^2}{16}q^2$
$\frac{1}{\sqrt{2}}(B_8-A_8)$	$\frac{g^2}{96}(16p^2+q^2)$
$\frac{1}{2\sqrt{10}}(\sqrt{3}(A_8+B_8)-3B_3+5A_3)$	$\frac{g^2}{10}(v^2+u^2)$
$\frac{1}{\sqrt{10}}(2B_3+\sqrt{3}(A_8+B_8))$	$\frac{5g^2}{32}q^2$

for these gauge bosons are

$$A^a \to -f^{abc}\omega_L^b A^c,$$
$$B^a \to -f^{abc}\omega_R^b B^c,$$

where we temporarily relax our assignment of a, b, c as $SU(3)_R$ indices and simply use them generically. The structure constants are defined through $[T^a, T^b] = if^{abc}T^c$. Under the electromagnetic gauge transformation (3.2) in particular,

$$A^a \to -f^{a3c}\omega A^c + f^{a8c}\frac{1}{\sqrt{3}}\omega A^c,$$
$$B^a \to -f^{a3c}\omega B^c + f^{a8c}\frac{1}{\sqrt{3}}\omega B^c,$$

The only nontrivial transformations are then

$$\delta A^1 = \omega A^2,$$
$$\delta A^2 = -\omega A^1,$$
$$\delta A^6 = -\omega A^7,$$
$$\delta A^7 = \omega A^6,$$

and similarly for B. It is clear that any gauge boson state containing none of $(A, B)^{1,2,6,7}$ are EM singlets. In analogy with the case of the Standard Model, we construct charge eigenstates out of the components with the same mass that transform into each other. Thus,

$$\delta(A^6 - iA^7) = -i\omega(A^6 - iA^7),$$
$$\delta(A^6 + iA^7) = +i\omega(A^6 + iA^7),$$

and

$$\delta(B^6 - iB^7) = -i\omega(B^6 - iB^7),$$
$$\delta(B^6 + iB^7) = +i\omega(B^6 + iB^7)$$

form the states $W_L'^{\pm}$ and $W_R'^{\pm}$ with masses $g\sqrt{p^2+q^2}/4$ and $gp/4$, respectively. In a similar vein, we find the states

$$\delta(A^1 - iA^2) = -i\omega(A^1 - iA^2),$$
$$\delta(A^1 + iA^2) = +i\omega(A^1 + iA^2),$$
$$\delta(B^1 - iB^2) = -i\omega(B^1 - iB^2),$$
$$\delta(B^1 + iB^2) = +i\omega(B^1 + iB^2)$$

which we denote $W_L^{\pm}$ and $W_R^{\pm}$, respectively. $W_R^{\pm}$ has the mass $gq/4$ and $W_L^{\pm}$ is identified as the SM $W^{\pm}$, with (SM scale) mass $g\sqrt{v^2+u^2}/4$.

3.5 Conclusions and outlook

Having concluded the initial analysis of the model, we are convinced that it deserves more detailed study. We will here collect a few comments on our results, and give a brief overview of the avenues of study which may be pursued.

First of all, it should be noted that, as we saw, $SU(3)_f$ is fully broken by this particular vacuum. We have, however, also been able to construct vacua which leave global $U(1)$ symmetries unbroken (placing the SM-breaking VEVs u, v on the off-diagonal instead of the diagonal). Although this particular vacuum spontaneously broke the Trinification gauge group down to a $U(1)$, the scalar components which get VEVs were not neutrally charged under it; the group is not electromagnetism. It should be possible to construct another vacuum (or perhaps change shift the representations of the fields) so that the charges are sensible, and there are unbroken global symmetries in addition. There are accidental global symmetries in the SM; $U(1)_{B-L}$ and the custodial $SU(2)$ symmetry on the scalar sector, for example. Unbroken remnants of the broken $SU(3)_f$ might be a compelling origin of these.

Furhtermore, the breaking of the global family symmetry should engender Goldstone bosons. We have not identified them, but doing so should be straightforward following the prescription we have developed.

As we saw, the tree-level masses for all but one scalar, all leptons and six quarks are zero. This means that mass terms need to be regenerated at the 1-loop level, via the radiative symmetry breaking process described in Section 1.2. Obviously, further investigation into whether or not this happens is needed. If the radiatively generated masses of the six quarks are small enough, they might correspond to the SM quarks, which is an interesting possibility.

An obvious area of study is the potential minimum at 1-loop and its stability. Work on this has commenced. A SARAH model file has also been developed for this trinified model; it is not included in this thesis since it is not completely done, but SARAH might help with such vacuum analysis.

Finally, we add that several Trinification variants are currently being studied by R. Pasechnik et al. Whether the model is supersymmetric or not; the form of the scalar potential and the various modes of symmetry breaking employed, all contribute to give the theory a promising array of different attributes.

4 Summary

We have studied two LR models; the $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ MLRM, and a trinified model with gauge group $SU(3)_L \otimes SU(3)_L \otimes SU(3)_R \otimes \mathbb{Z}_3$ with an additional, global, $SU(3)_f$ family symmetry. We have written the Lagrangian for each model, and derived masses, charges and representations of the particles. We have also implemented SARAH model files which should prove useful to future studies. The MLRM has been extensively studied previously; while the trinified model certainly requires further research, we find both models theoretically attractive and phenomenologically viable.

4.1 Acknowledgements

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A MLRM SARAH model file

```
Model`Name = "MLRM";
Model`NameLaTeX ="MLRM";
Model`Authors = "E. Corrigan";
Model`Date = "2015-03-08";

(* ---- Gauge Groups ---- *)

Gauge[[1]]={B, U[1], bminl, gBL, False};
Gauge[[2]]={WL, SU[2], left, g1, True};
Gauge[[3]]={WR, SU[2], right, g2, True};
Gauge[[4]]={G, SU[3], color, g3, False};

(* ---- Matter Fields ---- *)

FermionFields[[1]] = {qL, 3, {uL, dL}, 1/3, 2, 1, 3};
FermionFields[[2]] = {qR, 3, {conj[uR],conj[dR]},      -1/3, 1,
   -2, -3};
FermionFields[[3]] = {lL, 3, {vL, eL}, -1, 2, 1, 1};
FermionFields[[4]] = {lR, 3, {conj[vR],conj[eR]},      1, 1, -2,
   1};

ScalarFields[[1]] =  {phi, 1, {{phi10, phi1p},{phi2m, phi20}},
       0, 2, -2, 1};
ScalarFields[[2]] =  {delL, 1, {{delLp/Sqrt[2],delLpp},{delL0,-
   delLp/Sqrt[2]}},    2,  3, 1, 1};
ScalarFields[[3]] =  {delR, 1, {{delRp/Sqrt[2],delRpp},{delR0,-
   delRp/Sqrt[2]}},    2,  1, 3, 1};

NameOfStates={GaugeES,EWSB};

(* Before EWSB *)

DEFINITION[GaugeES][LagrangianInput]=
{
        {LagHC, {Overwrite->True, AddHC->True}},
        {LagNoHC,{Overwrite->True, AddHC->False}}
};
```

```
(* ----- Scalar potential ----- NOTE: SARAH warns about charge
   nonconservation for the vertices below, but they are
   contracted correctly.*)

LagNoHC = -(-mu12 phi.conj[phi] - mu22 epsTensor[lef2,lef1]
   epsTensor[rig2,rig1] conj[phi].conj[phi] - mu22 epsTensor[lef2
   ,lef1] epsTensor[rig2,rig1] phi.phi - mu32 delL.conj[delL] -
   mu32 delR.conj[delR]  + lambda1 phi.conj[phi].phi.conj[phi] +
   lambda2 epsTensor[lef2,lef1] epsTensor[rig2,rig1] epsTensor[
   lef4,lef3] epsTensor[rig4,rig3] conj[phi].conj[phi].conj[phi].
   conj[phi] + lambda2 epsTensor[lef1,lef2] epsTensor[rig2,rig1]
   epsTensor[lef3,lef4] epsTensor[rig4,rig3] phi.phi.phi.phi +
   lambda3 epsTensor[lef1,lef2] epsTensor[rig2,rig1] epsTensor[
   lef3,lef4] epsTensor[rig4,rig3] conj[phi].conj[phi].phi.phi +
   lambda4 epsTensor[lef2,lef1] epsTensor[rig2,rig1] Delta[lef3,
   lef4] Delta[rig3,rig4] conj[phi].conj[phi].phi.conj[phi] +
   lambda4 Delta[lef1,lef2] Delta[rig1,rig2] epsTensor[lef4,lef3]
    epsTensor[rig4,rig3] phi.conj[phi].phi.phi + rho1 delL.conj[
   delL].delL.conj[delL] + rho1 delR.conj[delR].delR.conj[delR] +
    rho2 delL.delL.conj[delL].conj[delL] + rho2 delR.delR.conj[
   delR].conj[delR] + rho3 delL.conj[delL].delR.conj[delR] + rho4
    delL.delL.conj[delR].conj[delR] + rho conj[delL].conj[delL].
   delR.delR + alpha1 phi.conj[phi].delL.conj[delL] + alpha1 phi.
   conj[phi].delR.conj[delR] + alpha2 epsTensor[lef2,lef1]
   epsTensor[rig2,rig1] Delta[rig3,rig4] Delta[rig3b,rig4b] phi.
   phi.delR.conj[delR] + alpha2 epsTensor[lef2,lef1] epsTensor[
   rig2,rig1] Delta[lef3,lef4] Delta[lef3b,lef4b] phi.phi.delL.
   conj[delL] + alpha2 epsTensor[lef2,lef1] epsTensor[rig2,rig1]
   Delta[rig3,rig4] Delta[rig3b,rig4b] conj[phi].conj[phi].delR.
   conj[delR] + alpha2 epsTensor[lef2,lef1] epsTensor[rig2,rig1]
   Delta[lef3,lef4] Delta[lef3b,lef4b] phi.phi.delL.conj[delL] +
   alpha3 Delta[lef1,lef4] Delta[lef2,lef3] Delta[lef3b,lef4b]
   Delta[rig1,rig2] phi.conj[phi].delL.conj[delL] + alpha3 Delta[
   lef1,lef2] Delta[rig1,rig3] Delta[rig3b,rig4b] Delta[rig2,rig4
   ] phi.conj[phi].delR.conj[delR] (* + beta1 Delta[lef1,lef4b]
   Delta[lef3,lef4] Delta[rig2,rig2b] Delta[rig1,rig3] phi.delR.
   conj[phi].conj[delL] + beta1 Delta[lef1,lef3] Delta[lef3b,lef3
   ] Delta[rig1,rig4b] Delta[rig3,rig4] conj[phi].delL.phi.conj[
   delR] + beta2 phitil.delR.conj[phi].conj[delL] + beta2 conj[
   phitil].delL.phi.conj[delR] + beta3 phi.delR.conj[phitil].conj
   [delL] + beta3 conj[phi].delL.phitil.conj[delR]*) );
```

```
(* ----- Yukawa Lagrangian ----- NOTE: SARAH warns about charge
   nonconservation for the vertices below, but they are
   contracted correctly.*)

LagHC=  -(hl conj[phi].lL.lR + hltil  phi.lL.lR + hqtil phi.qL.qR
    + hq conj[phi].qL.qR);

(* ----- VEVs ----- *)

DEFINITION[EWSB][VEVs]=
{
  {phi10, {vevk, 1/Sqrt[2]}, {Aphi10ps, I/Sqrt[2]},{hphi10s, 1/
     Sqrt[2]}},
  {phi20, {vevkprime, 1/Sqrt[2]}, {Aphi20ps, I/Sqrt[2]},{hphi20s,
      1/Sqrt[2]},{alphakprime}},
  {delL0, {vevL, 1/Sqrt[2]}, {AdelL0ps, I/Sqrt[2]},{hdelL0s, 1/
     Sqrt[2]},{betaL}},
  {delR0, {vevR, 1/Sqrt[2]}, {AdelR0ps, I/Sqrt[2]},{hdelR0s, 1/
     Sqrt[2]}}
};


(* ----- Matter sector mixing ----- *)

DEFINITION[EWSB][MatterSector]=
{
{{{dL}, {conj[dR]}}, {{DL,Vd}, {DR,Ud}}},
{{{uL}, {conj[uR]}}, {{UL,Vu}, {UR,Uu}}},
{{{eL}, {conj[eR]}}, {{EL,Ve}, {ER,Ue}}},
{{{vL}, {conj[vR]}}, {{NuL,VNu},{NuR, UNu}}},

{{hphi10s,hphi20s,hdelR0s,hdelL0s,Aphi10ps,Aphi20ps, AdelR0ps,
   AdelL0ps},{hrealim,Urealim}},
{{phi1p,conj[phi2m],delRp,delLp},{singchar,Usingchar}},
{{delRpp,delLpp},{doubchar,Udoubchar}}
};

(* ----- Gauge sector mixing ----- *)

DEFINITION[EWSB][GaugeSector] =
{
  {{VWL[3],VWR[3],VB},{VPP,VZ1,VZ2},UZ},
```

```
  {{VWL[1],VWL[2],VWR[1],VWR[2]},{VWp1, conj[VWp1],VWp2, conj[VWp2
      ]},UW}
};



(* ---- Dirac spinors ---- *)

DEFINITION[EWSB][DiracSpinors]=
{
  Fd ->{  DL,  conj[DR]},
  Fe ->{  EL,  conj[ER]},
  Fu ->{  UL,  conj[UR]},
  Fv ->{  NuL,  conj[NuR]}
};

DEFINITION[GaugeES][DiracSpinors]=
{
 Fd1 ->{  dL,  0},
 Fd2 ->{   0,  dR},
 Fu1 ->{  uL,  0},
 Fu2 ->{   0,  uR},
 Fe1 ->{  eL,  0},
 Fe2 ->{   0,  eR},
 Fv1 ->{  vL,  0},
 Fv2 -> {   0,vR}
};
```

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