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Chirally Symmetric Technicolor Model: A possible origin of the Higgs Boson

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Abstract

This thesis investigates a possible extension to the Standard Model: the Chirally Symmetric Technicolor Model. The motivation is to explain the origin of the electroweak interaction scale by introducing a new interaction. The thesis is based on the two-flavour Technicolor model presented in "Chiral-Symmetric Technicolor with Standard Model Higgs boson" by Pasechnik et. al. [1], and extends it to the three flavour case with a composite Higgs sector. The model is based on a low energy effective field theory known as the Linear Sigma Model, and borrows several properties from the approximate flavour $SU(3)$ chiral symmetry of Quantum Chromodynamics. A physical Lagrangian is derived and using this a possible signature for discovery in the $H \to \gamma\gamma$ channel is calculated.

Acknowledgements

This thesis would not have been made without the help, commitment and patience of my supervisor Roman Pasechnik. Thank you.

Populärvetenskaplig sammanfattning

Partikelfysiken utforskar de allra minsta beståndsdelarna av universum. Vissa partiklar — kvarkar och elektroner — är det som atomer består av, och de är stabila, men det finns många andra partiklar som är mycket svåra att skapa, och som bara kan existera under en mycket kort period innan de faller sönder. Allt vi vet om de här partiklarna har vi samlat ihop och satt samman till en teori som heter Standardmodellen. Standardmodellen kan förklara ganska mycket, till exempel varför partiklar får massa. Vissa partiklar är masslösa, medan andra är massiva, och de som är massiva är det för att de interagerar med Higgsfältet, som finns överallt. Till Higgsfältet hör en partikel: Higgsbosonen.

Vad vi vill veta är varför Higgsfältet har just det värde det har. Ett svar på denna fråga är att det skulle kunna finnas en helt ny grupp av partiklar som vi inte har upptäckt än, för att de kräver så mycket energi för att kunna bli upptäckta. Om de skulle finnas går det att beräkna vissa av deras egenskaper, till exempel hur de interagerar med varandra, och med de vanliga partiklarna, och hur det skulle se ut när man upptäcker dem. Just detta har gjorts i denna avhandling: jag har undersökt en teori om att det finns en oupptäckt sorts partiklar som kallas Technikvarkar, och försökt lista ut hur de skulle bete sig om de fanns.

Contents

1 Background

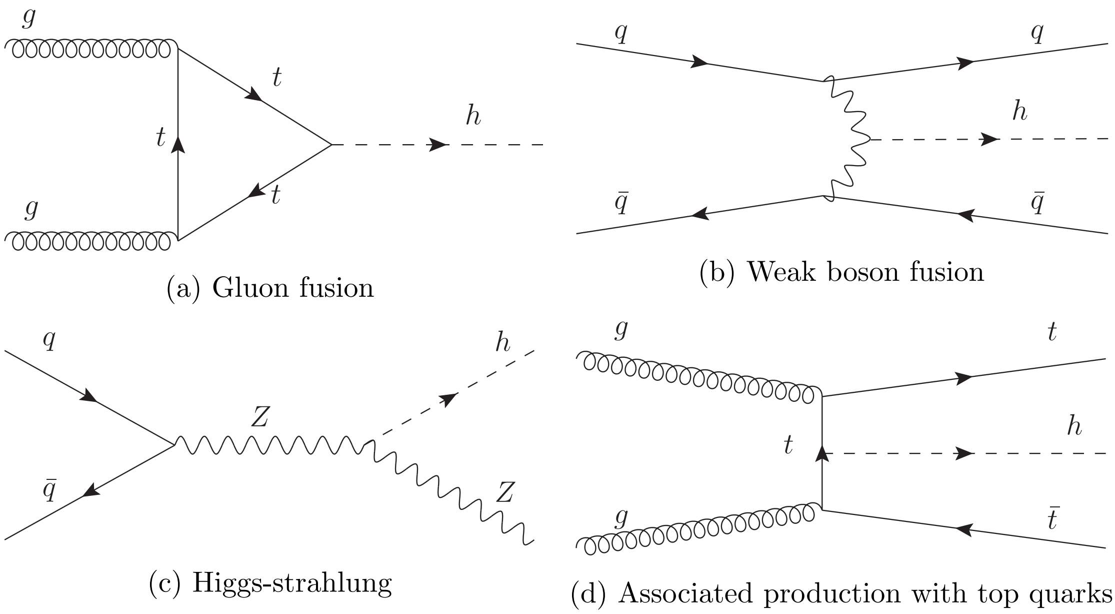
The Standard Model is currently the predominant theory used to describe the fundamental particles and how they interact with each other. The foundation of the Standard Model was laid out in the late fifties and early sixties. The quark model was proposed in 1964 by Murray Gell-Mann and George Zweig and got experimental support four years later. In the early seventies some theorists had predicted the existence of the charm quark and when it was found in 1974, the Standard Model quickly became the leading theory of particle physics. The model has over the years been very successful in making predictions, such as the existence of the top quark and the Higgs boson, and is very consistent with experimental results. It does however have some flaws and problems. For example, it is believed to be an effective field theory in the sense that it does only work well in energy scales we are able to measure, but breaks down at high enough energies. It does not explain the observed dark matter and baryon asymmetry in the universe. It does not include gravity, one of the four known fundamental forces, and there are plenty of free parameters which can not be derived from theory but have to be measured. Another unsolved question is what is called the problem of naturalness, which refers to the unexplained huge difference between the different interaction scales. The motivation of the Technicolor models comes from trying to find an origin of the electroweak interaction scale, from a new hidden strongly coupled dynamics at high energies.

1.1 The Standard Model Higgs boson

The massive particles of the Standard Model get their masses by means of the Higgs mechanism of spontaneous electroweak symmetry breaking, which is a breaking of the $SU(2)_L \otimes U(1)_Y$ symmetry down to electromagnetic $U(1)_{em}$ symmetry. The field responsible for this breaking is the Higgs field, and the particle associated with this field is the Higgs boson, a scalar boson with the approximate mass $m_H \approx 125$ GeV. The existence of such a field was first proposed in 1964 by several physicists, and in 2012 an announcement was made that a particle in the predicted mass range had been discovered at the ATLAS [2] and CMS [3] experiments at the LHC. This discovery led to the Nobel Prize in Physics being awarded to François Englert and Peter Higgs — two of the persons that first described the Higgs mechanism — in 2013. As of 2013 the measurements of the mass made at ATLAS [4] concluded it to be

$$m_h = 125.5 \pm 0.2(\text{stat})^{+0.5}_{-0.6}(\text{sys}) \text{ GeV}$$

In hadron colliders, there are four main mechanisms that produces the Higgs boson: gluon fusion, vector boson fusion, Higgs-strahlung or associated production with top quarks. These processes are all depicted in Feynman diagrams in figure 1. The mechanism with the largest cross section is the gluon fusion process, in which two gluons fuse and produce a Higgs boson via top quark loops. At a center of mass energy $\sqrt{s} = 14$ TeV, the cross section for this process is $\sigma = 49.8^{+9.96}_{-7.47}$ pb. Other quark loops contribute as well, but



(a) Gluon fusion (b) Weak boson fusion (c) Higgs-strahlung (d) Associated production with top quarks

Figure 1: The four Higgs boson production mechanisms.

the coupling between the quark and the Higgs boson is proportional to the mass, and the top quark mass is big enough that contributions from the other quarks are negligible. The second most common process is the vector boson fusion, in which a quark and an antiquark are scattered. They exchange a weak boson from which a Higgs boson is radiated. This process has the cross section $\sigma = 4.18^{+0.13}_{-0.13}$ pb. The Higgs-strahlung process contains a quark pair that collide and form a vector boson, from which the Higgs particle is radiated, and the fourth is a process in which the Higgs boson is radiated from top quarks. These two processes both have a cross section less than 2 pb. The Higgs boson could also be produced in e^+e^- colliders, mainly through the process $e^+e^- \to ZH$.

The total width of the Higgs boson is $\Gamma_H = 4.1 \cdot 10^{-3}$ GeV and there are several decay channels for the Higgs boson. The most common ones are $H \to b\bar{b}$ and $H \to W^+W^-$. The $H \to b\bar{b}$ channel, together with $H \to \tau^+\tau^-$, with branching ratios $5.8 \cdot 10^{-1}$ and $6.3 \cdot 10^{-2}$ respectively, are common decay channels but not very useful as these channels are disturbed by large background signals. The $H \to W^+W^-$ channel has the branching ratio $2.2 \cdot 10^{-1}$, and is thus the second most common decay channel, but as the W bosons decay, they produce neutrinos which can not be detected in the accelerators, making it hard to trace the process backwards to get an accurate measurement of the Higgs mass.

The most relevant decay channel for this thesis is $H \to \gamma\gamma$, with a branching ratio of $2.28^{+0.11}_{-0.11} \cdot 10^{-3}$ [5]. The cross section has a narrow peak embedded in a background consisting mainly of $\gamma\gamma$ events, particle jets and dijets. A typical background contributing is the quark loop decay depicted in figure 3. In this channel the boson decays via loops into two photons, mainly through W loops, as is shown in figure 2. These loops are sensitive to contributions from new physics; comparing the decay width predicted by the Standard

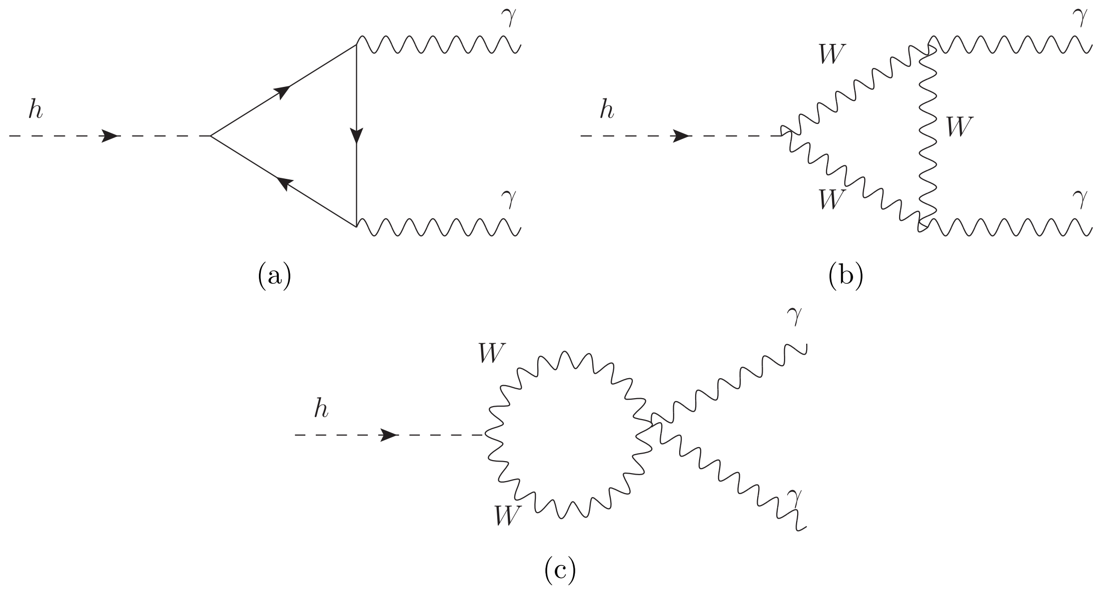


Figure 2: Three typical decay modes of the Higgs boson, via fermion and weak boson loops.

Model with predictions from Standard Model extensions is a good way of testing new theories [4, 5, 6]. Measurements of this channel has been made at both ATLAS and CMS. Results from the measurements made at ATLAS is displayed in figure 4.

1.2 The naturalness problem

The naturalness problem considers the unexpectedly vast differences in between the different energy scales, foremost the difference between the energy scale that characterises weak interactions, about the size of the Higgs vacuum expectation value (vev) $v = 246$ GeV [7], and the scale that characterises gravitational processes, the size of the Planck mass $M_P = 1.22 \cdot 10^{-19}$ GeV [7].

The masses of the Standard Model particles comes from interactions with the Higgs field. In vacuum, virtual particles are created and destroyed due to quantum fluctuations, and these interact with the Higgs field with a strength proportional to the available energy. The Higgs boson then receives an extra contribution to its mass which, if there is enough energy available, suggests that both the mass and the vev are unstable. Other parameters of the Standard Model could be perturbed by quantum fluctuations as well, but are "protected" by so called custodial symmetries which prevent radiative corrections to change its value. The Higgs mass is not protected by such a symmetry, but a solution to this problem would be to introduce this symmetry, together with new interactions at an energy scale about a TeV, which is what the technicolor models do [8, 9, 10]

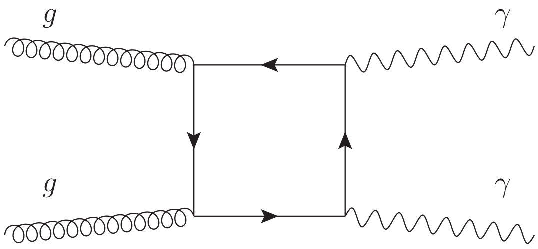


Figure 3: A box diagram showing a typical background contribution in the $H\gamma\gamma$ channel. The gluons fuse via fermion loops into photons.

1.3 The Technicolor Models

Technicolor models introduce new particles called technifermions and new interactions at the scale Λ_{TC} which in many aspects resemble the interactions in Quantum Chromodynamics (QCD). In a model consisting of N_{TC} techniquarks, a new gauge group, $SU(N_{TC})$, is introduced, and together with it $N_{TC}^2 - 1$ new gauge bosons called technigluons. The techniquarks are assumed to be confined, just as quarks, and form bound states: technimesons and technibaryons. In vacuum they will form a condensate

$$\langle \bar{Q}Q \rangle \sim \Lambda_{TC}^3$$

which gives rise to constituent techniquark masses of order Λ_{TC}. This condensate is analogous to condensates formed in QCD, which is described in reference [11]. There are several technicolor models with different components and which makes different predictions. Here follow a short review of a couple of them [8].

1.3.1 Minimal TC Model

The first of these models, introduced in 1977 by Weinberg and Susskind, was the Minimal Technicolor model, whose gauge group is $SU(N_{TC}) \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. The model includes at least two technicolor flavours which form a doublet in isospace. The Dirac doublet is split up in a left- and a right-handed chiral weak doublet.

Since the technimodel in much extent is analogous to QCD, some of the properties of this new sector can be found by rescaling. Starting from the scaling factors $f_\pi \approx 93$ MeV and $\Lambda_{QCD} \approx 200$ MeV, the scale on which the new interactions take place is approximately

$$\Lambda_{TC} \approx \Lambda_{QCD} \frac{\sqrt{3}v}{f_\pi \sqrt{N_D N_T}}$$

and the scales are connected to the electroweak scale as

$$v \approx f_\pi \sqrt{\frac{N_D N_T}{3}} \left(\frac{\Lambda_{TC}}{\Lambda_{QCD}} \right) \tag{1.1}$$

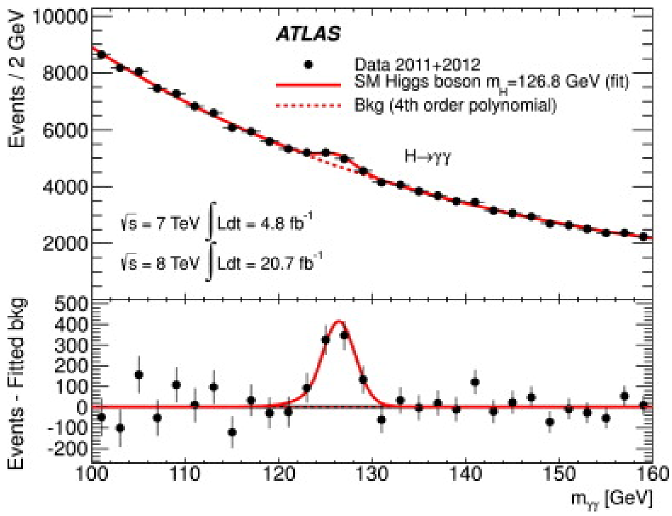


Figure 4: In the upper panel the invariant mass distribution of diphoton events measured at ATLAS is presented. The data consists of combined results from both 7 TeV and 8 TeV measurements. A computed Standard Model Higgs signal and a background is fitted to the data. In the lower panel, the deviation of the measurements from the fit is shown. The figure is taken from [4].

where N_D is the number of technicolor doublets and N_T is the number of triplets included in the model and v is the vacuum expectation value associated with the Higgs field, and which is introduced in section 2.1. Using $N_T = 4$ and $N_D = 1$ for example, the scale becomes $\Lambda_{TC} \approx 460$ GeV [8].

Performing a renormalisation of the running coupling of the technicolor sector to get the ratio between the technicolor scale and the QCD scale, and comparing to equation (1.1) predicts a vacuum expectation value that lies near the actual measured value. The biggest problem with this model is however that it does not contain any way for the techniquarks to decay into ordinary matter but suggests that the lightest techniquark hadrons are stable. Extensions of this model often include a way for the techniquarks to decay.

1.3.2 Extended TC models

There are several extended technicolor models which share some general traits. This section goes through the common properties of these models and then describe two specific ones. The shortcomings of the minimal techniquark model is mainly two: it does not allow technifermions to decay, and it does not contain any coupling to leptons or quarks. If technifermions were stable, they would form technibaryons which would be present on Earth in much larger densities than made measurements allow for [12]. In the extended models the quark and lepton masses rise from the electroweak symmetry breaking and the symmetry is broken by the technicolor condensate.

To allow for technifermion decay currents like

$$\bar{Q}_{L,R}\gamma^{\mu}\phi_{L,R}$$

are introduced. Here Q represent a techniquark and ϕ a quark, and the L and R denote their respectively helicity. Similar terms are introduced for the leptons. In order to contain these interactions a large gauge group is introduced. For example if one would like to extend the example of a minimal technicolor model above, with $N_D = 1$ and $N_{TC} = 4$, the gauge group accommodating all the techniquark-quark and techniquark-lepton interactions would be $SU(16)$. On top of that the chiral group must be included so the total gauge group would be $SU(16) \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_Y$. The fermions are arranged in two 16 multiplets in fundamental representation under $SU(16)$, where one is a doublet under $SU(2)_L$ and singlet under $SU(2)_R$ and vice versa [8].

A new theory must comport with available experimental data, and one way to decide how well a new model fits to actual measurements is to do an electroweak precision test. The Peskin-Tacheuki parameters, S, T and U, are used to parametrise contributions to electroweak radiative corrections that a new model would give rise to. They are defined from the mass of the top quark and the Higgs boson masses and have the values [13]

$$S = 0.00^{+0.11}_{-0.10}$$
$$T = 0.02^{+0.11}_{-0.12}$$
$$U = 0.08^{+0.11}_{-0.11}$$

New physics would alter these values but they must be kept within the margin of error. The S parameter is sensitive to the number of weak isodoublets and the number of technicolors, and both the minimal and extended models yield contributions to this parameter that put them outside the margin [8, 1].

2 Introduction

The Standard Model is formulated in terms of a Lagrangian density, or a Lagrangian as it will be referred to in this thesis. The concept of Lagrangians is briefly reviewed in appendix A.2 and the Standard Model is shortly summarised in appendix B.

The concept of symmetry breaking is something this thesis to a great extent is centered around, and this is therefore elaborated on below. First the idea behind the Higgs field is developed. This scalar field has a symmetry that is spontaneously broken which gives rise to massive bosons. This symmetry breaking and how it induces mixing in the W_μ^i and B_μ fields is explained as the same procedure later is used to find how the technimeson fields mix. Symmetry breaking is related to the Goldstone theorem which is gone through together with dynamical symmetry breaking which is of importance to this certain technicolor model. Finally the Linear Sigma Model is introduced, as an effective field theory on which the interactions of our technicolor model is based.

2.1 The Higgs field

The Lagrangian for the Higgs field is

$$\mathcal{L}_{Higgs} = (D_\mu \mathcal{H})^\dagger (D^\mu \mathcal{H}) - \mu^2 \mathcal{H}^\dagger \mathcal{H} - \lambda (\mathcal{H}^\dagger \mathcal{H})^2 \tag{2.1}$$

The Higgs field is a $SU(2)$ doublet

$$\mathcal{H} = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$$

The two components are complex fields that can be written as

$$H^+ = \tfrac{\phi_1 + i\phi_2}{\sqrt{2}} \quad H^0 = \tfrac{\phi_3 + i\phi_4}{\sqrt{2}}$$

which means that the term $\mathcal{H}^\dagger \mathcal{H}$ is

$$\mathcal{H}^\dagger \mathcal{H} = \begin{pmatrix} H^{+\dagger} & H^{0\dagger} \end{pmatrix} \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} = \frac{\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2}{2} \tag{2.2}$$

The above equation constitutes a symmetry in the four fields: exchanging two fields with each other would not affect the Lagrangian. The potential energy of the field is

$$V = \mu^2 \left(\mathcal{H}^\dagger \mathcal{H}\right) + \lambda \left(\mathcal{H}^\dagger \mathcal{H}\right)^2 \tag{2.3}$$

To find the minimum the above expression is derived and set to zero:

$$\frac{\partial V}{\partial \phi_1} = \left(\mu^2 + \lambda(\mathcal{H}^\dagger \mathcal{H})\right) \phi_1 = 0$$

Deriving with any other of the four field components will yield similar results. $\mathcal{H}^\dagger \mathcal{H}$ is positive definite, so the minimum of the potential depends on the value of μ^2. If $\mu^2 > 0$,

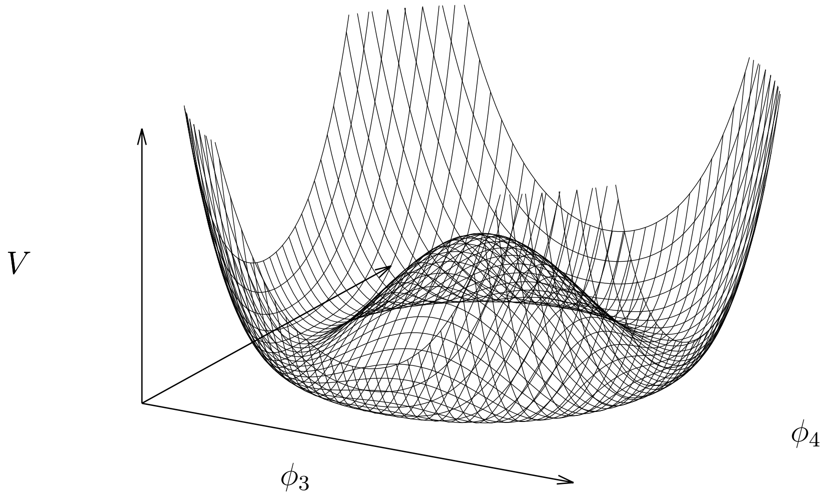


Figure 5: The general shape of the potential of the Higgs field plotted in two dimensions in arbitrary units. The minimum of the potential is not in $\phi_3 = \phi_4 = 0$ but in a circle around the origin.

then the only extremum is when $\phi_1 = \phi_2 = \phi_3 = \phi_4 = 0$, which is a minimum. However, if $\mu^2 < 0$ there is a minimum when

$$\mathcal{H}^\dagger\mathcal{H} = \frac{-\mu^2}{2\lambda} = \frac{v^2}{2} \tag{2.4}$$

Here the quantity $v = \sqrt{-\mu^2/\lambda}$ is introduced. This is the vacuum expectation value (vev) of the Higgs field. In this case, the potential will not have its minimum in the origin, but in infinetly many points forming a circle around it. The potential is shown plotted in two dimensions in figure 5. As there are four independent parameters in equation (2.4) we must choose a point in order to proceed. The choice will be $\phi_3 = v+h+i\eta$ and $\phi_1 = \phi_2 = \phi_4 = 0$. h and η are expansions around this minimum, so that

$$\mathcal{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h + i\eta \end{pmatrix} \tag{2.5}$$

Exitations of the h field corresponds to an increase of the potential. Exitations of the η field corresponds to a rotation along the minimum cirle. If the Higgs doublet, as it is written above, is reintroduced into the Higgs Lagrangian and expanded, one will see that the η field corresponds to a massless particle. This particle is known as a Goldstone boson. The Goldstone theorem states that whenever a symmetry is spontaneosly broken, there will be one Goldstone boson corresponding to every generator that breaks the symmetry. We are however free to choose which gauge to work in, and can choose to set $\eta = 0$. The Goldstone particle only has a logitudinal polarisation state, but by choosing this gauge, this degree of freedom is aquired by the massive vector bosons [14, 15]. Studying the kinetic term we see that

$$(D_\mu \mathcal{H})^\dagger (D^\mu \mathcal{H}) = \mathcal{H}^\dagger D_\mu^\dagger D^\mu \mathcal{H}$$

will yield new interaction terms from which gauge boson masses arise. These terms are

$$\mathcal{H}^\dagger \left(ig\frac{Y}{2}B_\mu + ig'\frac{\tau_i}{2}W_\mu^i \right)^\dagger \left(ig\frac{Y}{2}B_\mu + ig'\frac{\tau_i}{2}W_\mu^i \right) \mathcal{H}$$

Putting $Y = 1$ and using the field of equation (2.5), with $h \to 0$, the above expression could be rewritten into

$$\frac{1}{8}v^2 g'^2 \left((W_\mu^1)^2 + (W_\mu^2)^2 \right) + \frac{1}{8}v^2 \left(gB_\mu + g'W_\mu^3 \right)^2$$

If we use W_μ^1 and W_μ^2 to define charged states $W_\mu^\pm = (W_\mu^1 \mp +iW_\mu^2)/\sqrt{2}$ and a neutral state $W_\mu^0 = W_\mu^3$we see that the first term can be written as

$$\frac{1}{8}v^2 g'^2 \left((W_\mu^1)^2 + (W_\mu^2)^2 \right) = \left(\frac{vg'}{2} \right)^2 W_\mu^+ W^{-\mu}$$

The term $vg'/2$ is interpreted as a mass term, and thus the $W^\pm$ bosons have a mass $M_W = vg'/2$. The term $\frac{1}{8}v^2 \left(gB_\mu + g'W_\mu^3 \right)^2$ is a non-diagonal matrix, but can be diagonalised by a basis change. If we introduce a new field

$$Z_\mu = \frac{gB_\mu + g'W_\mu^0}{\sqrt{g^2 + g'^2}}$$

the term is diagonalised to

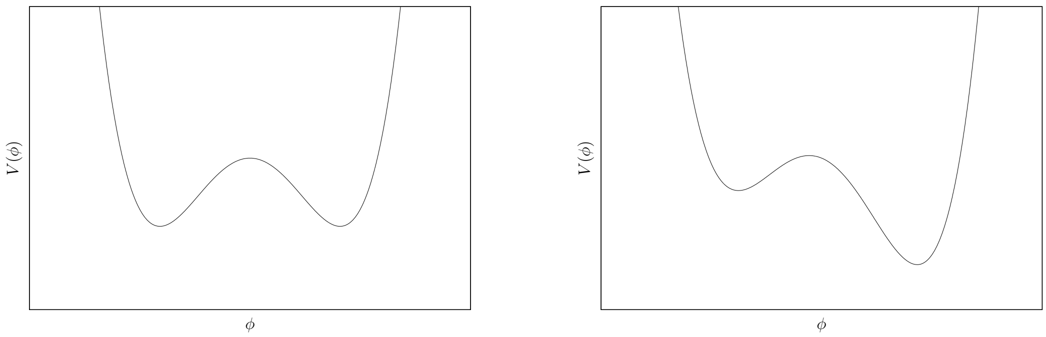
$$\frac{1}{8}v^2 \left(gB_\mu + g'W_\mu^3 \right)^2 = \frac{1}{2} \left(\frac{v}{2\sqrt{g^2 + g'^2}} \right)^2 Z_\mu Z^\mu$$

This field is the Z boson field. The mass of the particle is $M_Z = \frac{v}{2\sqrt{g^2+g'^2}}$. We may also construct the field who will be orthogonal to the Z_μ field as:

$$A_\mu = \frac{g'B_\mu - gW_\mu^0}{\sqrt{g^2 + g'^2}}$$

This is the field of the photon, the massless particle that mediates the electromagnetic force.

The change of basis between W_μ, B_μ and Z_μ, A_μ can be written as a rotation:



(a) Without source term. (b) With source term.

Figure 6: One dimensional representations of spontaneously broken potentials to arbitrary units. In figure 6a the potential does not contain any source term, making the number of minima points infinite. In figure 6b the source term is added which tilts the potential. This potential has only one minimum.

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W^0_\mu \\ B_\mu \end{pmatrix} \tag{2.6}$$

This angle θ_W is the Weinberg angle or the weak mixing angle. The ratio between the W and Z boson masses is

$$\frac{M_W}{M_Z} = \cos\theta_W$$

To sum up this is the electroweak symmetry breaking that give rise to the masses of the massive gauge bosons. The Higgs mechanism is also responsible for the masses of the fermions, through a different process: the Yukawa terms, which are reviewed in appendix B.3.

2.2 Dynamical symmetry breaking

In the electroweak symmetry breaking, the symmetry is broken when one of the minimum points are chosen, but we are however free to choose which point. A one dimentional representation of this potential is shown in figure 6a. This potential requires that $\mu^2 < 0$. The techniquarks form a vacuum condensate, which if using the Linear Sigma Model (section 2.4) corresponds to adding a linear term to the potential. In this way the scalar vev acquire a dynamical interpretation in terms of a techniquark condensate. In figure 6b the same potential is plotted as in figure 6a but with a linear term added to it. The result is that the whole potential is tilted and that there actually is only one single unique minimum point in the potential.

Earlier the symmetry breaking gave rise to Goldstone bosons, which were massless as exitations of them corresponded to movements along a minimum, requiring no extra energy. When this linear term is present, the Goldstone bosons aquire a small mass and are therefore refered to as pseudo-Goldstone bosons. This dynamical symmetry breaking also has the consequence that the constant μ^2 no longer is needed. It is introduced in equation (2.3) as it is required for the symmetry to break, and the Higgs vev squared is proportional to it. With the linear term, the potential is asymmetric regardless of whether μ^2 is positive, negative or zero, and particles that gain mass from a dynamicaly broken symmetry does so even without it.

2.3 The chiral transform

The most important transform for this thesis is the chiral transform [16]

$$\mathbf{U}_5 = e^{-ig_5\gamma^5\frac{1}{2}\tau_i\epsilon_i} \tag{2.7}$$

which is a transform that operates in flavour space, γ^5 is given by equation (A.5) and g_5 is some constant. If a Lagrangian is invariant under a chiral transform we have a chiral symmetry. The transform matrix U_5 satisfy the relation

$$\gamma^\mu\mathbf{U}_5 = \mathbf{U}_5^\dagger\gamma^\mu$$

which means that the field transform as

$$\begin{aligned}
&\psi \to \psi' = \mathbf{U}_5\psi \\
&\bar{\psi} \to \bar{\psi}' = \psi^\dagger\mathbf{U}_5^\dagger\gamma^0 = \psi^\dagger\gamma^0\mathbf{U}_5 = \bar{\psi}\mathbf{U}_5
\end{aligned}$$

and the kinetic term of the Dirac Lagrangian in equation (A.4) will transform as

$$i\bar{\psi}\mathbf{U}_5\gamma^\mu\partial_\mu\mathbf{U}_5\psi = i\bar{\psi}\mathbf{U}_5\mathbf{U}_5^\dagger\gamma^\mu\partial_\mu\psi = i\bar{\psi}\gamma^\mu\partial_\mu\psi \tag{2.8}$$

Thus the kinetic term is invariant under chiral transforms. The mass term is however not invariant under this transform:

$$m\bar{\psi}'\psi' = m\bar{\psi}\mathbf{U}_5\mathbf{U}_5\psi \neq m\bar{\psi}\psi$$

In QCD the Dirac Lagrangian is therefore not chirally symmetric for massive particles. One can though consider the so called chiral limit, where the kinetic term is much bigger than the potential so that the mass term can be neglected, in which case the symmetry is conserved. This is why the chiral symmetry only is considered to be an approximate symmetry.

The QCD Lagrangian, equation (B.9), contains a Dirac term which is rewritten as

$$\mathcal{L} = \bar{q}i\gamma^\mu D_\mu q - \bar{q}\mathcal{M}q$$

where the quarks are put together in a column vector:

$$q = \begin{pmatrix} u \\ d \end{pmatrix} \quad \text{or} \quad q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

for the two and three flavour case (more flavours could also be added). The mass matrix is in the three flavour case

$$\mathcal{M} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}$$

If we rewrite equation (B.9) with the use of the operators in equation (A.6) we split the quark fields into their left and right handed projections:

$$\mathcal{L} = \bar{q}_L i\gamma^\mu D_\mu q_L + \bar{q}_R i\gamma^\mu D_\mu q_R - \bar{q}_L \mathcal{M} q_R - \bar{q}_R \mathcal{M} q_L \tag{2.9}$$

In the chiral limit, when $\mathcal{M} = 0$, we see that the symmetry of equation (2.8) is maintained in equation (2.9), so the chiral symmetry group can be divided into two subgroups. We denote the chiral symmetry group G_χ and divide it into one left handed and one right handed group:

$$G_\chi = SU(n_f)_L \otimes SU(n_f)_R$$

with n_f being the number of flavours considered. In the two flavour case, the quark doublet is symmetric under the $SU(2)_L \otimes SU(2)_R$ global symmetry, and in the three flavour case, the symmetry group is $SU(3)_L \otimes SU(3)_R$. The chiral transforms will now be denoted as

$$g_L \in SU(n_f)_L, \quad g_R \in SU(n_f)_R$$

and the quark fields transform as $q_L \to g_L q_L$ and $q_R \to g_R q_R$.

2.4 The Linear Sigma Model

As seen in section 2.3 the Dirac Lagrangian is not invariant under chiral transforms when the particle is massive. It is however possible to construct a theory where the particle is massless at the beginning and gains a mass due to an interaction leading to a spontaneous symmetry breaking similar to the symmetry breaking of the Higgs field described in section 2.1. One of the theories constructed in this ways is the Linear Sigma Model (LσM).

In the LσM the mass is assumed to rise from a fermion-meson interaction. In the low-energy spectrum, the interaction term between fermions and a meson can be written as

$$\mathcal{L}_{int} = -g\bar{\psi}\Sigma\psi$$

where ψ is the fermion field and Σ is a linear combination of two fields:

$$\Sigma = \sigma \mathbb{1} + i\gamma^5 \tau_i \pi_i$$

The σ is chosen to be a scalar field, and π_i are three pseudoscalar fields [16]. For chiral symmetry we now require that

$$\mathcal{L}'_{int} = -\bar{\psi}\mathbf{U}_5 \Sigma' \mathbf{U}_5 \psi = -\bar{\psi}\Sigma\psi = \mathcal{L}_{int}$$

or in other words that Σ transforms as $\Sigma' = \mathbf{U}_5^\dagger \Sigma \mathbf{U}_5^\dagger$. By expanding the chiral transform as a series and performing a series of algebraic manipulations described in reference [16], it can be shown that the fields transform as

$$\begin{aligned}
\sigma' &= \sigma \cos 2\theta - (\pi \cdot \hat{\epsilon}) \sin 2\theta \\
\pi'_i &= \pi_i - (\pi \cdot \hat{\epsilon})\hat{\epsilon}_i + \hat{\epsilon}_i \left(\sigma \sin 2\theta + (\pi \cdot \hat{\epsilon}) \cos 2\theta\right)
\end{aligned}$$

Here π is the triplet containing the three π_i fields, $\hat{\epsilon}$ is the (normalised) vector containing the three parameters ϵ_i of the chiral transform, equation (2.7) and $\theta = |\epsilon|/2$. The interpretation of the chiral transform is a rotation in the plane constituted by σ and $\hat{\epsilon}$, and the components perpendicular to π are unchanged. The transforms above can be expanded as a series with respect to ϵ and keeping only the first order terms yields the infinitesimal transform:

$$\sigma' = \sigma - \pi_i \epsilon_i, \quad \pi'_i = \pi_i - \sigma \epsilon_i$$

Using this we can construct a kinetic term for the Σ field that is invariant under the infinitesimal transform:

$$\mathcal{L}_{KE} = \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma + \frac{1}{2}\partial_\mu \pi_i \partial^\mu \pi_i$$

Using the relation

$$1/2\text{tr}(\Sigma\Sigma^\dagger) = \frac{1}{2}\text{tr}\left((\sigma + i\gamma^5 \tau_i \pi_i)(\sigma - i\gamma^5 \tau_i \pi_i)\right) = \sigma^2 + \pi^2 = |\Sigma|^2$$

we can rewrite the kinetic term as

$$\mathcal{L}_{KE} = \frac{1}{4}\text{tr}\left(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\right)$$

We can now write the complete Lagrangian of the LσM of the two-flavour $SU(2)_L \otimes SU(2)_R$ Technicolor model with an elementary Higgs boson [1]:

$$\mathcal{L}_{L\sigma M} = \frac{1}{2}\bar{\psi} i\gamma^\mu \partial^\mu \psi - \bar{\psi}\Sigma\psi + \frac{1}{2}\text{tr}(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger) - \mathcal{L}_{self} + \mathcal{L}_{source} \tag{2.10}$$

with the self interaction term

$$\mathcal{L}_{self} = \frac{1}{2}\mu^2|\Sigma|^2 - \frac{1}{4}\lambda_{SP}|\Sigma|^4 + \mu_H\mathcal{H}^2 - \lambda_H\mathcal{H}^4 + \lambda\mathcal{H}^2|\Sigma|^2$$
$$= \frac{1}{2}\mu^2(\sigma^2+\pi^2) - \frac{1}{4}\lambda_{SP}(\sigma^2+\pi^2)^2 + \mu_H\mathcal{H}^2 - \lambda_H\mathcal{H}^4 + \lambda\mathcal{H}^2(\sigma^2+\pi^2)$$

where a Higgs field $\mathcal{H}$ has been added and

$$\mathcal{L}_{source} = -gs\left\langle\bar{\psi}\psi\right\rangle$$

We have now added the self-interaction terms of the Σ field, the two last terms above, which looks exactly like the potential part of the Higgs field Lagrangian in equation (2.1). We have also added the source term $-gs\left\langle\bar{\psi}\psi\right\rangle$. The meaning of the new field s will be explained later. We may break the chiral symmetry of the LσM Lagrangian in a way similar to the steps of equation (2.3) to (2.4). The σ field then gains a vev. Just as we had to choose a field to expand around in equation (2.2) a choice must be made here as well. The choice is made so that the π fields are zero. We now define a new field s so that

$$\sigma = s + u$$

In the vacuum the s field disappears and $\sigma = u$. In section 2.1 it is shown that the Higgs field can be written as

$$\mathcal{H} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+H \end{pmatrix}$$

where H is some expansion around the Higgs vev. To see that the vacuum is indeed the minimum of the self interaction Lagrangian it has to fulfill the vacuum stability conditions:

$$\left\langle\frac{\delta\mathcal{L}_{self}}{\delta\sigma}\right\rangle = \left\langle\frac{\delta\mathcal{L}_{self}}{\delta\mathcal{H}}\right\rangle = 0$$

The calculations are carried out, using $\langle\sigma\rangle = u$, $\langle\pi\rangle = 0$ and $\langle\mathcal{H}\rangle = 1/\sqrt{2}\begin{pmatrix} 0 & v \end{pmatrix}^T$.

$$\left\langle\frac{\delta\mathcal{L}_{self}}{\delta\sigma}\right\rangle = \mu^2\langle\sigma\rangle - \lambda_{SP}\langle\sigma\rangle\left(\langle\sigma\rangle^2 + \langle\pi\rangle^2\right) + 2\lambda\langle\mathcal{H}\rangle^2\langle\sigma\rangle - g\left\langle\bar{\psi}\psi\right\rangle$$
$$= u\left(\mu^2 - \lambda_{SP}u^2 + \lambda v^2 - g\frac{\left\langle\bar{\psi}\psi\right\rangle}{u}\right) = 0$$
$$\left\langle\frac{\delta\mathcal{L}_{self}}{\delta\mathcal{H}}\right\rangle = 2\mu_H\langle\mathcal{H}\rangle - 4\lambda_H\langle\mathcal{H}\rangle^3 + 2\lambda\langle\mathcal{H}\rangle\left(\langle\sigma\rangle^2 + \langle\pi\rangle^2\right)$$
$$= \sqrt{2}v\left(\mu_H - \lambda_H v^2 + \lambda u^2\right) = 0$$

Here we identify $m_\pi^2 = -g\frac{\langle\bar{\psi}\psi\rangle}{u} = \mu^2 - \lambda_{SP}u^2 + \lambda v^2$. This is motivated later. As neither u or v are zero, the condition for vacuum stability is that

$$\begin{cases} \mu^2 - \lambda_{SP}u^2 + \lambda v^2 - m_\pi^2 &= 0 \\ \mu_H - \lambda_H v^2 + \lambda u^2 &= 0 \end{cases}$$

Here it can be shown that $\langle\pi\rangle$ has to be zero; assigning a vev different from zero to the pions would violate the vacuum stability conditions. From the equation system above we get expressions for the vevs:

$$u^2 = \frac{\lambda_H(\mu^2 + m_\pi^2) + \lambda\mu_H^2}{\lambda_{SP}\lambda_H - \lambda^2}$$
$$v^2 = \frac{\lambda(\mu^2 + m_\pi^2) + \lambda_{SP}\mu_H^2}{\lambda_{SP}\lambda_H - \lambda^2}$$

In the self interaction Lagrangian we may extract the coefficients in front of the terms σ^2, $\mathcal{H}^2$ and $\sigma\mathcal{H}$. Writing the fields as their vevs plus an expansion, we get a quadratic form looking as

$$\frac{1}{2}\left(-m_\pi^2 - 2\lambda_{SP}s^2 + 4\lambda uvHs - 2\lambda_H v^2H^2\right) = -\frac{1}{2}\begin{pmatrix} s & H \end{pmatrix}\begin{pmatrix} m_{11}^2 & m_{12}^2 \\ m_{21}^2 & m_{22}^2 \end{pmatrix}\begin{pmatrix} s \\ H \end{pmatrix}$$

where

$$\begin{pmatrix} m_{11}^2 & m_{12}^2 \\ m_{21}^2 & m_{22}^2 \end{pmatrix} = \begin{pmatrix} m_\pi^2 + 2\lambda_{SP}u^2 & -2\lambda uv \\ -2\lambda uv & 2\lambda_H v^2 \end{pmatrix}$$

As the matrix above is symmetric, the spectral theorem states that it can be diagonalised. The eigenvalues are found by solving the characteristic equation:

$$\begin{vmatrix} m_{11}^2 - \lambda & m_{12}^2 \\ m_{12}^2 & m_{22}^2 - \lambda \end{vmatrix} = \lambda^2 - (m_{11}^2 + m_{22}^2)\lambda + (m_{11}^2 m_{22}^2 - m_{12}^4) = 0$$

The solutions to this equation is

$$M_H^2 = \frac{1}{2}m_\pi^2 + \lambda_{SP}u^2 + \lambda_H v^2 - \sqrt{(\frac{1}{2}m_\pi^2 + \lambda_{SP} + \lambda_H v^2)^2 - 2\lambda_H m_\pi^2 v^2 - 4(\lambda_{SP}\lambda_H - \lambda^2)u^2v^2}$$

$$M_\sigma^2 = \frac{1}{2}m_\pi^2 + \lambda_{SP}u^2 + \lambda_H v^2 + \sqrt{(\frac{1}{2}m_\pi^2 + \lambda_{SP} + \lambda_H v^2)^2 - 2\lambda_H m_\pi^2 v^2 - 4(\lambda_{SP}\lambda_H - \lambda^2)u^2v^2}$$

which mean that we now have found the masses of the physical Higgs and sigma particles in terms of the constants of the Lagrangian in (2.10). The squared masses above are positive definite, which they have to be if the vacuum is stable.

3 Method

The work in this thesis has been divided into several stages, each stage beeing a step in the process of going from a purely theoretical foundation to the results consisting of observables which can be meassured in an particle accelerator. The first stage has been to review the mathematical and conceptual basis on which the Standard Model and its extensions is constructed, including group theory, the concept of covariant derivatives, the Dirac equation and the chiral transform. This has also included a short review of the Standard Model itself and its different sectors.

The second stage has been to construct the chirally symmetric technicolor model. This was done by reproducing the results from unpublished notes by Roman Pasechnik [17]. Starting from the LσM Lagrangian (2.10), an effective model was built on interactions between the techniquarks and scalar technimesons. From here, the basis of the Lagrangian was changed from gauge to mass basis. The diagonalisation of the mass matrix was done using the computational software Mathematica [18].

The third stage involved a Mathematica package called FeynRules [19], which was used to derive all interaction vertices of the physical Lagrangian. The output file of this package was processed by three other packages: FeynArts [20], FormCalc [21] and LoopTools [22]. FeynArts generated the Feynman diagrams and amplitudes of the model, FormCalc calculated the tree-level and one-loop diagrams and LoopTools evaluated the one-loop integrals. A description of how these packages work is given in appendix C.

The first model file for FeynRules was made using the results aquired from the diagonalisation. There was problem here however. For some reason the chosen linear combination triggered a bug somewhere. Much time was spent on trying to get this model to work, and then later to identify the bug. Whether the bug was in the model file or in the FeynRules package was not decided. When the discovery was made where the bug was triggered, the choice of parameters was changed to another equivalent set suggested by Roman Pasechnik. The latter choice has enabled us to perform numerical analysis of the Higgs boson observables in the $H \to \gamma\gamma$ channel at the LHC.

4 Three techniquarks

In this section the fundamental properties of the three flavour $SU(3_L)\otimes SU(3)_R$ Technicolor model is laid out and from this the physical states are derived. First the transformation properties of the techniquarks is gone through and it is shown that they indeed are chirally symmetric. Then the Lagrangian of the model is constructed from the LσM gone through above. The chiral symmetry is broken, and then the electroweak symmetry, which gives rise to mixings of the fields. Finally the basis is changed into mass basis and the gauge fields are expressed in terms of the physical fields.

4.1 Transformation properties

The three flavour techniquark model contains three techniquarks divided into two families. The left handed techniquarks are arranged in a bi-doublet $Q^{a\alpha}_{L(A)}$. The index $a = 1, 2$ is the index of the fundamental $SU(2)_W$ representation and $\alpha = 1, 2$ is the index of $SU(2)_{TC}$. $A = 1, 2$ represent the generation of T-quarks considered. The right handed T-quarks are singlets in the $SU(2)_W$ representation but doublets under $SU(2)_{TC}$.

The infinitesimal group transform of the bi-doublets are

$$Q^{a\alpha\prime}_{L(A)} = Q^{a\alpha}_{L(A)} + \frac{i}{2}g_W\theta_k\tau^{ab}_k Q^{b\alpha}_{L(A)} + \frac{i}{2}g_{TC}\phi_k\tau^{\alpha\beta}_k Q^{a\beta}_{L(a)} \tag{4.1}$$

where the second term represents a transform in the electroweak space and the third in techniflavour space. θ_k and ϕ_k are the transform parameters of each transform and g_W and g_{TC} are some constants. The right hand techniquarks transform as

$$\begin{aligned} U^{\alpha\prime}_{R(A)} &= U^{\alpha}_{R(A)} - \frac{i}{2}g_1\theta U^{\alpha}_{R(A)} + \frac{i}{2}g_{TC}\phi_k\tau^{\alpha\beta}_k U^{\beta}_{R(A)} \\ D^{\alpha\prime}_{R(A)} &= D^{\alpha}_{R(A)} + \frac{i}{2}g_1\theta D^{\alpha}_{R(A)} + \frac{i}{2}g_{TC}\phi_k\tau^{\alpha\beta}_k D^{\beta}_{R(A)} \end{aligned} \tag{4.2}$$

where the terms containing θ are transforms in $U(1)_Y$ space. These transform terms have opposite sign since the hypercharge of $U^a_{R(A)}$ and $D^a_{R(A)}$ is of equal magnitude but of opposite sign. The charge conjugated bi-doublets will be denoted $\hat{\mathbf{C}}Q^{a\alpha}_{L(A)} = Q^{Ca\alpha}_{L(A)}$ and the $SU(2)_W$ singlets $\hat{\mathbf{C}}Q^{\alpha}_{L(A)} = Q^{C\alpha}_{L(A)}$. Charge conjugating a Weyl spinor changes the chirality of the fields, so turning to the second generation fields we get the relation

$$Q^{b\beta}_{R(2)} = \epsilon^{ab}\epsilon^{\alpha\beta}Q^{Ca\alpha}_{L(2)}$$

where the Levi-Civita symbol is denoted as

$$\epsilon^{ab} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Note now how the right handed fields of the second generation is in bi-fundamental representation. Performing the charge conjugation on the transform (4.1) and then inserting the above relation yields:

$$\begin{aligned} Q^{Ca\alpha\prime}_{L(2)} &= Q^{Cb\beta}_{L(2)} - \frac{i}{2}g_W\theta_k(\tau_k^{ab})^* Q^{Cb\alpha}_{L(2)} - \frac{i}{2}g_{TC}\phi_k(\tau_k^{\alpha\beta})^* Q^{Ca\beta}_{L(2)} \\ \epsilon^{ab}\epsilon^{\alpha\beta}Q^{Cb\beta\prime}_{L(2)} &= \epsilon^{ab}\epsilon^{\alpha\beta}Q^{Cb\beta}_{L(2)} - \frac{i}{2}g_W\theta_k\epsilon^{ab}(\tau_k^{bc})^*\epsilon^{\alpha\beta}Q^{Ccb}_{L(2)} - \frac{i}{2}g_TC\phi_k\epsilon^{\alpha\beta}(\tau_k^{\beta\gamma})^*\epsilon^{ab}Q^{b\gamma}_{L(2)} \end{aligned}$$

Using the relations $\epsilon^{ac}\epsilon^{bc} = \delta^{ab}$, δ being the Kronecker delta, the antisymmetric property of the Levi-Civita symbol and the commutation relations of equation (A.2), the above equations will yield

$$Q^{a\alpha\prime}_{R(2)} = Q^{a\alpha}_{R(2)} + \frac{i}{2}g_W\theta_k\tau_k^{ab}Q^{b\alpha}_{R(2)} + \frac{i}{2}g_{TC}\phi_k\tau_k^{\alpha\beta}Q^{a\beta}_{R(2)}$$

Thus the right handed bi-doublet of the second generation transform in the same way as the left handed bi-doublet of the first generation. In the same way the right handed $SU(2)_{EW}$ singlet $U^{\alpha}_{R(A)}$ of equation (4.2) can be charge conjugated:

$$\begin{aligned} U^{C\alpha\prime}_{R(A)} &= U^{C\alpha}_{R(A)} + \frac{i}{2}g_1\theta U^{C\alpha}_{R(A)} - \frac{i}{2}g_{TC}\phi_k(\tau_k^{ab})^* U^{C\alpha}_{R(A)} \\ -\epsilon^{ab}U^{Cb\prime}_{R(A)} &= -\epsilon^{ab}U^{Cb}_{R(A)} + \frac{i}{2}g_1\theta(-\epsilon^{ab})U^{Cb}_{R(A)} + \frac{i}{2}g_{TC}\phi_k\epsilon^{ab}(\tau_k^{bc})^*\epsilon^{cf}(-\epsilon^{fd}U^{Cd}_{R(A)}) \end{aligned} \quad (4.3)$$

Just as above the charge conjugation changes the chirality so we introduce the field

$$S^a_L = -\epsilon^{ab}U^{Cb}_{R(A)}$$

which inserted in equation (4.3) yields the transformation property of this new right handed field

$$S^{a\prime}_L = S^a_L + \frac{i}{2}g_1\theta S^a_L + \frac{i}{2}g_{TC}\phi_k\tau_k^{ab}S^b_L$$

We now have two chirally symmetric generations, the first being a doublet and the second a singlet under $SU(2)_W$:

$$\begin{aligned} Q^{a\alpha} &= Q^{a\alpha}_{L(1)} + Q^{a\alpha}_{R(2)} = Q^{Ca\alpha}_{L(1)} + \epsilon^{ab}\epsilon^{\alpha\beta}Q^{b\beta}_{L(1)} \\ S^a &= S^a_L + S^a_R = -\epsilon^{ab}U^{Cb}_R + D^a_R \end{aligned}$$

where Q and S are Dirac fields with respect to weak interactions. Therefore the interaction between techniquarks and electroweak bosons is vectorlike and vectorlike weak interactions protect the considered technicolor model from large techniquark contributions to the Peskin parameters. Models in which the interactions are non-vectorlike has been ruled out by the precision tests. Another important feature of the Dirac techniquarks is that the fundamental techniquark Lagrangian has an arbitrary mass term consistent with the chiral symmetry. This is the main difference from ordinary QCD where the left-right symmetry is broken by weak interactions.

4.2 The new Lagrangian

Introducing a The Lagrangian of the three flavour LσM can now be written as

$$\begin{aligned}\mathcal{L} =& i\bar{Q}\gamma^\mu\partial_\mu Q + i\bar{S}\gamma^\mu\partial_\mu S + \partial_\mu\Sigma^\dagger\partial^\mu\Sigma \\ &- \sqrt{6}\kappa(\bar{Q}_L\Sigma Q_R + \bar{Q}_R\Sigma^\dagger Q_L) + \mu^2\Sigma^\dagger\Sigma - \lambda_1(\Sigma^\dagger\Sigma)^2 \\ &- 3\lambda_2(\Sigma^\dagger\Sigma)^2 + \Lambda_3\mathrm{Re}\,(\det(\Sigma))\end{aligned} \tag{4.4}$$

where the first row contains the kinetic terms and the second and third row contains the potential terms. This Lagrangian is a straight forward extension of the $SU(2)$ LσM, which has been used to describe the lightest mesons in QCD [23]. Here $\Sigma = \frac{\lambda_a}{2}\sigma_a - i\frac{\lambda_a}{2}\pi_a$, analogous to the two flavour case with

$$\frac{\lambda_a}{2}\sigma_a = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{1}{\sqrt{2}}a^0 + \frac{1}{\sqrt{6}}f + \frac{1}{\sqrt{3}}\sigma & a^+ & H^+ \\ a^- & -\frac{1}{\sqrt{2}}a^0 + \frac{1}{\sqrt{6}}f + \frac{1}{\sqrt{3}}\sigma & H^0 \\ H^- & \bar{H}^0 & -\sqrt{\frac{2}{3}}f + \frac{1}{\sqrt{3}}\sigma \end{pmatrix}$$

$$\frac{\lambda_a}{2}\pi_a = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta + \frac{1}{\sqrt{6}}\eta' & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta + \frac{1}{\sqrt{3}}\eta' & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta + \frac{1}{\sqrt{3}}\eta' \end{pmatrix}$$

These fields are all technimesons. These are considered to behave as fundamental particles at low energies, as we consider an effective field theory in which these mesons are described in terms of interactions with Standard Model particles — quarks, leptons, bosons — as well as in terms of self-interactions. The σ_a matrix contains scalar mesons and the π_a field contains pseudoscalar mesons, and they are classified as follows: the π_i and a_i triplets have no hypercharge and are in adjoint representation under $SU(2)_W$, the doublets $K = (K^+, K^0)$ and $\mathcal{H} = (H^+, H^0)$ have hypercharge 1/2 and are in fundamental representation, and the $SU(2)_W$ singlets σ, η, η' and f have neither hypercharge nor isospin [23, 24].

4.3 Breaking the chiral symmetry

In vacuum, the σ field gains a vev, $\langle\sigma\rangle = u$ while the other fields get no vev. Going back to the potential part of equation (4.4), setting the σ field to u and the rest to zero in Σ, will yield

$$\langle U \rangle = -\frac{\mu^2}{2}u^2 + \frac{1}{4}(\lambda_1 + \lambda_2)u^4 - \frac{1}{3}\Lambda_3 u^3 + \kappa u \left\langle \bar{Q}Q \right\rangle$$

The vacuum stability conditions are

$$\begin{aligned}
\left\langle \frac{\delta U}{\delta u} \right\rangle &= u \left(-\mu^2 + (\lambda_1 + \lambda_2)u^2 - \Lambda_3 u + \frac{\kappa}{u} \left\langle \bar{Q}Q \right\rangle \right) = 0 \\
\left\langle \frac{\delta^2 U}{\delta u^2} \right\rangle &= -\mu^2 + 3(\lambda_1 + \lambda_2)u^2 - 2\Lambda_3 u \\
&= 2(\lambda_1 + \lambda_2)u^2 - \Lambda_3 u - \frac{\kappa}{u} \left\langle \bar{Q}Q \right\rangle > 0
\end{aligned}$$

The potential part of the Lagrangian can be explicitly expanded and from this mass terms for the fields contained in Σ can be identified. These are

$$\begin{aligned}
M^2_{\pi(0)} &= M^2_{K(0)} = M^2_{\eta(0)} = -\frac{\kappa}{u} \left\langle \bar{Q}Q \right\rangle \\
&= -\mu^2 + (\lambda_1 + \lambda_2)\, u^2 - \Lambda_3 u \\
M^2_{a(0)} &= M^2_{H(0)} = M^2_{f(0)} = 2\lambda_2 u^2 + 2\Lambda_3 u + M^2_{\pi(0)} \\
M^2_{\sigma(0)} &= 2(\lambda_1 + \lambda_2)u^2 - \Lambda_3 u + M^2_{\pi(0)} \\
M^2_{\eta'(0)} &= 3\Lambda_3 u + M^2_{\pi(0)}
\end{aligned}$$

It is here apparent that the π, K and η particles are pseudo-Goldstone bosons, as they have gained a small mass only through the vacuum condensate. The vacuum stability conditions set constraints on the possible choices of parameter values; the masses must be positive definite. This system of equations may be solved so that the constants $\lambda_1 u^2$, $\lambda_2 u^2$ and $\Lambda_3 u$ are expressed as linear combinations of the different masses. Doing so yields

$$\begin{aligned}
\lambda_1 u^2 &= \frac{1}{2} \left(M^2_{\eta'(0)} + M^2_{\sigma(0)} - M^2_{a(0)} - M^2_{\pi(0)} \right) && (4.5) \\
\lambda_2 u^2 &= \frac{1}{6} \left(3M^2_{a(0)} - 2M^2_{\eta'(0)} - M^2_{\pi(0)} \right) \\
\Lambda_3 u &= \frac{1}{3} \left(M^2_{\eta'(0)} - M^2_{\pi(0)} \right) && (4.6)
\end{aligned}$$

The full Lagrangian of this theory can now be expressed as

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{TC}$$

with $\mathcal{L}_{TC}$ being these new terms of the techniquark model and $\mathcal{L}_{SM}$ is the Lagrangian of equation (B.8) *except for* the Higgs term $\mathcal{L}_{Higgs}$ which is replaced by this new techniquark sector. The meson fields have different isospin and hypercharge and so the Lagrangian is rewritten with the proper covariant derivatives:

$$
\begin{aligned}
\mathcal{L}_\sigma = & \, i\bar{Q}\gamma^\mu\left(\partial_\mu - \frac{i}{2}g_W W^a_\mu \tau_a\right)Q + i\bar{S}\gamma^\mu\left(\partial_\mu + \frac{i}{2}g_1 B_\mu\right)S \\
& - \sqrt{6}\kappa\left(\bar{Q}_L\Sigma Q_R + \bar{Q}_R\Sigma^\dagger Q_L\right) + \frac{1}{2}\left(D^\mu\pi_a D_\mu\pi_a + D^\mu a_a D_\mu a_a\right) \\
& + \left(\mathcal{D}^\mu K\right)^\dagger \mathcal{D}_\mu K + \left(\mathcal{D}^\mu H\right)^\dagger \mathcal{D}_\mu H \\
& + \frac{1}{2}\left(\partial_\mu\eta\partial^\mu\eta + \partial_\mu\eta'\partial^\mu\eta' + \partial_\mu f\partial^\mu f + \partial_\mu\sigma\partial^\mu\sigma\right) \\
& + \mu^2\Sigma^\dagger\Sigma - \lambda_1(\Sigma^\dagger\Sigma)^2 - 3\lambda_2(\Sigma^\dagger\Sigma)^2 + \Lambda_3 \mathrm{Re}\left(\det\left(\Sigma\right)\right)
\end{aligned}
$$

and the different derivatives are

$$
\begin{aligned}
D_\mu\pi_a &= \partial_\mu\pi_a + g_W\epsilon_{abc}W^b_\mu\pi_c \\
D_\mu a_a &= \partial_\mu a_a + g_W\epsilon_{abc}W^b_\mu a_c \\
\mathcal{D}_\mu K &= \partial_\mu K - \frac{i}{2}g_1 B_\mu K - \frac{i}{2}g_W W^a_\mu\tau_a K \\
\mathcal{D}_\mu \mathcal{H} &= \partial_\mu \mathcal{H} - \frac{i}{2}g_1 B_\mu \mathcal{H} - \frac{i}{2}g_W W^a_\mu\tau_a \mathcal{H}
\end{aligned}
$$

Note that the Q that interacts with the weak gauge field is a Dirac spinor and not a Weyl spinor. This chirally symmetric interaction is necessary in order for the model to be consistent with the electroweak precision tests [1].

4.4 The electroweak symmetry breaking

In order to go to the physical fields the electroweak symmetry must be broken just as the chiral. The techniquark condensates of the unbroken electroweak phase $\langle\bar{Q}Q\rangle$ will still be present, but there will be a new condensate added, the non-diagonal $\langle\bar{D}S + \bar{S}D\rangle$ state. After the symmetry breaking σ keeps the vev $\langle\sigma\rangle = u$ and the lower component of the Higgs field gets the vev $H^0 = v/\sqrt{2}$. Inserting this in the potential part of the Lagrangian yields

$$
\begin{aligned}
\langle U\rangle = & -\frac{\mu^2}{2}\left(u^2+v^2\right) + \frac{1}{4}\left(\lambda_1+\lambda_2\right)\left(u^2+v^2\right)^2 + \lambda_2 v^2\left(u^2 + \frac{1}{8}v^2\right) \\
& - \Lambda_3\left(\frac{1}{3}u^2 - \frac{1}{2}v^2\right)u + \kappa u\left\langle\bar{Q}Q\right\rangle + \sqrt{\frac{3}{2}}\kappa v\left\langle\bar{D}S + \bar{S}D\right\rangle
\end{aligned}
$$

Again the vacuum stability condition is that the potential above is in its minimum when $\sigma = u$ and $H^0 = v/\sqrt{2}$:

$$
\begin{aligned}
&\left\langle \frac{\delta U}{\delta \sigma} \right\rangle = 0 \\
&= u \left(-\mu^2 + (\lambda_1 + \lambda_2)\left(u^2 + v^2\right) + 2\lambda_2 v^2 - \Lambda_3 \left(u - \frac{v^2}{2u} \right) + \frac{\kappa}{u} \left\langle \bar{Q} Q \right\rangle \right) \\
&\left\langle \frac{\delta U}{\delta H^0} \right\rangle = 0 \\
&= v \left(-\mu^2 + (\lambda_1 + \lambda_2)\left(u^2 + v^2\right) + \lambda_2 \left(2u^2 + \frac{1}{2} v^2 \right) + \Lambda_3 u + \sqrt{\frac{3}{2}} \frac{\kappa}{v} \left\langle \bar{D} S + \bar{S} D \right\rangle \right)
\end{aligned}
$$

To make sure that the extremum is a minimum the second derivatives must fulfill the condition

$$
\left\langle \frac{\delta^2 U}{\delta \sigma^2} \right\rangle \left\langle \frac{\delta^2 U}{(\delta H^0)^2} \right\rangle - \left\langle \frac{\delta^2 U}{\delta \sigma \delta H^0} \right\rangle^2 > 0
$$

where the second derivatives are

$$
\begin{aligned}
\left\langle \frac{\delta^2 U}{\delta \sigma^2} \right\rangle &= \left(-\mu^2 + (\lambda_1 + \lambda_2)(3u^2 + v^2) + 2\lambda_2 v^2 - 2\Lambda_3 u \right) \\
\left\langle \frac{\delta^2 U}{(\delta H^0)^2} \right\rangle &= \left(-\mu^2 + (\lambda_1 + \lambda_2)(u^2 + 3v^2) + \lambda_2 \left(2u^2 + \frac{3}{2} v^2 \right) + \Lambda_3 u \right) \\
\left\langle \frac{\delta^2 U}{\delta \sigma \delta H^0} \right\rangle &= \left(2(\lambda_1 + 2\lambda_2) uv + \Lambda_3 v \right)
\end{aligned}
$$

These conditions sets constraints on the vevs. In order to keep the vacuum stable and to keep the masses positive definite, the Higgs vev must be much smaller than the sigma vev. Therefore it will be assumed that $v \ll u$. After the electroweak symmetry breaking the different fields mix and the mass terms can be extracted in the gauge basis. The fields can be divided into four groups of fields: neutral scalar, neutral pseudoscalar, charged scalar and charged pseudoscalar fields. The mass terms of the neutral scalar fields are

$$
\begin{aligned}
&M^2_{H(0)} \left((a^0)^2 + (H^0)^2 + f^2 \right) + M^2_{\sigma(0)} \sigma^2 + \left((\lambda_1 + 3\lambda_2)\, uv + \frac{\Lambda_3}{2} v \right) H^0 \sigma \\
&+ \frac{1}{2\sqrt{2}} \left(\Lambda_3 v - 3\lambda_2 uv \right) \left(H^0 f - \sqrt{3} H^0 a^0 \right) - \frac{3}{2\sqrt{2}} \lambda_2 v^2 \left(\sigma f + \sqrt{3} \sigma a^0 \right)
\end{aligned}
$$

and those of the charged scalar fields are

$$
M^2_{H(0)} \left(H^+ H^- + a^+ + a^- \right) + \sqrt{\frac{3}{2}} \left(3\lambda_2 uv - \Lambda_3 v \right) \left(H^+ a^- + H^- a^+ \right)
$$

while the mass terms of the neutral pseudoscalar fields are

$$
\begin{aligned}
&M^2_{\pi(0)}\left((\pi^0)^2+\eta^2+K^0\bar{K}^0\right)+M^2_{\eta'(0)}\eta'^2-\lambda_2 v^2\left(\frac{\sqrt{3}}{4}\pi^0\eta+\frac{\sqrt{3}}{2\sqrt{2}}\pi^0\eta'+\frac{1}{2\sqrt{2}}\eta\eta'\right)\\
&-\frac{1}{2}\left(\lambda_2 uv+\Lambda_3 v\right)\left(\eta\left(K^0+\bar{K}^0\right)+\sqrt{3}\pi^0\left(K^0+\bar{K}^0\right)\right)\\
&+\left(\sqrt{2}\lambda_2 uv-\frac{1}{\sqrt{2}}\Lambda_3 v\right)\eta'\left(K^0+\bar{K}^0\right)
\end{aligned}
\tag{4.7}
$$

and the terms of the charged pseudoscalar fields are

$$
M^2_{\pi(0)}\left(\pi^+\pi^-+K^+K^-\right)+\sqrt{\frac{3}{2}}\left(\lambda_2 uv+\Lambda_3 v\right)\left(\pi^+K^-+\pi^-K^+\right)
$$

These terms can all be expressed using symmetric matrices. By diagonalising them, the physical mass spectrum will be found just as in the two techniquark case. Here the notation $\delta = v/u$ is introduced. The mixing matrices will be somewhat complicated and hard to diagonalise, but since $v \ll u$, δ will be small and therefore we will expand the mass terms as a series in δ and neglect quadratic and higher terms.

4.5 Transforming into the mass basis

In the case of the neutral scalar fields and the neutral pseudoscalar fields, the mass terms expressed in gauge basis consists of a four-by-four matrix with diagonal terms and cross terms. In both the matrix (4.8) and (4.9) each diagonal term contains a constant and an extra term dependent on δ. When $\delta \to 0$, corresponding to no electroweak symmetry breaking, the matrices turn diagonal, and three of the fields get the same mass, while the fourth get another mass. If we extract these three fields we might write it as

$$
\begin{pmatrix}\phi_1 & \phi_2 & \phi_3\end{pmatrix}\begin{pmatrix}M^2 & 0 & 0\\ 0 & M^2 & 0\\ 0 & 0 & M^2\end{pmatrix}\begin{pmatrix}\phi_1\\ \phi_2\\ \phi_3\end{pmatrix}=M^2\left(\phi_1^2+\phi_2^2+\phi_3^2\right)
$$

where ϕ_i represent the three real fields with the same mass term and M^2 is this mass term. The left hand side above is however not a unique way of writing the right hand side in matrix form. The group of rotations in three-dimensional real space is the $SO(3)$ group. A rotation in three dimensions can in general be expressed in terms of three angles, call them ψ, θ, and ξ, combined into a rotation matrix [25]. The convention is chosen in which the rotation matrix looks like

$$
\begin{pmatrix}
\cos\psi\cos\xi-\cos\theta\sin\xi\sin\psi & \cos\psi\sin\xi+\cos\theta\cos\xi\sin\psi & \sin\psi\sin\theta\\
-\sin\psi\cos\xi-\cos\theta\sin\xi\cos\psi & -\sin\psi\sin\xi+\cos\theta\cos\xi\cos\psi & \cos\psi\sin\theta\\
\sin\theta\sin\xi & -\sin\theta\cos\xi & \cos\theta
\end{pmatrix}
$$

Call this rotation matrix R and set $z = \begin{pmatrix} \phi_1 & \phi_2 & \phi_3 \end{pmatrix}$. Now consider a change of basis by rotating the vectors, $z \to z' = Rz$. Write the mass matrix as $M^2\mathbb{1}$, a scalar times the unit matrix. Then

$$z'^T M^2 \mathbb{1} z' = z^T R^T M^2 \mathbb{1} R z = z^T R^T R M^2 \mathbb{1} z = z^T M^2 \mathbb{1} z$$

where it is noted that all three dimensional rotational matrices are orthogonal, which means that $R^T R = \mathbb{1}$ [26]. Thus we have an $SO(3)$ symmetry in the mass spectrum which is softly broken by the Higgs vev, allowing physical fields to mix even before the symmetry breaking, and we can freely choose how the gauge fields are expressed in terms of these three new parameters. When the symmetry breaks though, the choice of angles will affect the resulting physics.

4.5.1 The neutral scalars

The mass matrix for the neutral scalar multiplet $\begin{pmatrix} a^0 & h & f & \sigma \end{pmatrix}^T$ is

$$\begin{pmatrix} M^2_{H(0)} & \frac{1}{2\sqrt{6}}\kappa_1\delta & 0 & 0 \\ \frac{1}{2\sqrt{6}}\kappa_1\delta & M^2_{H(0)} & \frac{1}{6\sqrt{2}}\kappa_1\delta & \frac{1}{3}\kappa_2\delta \\ 0 & \frac{1}{6\sqrt{2}}\kappa_1\delta & M^2_{H(0)} & 0 \\ 0 & \frac{1}{3}\kappa_2\delta & 0 & M^2_{\sigma(0)} \end{pmatrix} \tag{4.8}$$

where the constants are

$$\begin{aligned} \kappa_1 &= 8M^2_{\eta'(0)} - 9M^2_{H(0)} + M^2_{\pi(0)} \\ \kappa_2 &= -2M^2_{\eta'(0)} + 6M^2_{H(0)} - 7M^2_{\pi(0)} + 3M^2_{\sigma(0)} \\ \kappa_3 &= M^2_{H(0)} - M^2_{\sigma(0)} \end{aligned}$$

The eigenvalues of the mass matrix are

$$\begin{aligned} M^2_h &= M^2_{H(0)} + \mathcal{O}(\delta^2) & M^2_\sigma &= M^2_{\sigma(0)} + \mathcal{O}(\delta^2) \\ M^2_f &= M^2_{H(0)} + \tfrac{1}{3\sqrt{2}}\kappa_2\delta + \mathcal{O}(\delta^2) & M^2_{a_0} &= M^2_{H(0)} - \tfrac{1}{3\sqrt{2}}\kappa_2\delta + \mathcal{O}(\delta^2) \end{aligned}$$

Following the reasoning above, the most general way of expressing the gauge fields in terms of physical fields, before symmetry breaking and noting the physical fields in Fraktur, would be

$$\begin{aligned} \sigma &= \mathfrak{s} \\ h &= (\cos\psi\cos\xi - \cos\theta\sin\xi\sin\psi)\,\mathfrak{h} + (\cos\psi\sin\xi + \cos\theta\cos\xi\sin\psi)\,\mathfrak{f} + (\sin\psi\sin\theta)\,\mathfrak{a} \\ f &= (-\sin\psi\cos\xi - \cos\theta\sin\xi\cos\psi)\,\mathfrak{h} + (-\sin\psi\sin\xi + \cos\theta\cos\xi\cos\psi)\,\mathfrak{f} + (\cos\psi\sin\theta)\,\mathfrak{a} \\ a_0 &= \sin\theta\sin\xi\mathfrak{h} - \sin\theta\cos\xi\mathfrak{f} + \cos\theta\mathfrak{a} \end{aligned}$$

One solution that was found, corresponding to the parameters

$$
\begin{aligned}
\psi &= 140.768^\circ \\
\xi &= 63.435^\circ \\
\theta &= 131.694^\circ
\end{aligned}
$$

is

$$
\begin{aligned}
\sigma &= \mathfrak{s} + \frac{1}{3}\frac{\kappa_1\delta}{\kappa_3}\mathfrak{h} \\
h &= \frac{1}{2}\mathfrak{a} - \frac{\sqrt{3}}{2}\mathfrak{f} \\
a_0 &= \left(\frac{1}{\sqrt{2}} - \frac{\kappa_1^2}{12\kappa_2\kappa_3}\delta\right)\mathfrak{h} - \left(\frac{1}{2\sqrt{2}} + \frac{\kappa_1^2}{24\kappa_2\kappa_3}\delta\right)\left(\mathfrak{f} + \sqrt{3}\mathfrak{a}\right) - \frac{1}{3\sqrt{2}}\frac{\kappa_1\delta}{\kappa_3}\mathfrak{s} \\
f &= -\left(\frac{1}{\sqrt{2}} + \frac{\kappa_1^2}{12\kappa_2\kappa_3}\delta\right)\mathfrak{h} - \left(\frac{1}{2\sqrt{2}} - \frac{\kappa_1^2}{24\kappa_2\kappa_3}\delta\right)\left(\mathfrak{f} + \sqrt{3}\mathfrak{a}\right) + \frac{1}{3\sqrt{2}}\frac{\kappa_1}{\kappa_3}\delta\mathfrak{s}
\end{aligned}
$$

In this configuration h does not mix with $\mathfrak{s}$. If δ tends to zero, the fields become

$$
\begin{aligned}
\sigma &= \mathfrak{s} \\
h &= \frac{1}{2}\mathfrak{a} - \frac{\sqrt{3}}{2}\mathfrak{f} \\
a_0 &= \frac{1}{\sqrt{2}}\mathfrak{h} - \frac{1}{2\sqrt{2}}\mathfrak{f} - \frac{\sqrt{3}}{2\sqrt{2}}\mathfrak{a} \\
f &= -\frac{1}{\sqrt{2}}\mathfrak{h} - \frac{1}{2\sqrt{2}}\mathfrak{f} - \frac{\sqrt{3}}{2\sqrt{2}}\mathfrak{a}
\end{aligned}
$$

Another possible way of choosing these fields is by the parametrisation

$$
\begin{aligned}
\psi &= -63.435^\circ \\
\xi &= 39.2315^\circ \\
\theta &= -127.761^\circ
\end{aligned}
$$

which yields the result

$$\sigma = \mathfrak{s} - \frac{\varpi\delta}{3\sqrt{2}M^2_{\sigma(0)}}(\mathfrak{a} + \mathfrak{f})$$
$$h = \frac{\mathfrak{a} + \mathfrak{f}}{\sqrt{2}} + \frac{2\varpi\delta}{3M^2_{\sigma(0)}}\mathfrak{s}$$
$$f = \frac{\sqrt{3}}{2}\mathfrak{h} - \frac{1}{2\sqrt{2}}(\mathfrak{a} - \mathfrak{f})$$
$$a = -\frac{1}{2}\mathfrak{h} - \frac{1}{2}\sqrt{\frac{3}{2}}(\mathfrak{a} - \mathfrak{f})$$

in which the constant $\varpi = M^2_{\eta'(0)} - M^2_{\pi(0)} + 6M^2_{H(0)}$ is introduced. Remember though that this solution has been found with $\mu^2 = 0$. With these parameters, the $\mathfrak{a}$ and $\mathfrak{f}$ is maximally mixed, with a 45° mixing angle, in the h field.

4.5.2 The charged scalars

The terms of the charged scalars can be written as

$$\begin{pmatrix} H^+ & a^+ \end{pmatrix} \begin{pmatrix} M^2_{H(0)} & \sqrt{\frac{3}{2}}(3\lambda_2 uv - \Lambda_3 v) \\ \sqrt{\frac{3}{2}}(3\lambda_2 uv - \Lambda_3 v) & M^2_{H(0)} \end{pmatrix} \begin{pmatrix} H^- \\ a^- \end{pmatrix}$$

Diagonalising this matrix, and changing the basis of the doublets accordingly, two new fields is identified as

$$\tilde{H}^\pm = \frac{1}{2}\left(-H^\pm + a^\pm\right)$$
$$\tilde{a}^\pm = \frac{1}{2}\left(H^\pm + a^\pm\right)$$

and the masses of these fields are

$$M^2_{H^\pm} = M^2_{\pi(0)} + \frac{1}{2\sqrt{6}}\left(3M^2_{H(0)} + M^2_{\pi(0)} - 4M^2_{\eta'(0)}\right)\delta + \mathcal{O}(\delta^2)$$
$$M^2_{a^\pm} = M^2_{\pi(0)} - \frac{1}{2\sqrt{6}}\left(3M^2_{H(0)} + M^2_{\pi(0)} - 4M^2_{\eta'(0)}\right)\delta + \mathcal{O}(\delta^2)$$

4.5.3 The neutral pseudoscalars

Among the terms of the neutral pseudoscalars, the term $K^0 + \bar{K}^0$ is present several times. The new fields

$$\zeta = \frac{1}{\sqrt{2}}\left(K^0 + \bar{K}^0\right)$$
$$\xi = \frac{i}{\sqrt{2}}\left(K^0 - \bar{K}^0\right)$$

are therefore introduced, so that $K^0\bar{K}^0 = \zeta^2 + \xi^2$. When inserting this into the expression (4.7), one will see that the field ξ does not mix with the other fields but only have the mass $M^2_{\pi(0)}$. Constructing the multiplet $\begin{pmatrix} \eta' & \zeta & \eta & \pi^0 \end{pmatrix}^T$ the mass matrix can be written as

$$\begin{pmatrix} M^2_{\eta'(0)} & \chi_1\delta & 0 & 0 \\ \chi_1\delta & M^2_{\pi(0)} & \frac{1}{2\sqrt{2}}\chi_2\delta & \frac{1}{2}\sqrt{\frac{3}{2}}\chi_2\delta \\ 0 & \frac{1}{2\sqrt{2}}\chi_2\delta & M^2_{\pi(0)} & 0 \\ 0 & \frac{1}{2}\sqrt{\frac{3}{2}}\chi_2\delta & 0 & M^2_{\pi(0)} \end{pmatrix} \tag{4.9}$$

where the constants have been rewritten in terms of equation (4.5) - (4.6) and

$$\chi_1 = M^2_{H(0)} - M^2_{\eta'(0)}$$
$$\chi_2 = M^2_{\pi(0)} - M^2_{H(0)}$$
$$\chi_3 = M^2_{\eta'(0)} - M^2_{\pi(0)}$$

The eigenvalues of this matrix are

$$M^2_{\pi_0} = M^2_{\pi(0)} + \mathcal{O}(\delta^2) \qquad M^2_{\eta'} = M^2_{\eta'(0)} + \mathcal{O}(\delta^2)$$
$$M_\zeta = M^2_{\pi(0)} + \tfrac{1}{\sqrt{2}}\chi_2\delta + \mathcal{O}(\delta^2) \quad M^2_\eta = M^2_{\pi(0)} - \tfrac{1}{\sqrt{2}}\chi_2\delta + \mathcal{O}(\delta^2)$$

The most general way of writing linear combinations of these fields would be

$$\eta' = \mathfrak{e}$$
$$\pi_0 = (\cos\psi\cos\xi - \cos\theta\sin\xi\sin\psi)\,\mathfrak{p} + (\cos\psi\sin\xi + \cos\theta\cos\xi\sin\psi)\,\mathfrak{y} + (\sin\psi\sin\theta)\,\mathfrak{z}$$
$$\eta = (-\sin\psi\cos\xi - \cos\theta\sin\xi\cos\psi)\,\mathfrak{p} + (-\sin\psi\sin\xi + \cos\theta\cos\xi\cos\psi)\,\mathfrak{y} + (\cos\psi\sin\theta)\,\mathfrak{z}$$
$$\zeta = \sin\theta\sin\xi\mathfrak{p} - \sin\theta\cos\xi\mathfrak{y} + \cos\theta\mathfrak{z}$$

One possible set of parameters would be

$$\psi = 90.000^\circ$$
$$\xi = 120.000^\circ$$
$$\theta = 45.000^\circ$$

which yields the solution

$$\eta' = \mathfrak{e} + \frac{\chi_1\delta}{\chi_3}\mathfrak{z}$$
$$\pi_0 = \frac{1}{2}\mathfrak{p} - \frac{\sqrt{3}}{2}\mathfrak{y}$$
$$\eta = \left(\frac{1}{2\sqrt{2}} - \frac{\chi_1^2}{\chi_2\chi_3}\delta\right)\left(\mathfrak{y} + \sqrt{3}\mathfrak{p}\right) - \left(\frac{1}{\sqrt{2}} + \frac{\chi_1^2}{4\chi_2\chi_3}\delta\right)\mathfrak{z} + \frac{1}{\sqrt{2}}\frac{\chi_1\delta}{\chi_3}\mathfrak{e}$$
$$\zeta = \left(\frac{1}{2\sqrt{2}} + \frac{\chi_1^2}{\chi_2\chi_3}\delta\right)\left(\mathfrak{y} + \sqrt{3}\mathfrak{p}\right) + \left(\frac{1}{\sqrt{2}} - \frac{\chi_1^2}{4\chi_2\chi_3}\delta\right)\mathfrak{z} - \frac{1}{\sqrt{2}}\frac{\chi_1\delta}{\chi_3}\mathfrak{e}$$

and just as in the case of the scalar fields, there is one field here, η', which does not mix with the others when $\delta \to 0$. Here π_0 is only composed by two fields. The relations before symmetry breaking looks like

$$\eta' = \mathfrak{e}$$
$$\pi_0 = \frac{1}{2}\mathfrak{p} - \frac{\sqrt{3}}{2}\mathfrak{y}$$
$$\eta = \frac{1}{2\sqrt{2}}\left(\mathfrak{y} + \sqrt{3}\mathfrak{p}\right) - \frac{1}{\sqrt{2}}\mathfrak{z}$$
$$\zeta = \frac{1}{2\sqrt{2}}\left(\mathfrak{y} + \sqrt{3}\mathfrak{p}\right) + \frac{1}{\sqrt{2}}\mathfrak{z}$$

As above these fields can be combined in another way. By again choosing the maximum mixing parameters

$$\psi = -63.435^\circ$$
$$\xi = 39.232^\circ$$
$$\theta = -127.7610^\circ$$

one solution where the $\mathfrak{y}$ and $\mathfrak{p}$ fields are maximally mixed, with 45°, is

$$\eta' = \mathfrak{e} + \frac{\chi_1\delta}{\sqrt{2}M^2_{\eta'(0)}}\left(\mathfrak{y} + \mathfrak{p}\right)$$
$$\zeta = \frac{\mathfrak{y} + \mathfrak{p}}{\sqrt{2}} - \frac{\delta}{M^2_{\pi(0)}}\left(2\chi_1\mathfrak{e} + \frac{\chi_2}{2\sqrt{2}}\mathfrak{z}\right)$$
$$\eta = -\frac{1}{2}\mathfrak{z} - \frac{1}{2}\sqrt{\frac{3}{2}}\left(\mathfrak{y} - \mathfrak{p}\right)$$
$$\pi_0 = \frac{\sqrt{3}}{2}\mathfrak{z} - \frac{1}{2\sqrt{2}}\left(\mathfrak{y} - \mathfrak{p}\right)$$

Also here, these results has been calculated assuming $\mu^2 = 0$.

4.5.4 The charged pseudoscalars

The mass terms of the charged scalar fields can be written in matrix form as

$$\begin{pmatrix} \pi^+ & K^+ \end{pmatrix} \begin{pmatrix} M^2_{\pi(0)} & \sqrt{\frac{3}{8}}\left(\lambda_2 uv + \Lambda_3 v\right) \\ \sqrt{\frac{3}{8}}\left(\lambda_2 uv + \Lambda_3 v\right) & M^2_{\pi(0)} \end{pmatrix} \begin{pmatrix} \pi^- \\ K^- \end{pmatrix}$$

The mass matrix can be transformed into the base where it is diagonalised. Applying the same transform of the field doublets the physical states can be constructed:

$$\tilde{\pi}^\pm = \frac{1}{2}\left(-\pi^\pm + K^\pm\right)$$
$$\tilde{K}^\pm = \frac{1}{2}\left(\pi^\pm + K^\pm\right)$$

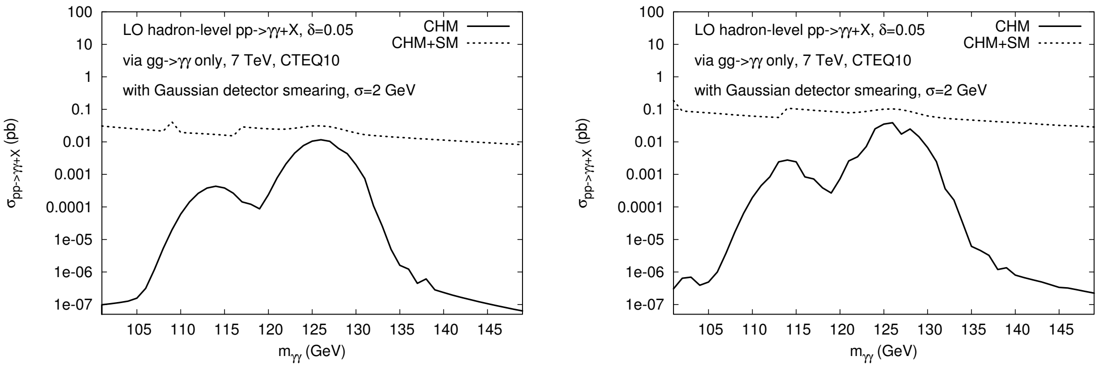
and the masses are

$$M^2_{\tilde{\pi}} = M^2_{\pi(0)} - \sqrt{\frac{3}{8}}\left(M^2_{H(0)} - M^2_{\pi(0)}\right)\delta + \mathcal{O}(\delta^2)$$
$$M^2_{\tilde{K}} = M^2_{\pi(0)} + \sqrt{\frac{3}{8}}\left(M^2_{H(0)} - M^2_{\pi(0)}\right)\delta + \mathcal{O}(\delta^2)$$

4.6 Technimeson production cross sections at the LHC

Using the results above, the cross sections for different choices of parameters were calculated and plotted. There are eleven free parameters in the model. The mixing angles of the neutral scalar fields are: $\psi = -63.435°, \xi = 39.2315°$ and $\theta = -127.761°$. The mixing angles of the neutral pseudoscalar fields are chosen to be the same as for the scalar fields. The Higgs mass $M^2_{H(0)}$ is set to be 126 GeV, as has been measured at the LHC. The mass of the pion $M^2_{\pi(0)}$ is set to be 114 GeV which is the smallest value consistent with measurements made at LEP II; if it was lighter, it would have been already discovered. The $M^2_{\eta'(0)}$ is set to be 135 GeV and the $M^2_{\sigma(0)}$ is 202 GeV. The δ variable has been changed between the different calculations, but, as has been shown by R. Pasechnik, electroweak precision tests forces the δ to be less than 0.08.

In figure 7a the cross section of the $gg \to \gamma\gamma$ process is shown evaluated at a beam energy of 7 TeV, which corresponds to the energy available at LHC during their first run. In figure 7b the same process is shown, but at 14 TeV, which corresponds to the energy scale the new measurements will be at during the second run, starting the spring of 2015. In these two calculations $\delta = 0.05$, which corresponds to $u = 4.92$ TeV. The cross section has been smeared out so that the diagrams are shown with the resolution of the LHC detectors. The plots suffer from some numerical deviations at approxemately 110 GeV, which might be caused by the integrator used in the software or the rather big step sizes of the invariant mass.



(a) 7 TeV

(b) 14 TeV

Figure 7: The cross section as a function of the gluon center-of-momentum energy with $\delta = 0.05$. The continous line represents the cross section of the technimeson resonances, while the dashed line is the combined cross section of the signal and the background.

In figure 8, the cross section of the $H \to \gamma\gamma$ process is displayed at parton level, with the combined signal and background shown as a dashed line. This plot shows the fine structure of the signal, with the pseudo-Goldstone bosons split into three peaks, and the peak at 126 GeV split into two.

In figures 9 the same process as in 7a is shown, at 7 TeV, but with different values of δ. In figure 9a $\delta = 0.1$ corresponding to $u = 2.46$ TeV and in figure 9b $\delta = 0.01$ corresponding to $u = 24.6$ TeV. Electroweak precision tests sets the limit $\delta < 0.08$, excluding the cross section of figure 9a.

This model introduces eighteen technimesons which have been modeled on the corresponding QCD mesons. These are represented in the figures 7 and 8, allthough some of them have the same mass and are therefore overlapping. The figure 8 shows the cross section on parton level, displaying six distinct peaks grouped at three different energy scales. The leftmost group shows three peaks. These are the pseudo-Goldstone bosons. There are actually six particles represented here, but some of them overlap. There are two peaks at about $\sim$ 125 GeV, corresponding to the Higgs particle and the other scalar particles whose originates from the Higgs mass. At 135 GeV there is one sharp peak corresponding to the $\mathfrak{e}$ particle. The only one of the eighteen mesons not represented in this graph is the $\mathfrak{s}$ which is heavier than the others, 202 GeV.

The plots displayed in figures 7 show one bigger peak at $\sim$ 125 GeV which is in the Higgs boson mass range. It also shows a lower peak at $\sim$ 113 GeV. Here there are several particles with approxemately the same mass. These are the pseudo-Goldstones. In the combined signal-background plot in the 7 TeV case, the second peak does not seem to be obviously present. In the 14 TeV case there is a faint peak in the combined signal, suggesting that this process might be more visible if the experiments are carried out at a higher energy scale. There is also a hint that the big peak starts to split up into two. Compared to figure 8 it seems as if the split between the somewhat lighter and the somewhat heavier

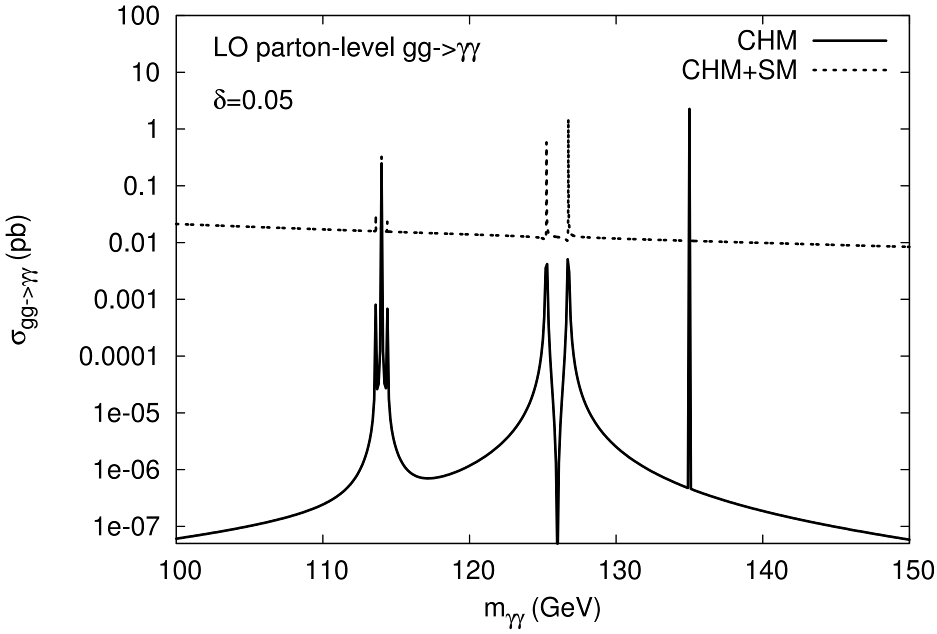
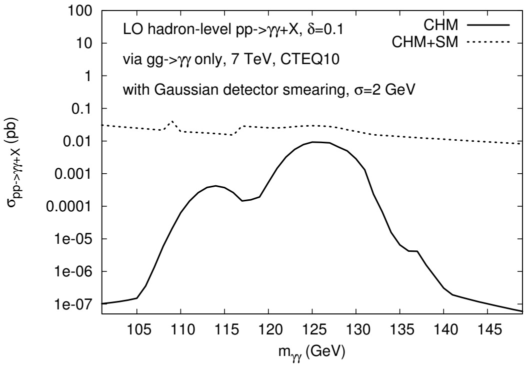


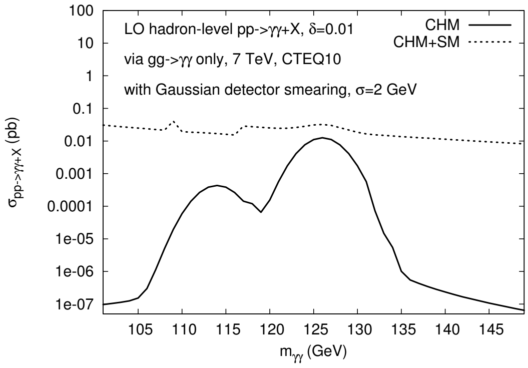
Figure 8: The cross section of the pure signal (continous lines) and the signal added with the background (dashed lines) at parton level.

scalar bosons appear clearer in the detector at higher energies. The very sharp and distinct peak of the $\mathfrak{e}$ boson does not appear in neither of the figures 7.

In figure 10 the signal-to-background ratio is shown for the $H \to \gamma\gamma$ process at 7 TeV. Together with this is the results from measurements done at CMS at 7 and 8 TeV [27].

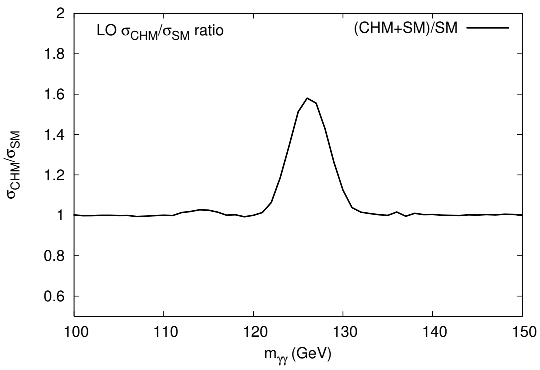


(a) $\delta = 0.1$

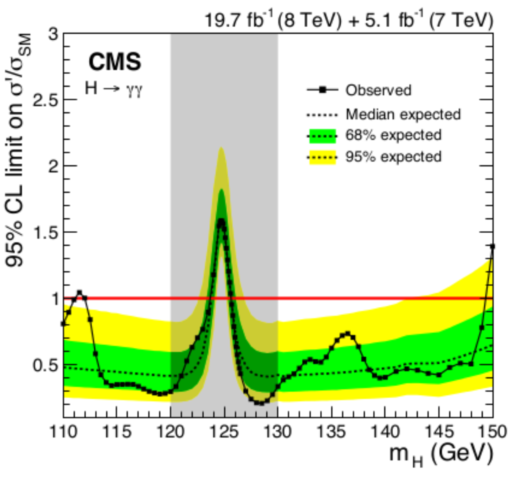


(b) $\delta = 0.01$

Figure 9: The 7 TeV cross section evaluated with different values of δ.



(a) Signal-to-background ratio at 7 TeV.



(b) Corresponding results from measurements at CMS.

Figure 10: The signal-to-background ratio of the $H \to \gamma\gamma$, displayed together with measurements made at CMS, with $\delta = 0.05$. The CMS data is Figure 27 of reference [27]

5 Conclusions

In this thesis, a chirally symmetric technicolor model containing three techniflavours has been constructed as an effective low energy field theory and the physical Lagrangian has been derived and implemented in a few processes. This model predicts a fine structure of the Higgs signal: the peak measured at 126 GeV should actually be divided into two. This split is one of the signatures that would verify this model.

Six new parameters have been introduced. The angles ψ, θ and ξ introduced in section 4.5, three corresponding to the neutral scalars and three corresponding to the neutral pseudoscalars, are free parameters whose value must be decided by measurements. These angles play the same role in the chiral symmetry breaking as the Weinberg angle does in Standard Model electroweak symmetry breaking (equation (2.6)). In the limit $\delta \to 0$ there is a $SO(3)$ symmetry in the mass spectrum, which is broken by the vevs u and v. The choice of parameters in this thesis has been arbritrary, but we had to choose a set of parameters in order to proceed with the phenomenological investigations. There is of course two sets of angles that correspond to each given configuration of fields, and the parameters given in section 4.5 is only one of these. The affect this has to the mass spectrum is not dramatic: it only changes the linear δ terms with prefactors. How it affects higher order terms in δ has not been investigated.

The model predicts plenty of new composite Higgs-like states in the energy range that has been investigated. These states have all similar masses and their peaks are very narrow. In order to separate these peaks, the resolution of the detectors must be increased. It would be natural to carry on with the work of this thesis by deciding how high the resolution must be in order to identify the separate peaks. In this model the physical mass spectrum has only been expressed to the first order in δ, which causes some of the particles to have the same masses. Redoing the analysis to higher orders might reveal further separation of the particle masses. The model also suggests that there will be a range of technibaryons, which would be more massive than the lightest technimesons. The model could be modified to account for baryons as well. Including baryons would make the Technicolor model more advanced, as this would include another symmetry group which accounts for baryon number.

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Appendix A The Mathematical Formulation

A.1 Group theory

A group G is defined as a set of elements g_i and a product operation $*$ with the following properties [26, 28, 29]:

1. Given two elements of the group, the product of the two are an element of the group as well:
$$g_i, g_j \in G \Rightarrow g_i * g_j \in G$$
2. There is an unit or identity element $\mathbb{1} \in G$ such that
$$\forall g \in G : \mathbb{1} * g = g * \mathbb{1} = g$$
3. For every element in the group, there is an inverse:
$$\forall g \in G; \exists g^{-1} : g * g^{-1} = g^{-1} * g = \mathbb{1}$$
4. The product operator must be associative:
$$g_i, g_j, g_k \in G \Rightarrow (g_i * g_j) * g_k = g_i * (g_j * g_k)$$

A group may be discrete or continuous. One also distinguish between abelian and non-abelian groups. A group is said to be abelian if all elements of the group commute under the product operation:

$$g_i, g_j \in G \Rightarrow [g_i, g_j] = 0$$

If this is not the case, the group is non-abelian.

A.1.1 Representation

Suppose that for a group G there is a set of $n \times n$-matrices $\{D(g)\}$ so that

$$\forall g_i, g_j \in G; D(g_i)D(g_j) = D(g_i g_j)$$

The set of matrices is said to form a representation of G. There exists some element $e \in G$ so that $D(e) = \mathbb{1}$, and from this we get

$$D(g^{-1}) = D^{-1}(g)$$

We can let each element of a set G represent itself so that $D(g) = g$. This representation is the fundamental representation of the group. If $D(g)$ is a matrix representation of a group, we may inverse and transpose the elements and get a representation $\bar{D}(g) = (D^T(g))^{-1}$. This representation is called the adjoint representation. If the fundamental representation is a column vector with n rows, the adjoint representation will be a $n \times n$ matrix.

A.1.2 Direct product

Given two separate groups G_1 and G_2, with the product operators $\cdot$ and $*$ respectively, one may form elements in pairs as $\{(g_{1i}, g_{2j})\}$. These elements form a new group G_3 that is a direct product of its constituent groups, written as $G_3 = G_1 \otimes G_2$. The product operation of G_3 is

$$(g_{1i}, g_{2k})(g_{1j}, g_{2l}) = (g_{1i} \cdot g_{1j}, g_{2k} * g_{2l})$$

A.1.3 Lie groups

Assume a group G whose elements g can be characterised by some parameter a. The product of two elements are a third so given the parameters a and b there must exist a third parameter c so that

$$g(a) * g(b) = g(c)$$

Since this must be true for all elements of the group, each pair of parameters a and b correspond to some c and there must be some functional ϕ establishing the relation between these:

$$c = \phi(a, b)$$

For every parameter a there is also a parameter $\bar{a}$ so that $g(a) * g(\bar{a}) = \mathbb{1}$. If the functional is differentiable an arbitrary number of times, and if ϕ is analytic, the group is said to be a Lie group. Although the elements above have been written as if they were dependent on only one parameter, they may as well be dependent on any number of them.

It can be shown that every element of a Lie group can be written as

$$g(a_1, a_2, ..., a_n) = \exp\left(\sum_{j=1}^{n} i a_j T_j\right) \tag{A.1}$$

Here $a_1, ..., a_n$ are the parameters of the element and T_j are the generators of the group. The generators are Hermitian matrices. The sum $\sum_j^n a_j T_j$ forms a space known as the Lie algebra of the group.

A.1.4 The groups of the Standard Model

The Standard Model is based on the direct product of three groups: $U(1) \otimes SU(2) \otimes SU(3)$. These are all Lie groups and the most important properties of each group is gone through bellow. All these groups are unitary, meaning that the inverse of each element is its Hermitian conjugate, or

$$U^\dagger U = U U^\dagger = \mathbb{1}$$

The U(1) group, with U standing for Unitary, is the group of all complex phase factors, on the form $U = e^{i\epsilon}$, under multiplication. This is a continuous group and the elements form the unit circle around the origin in the complex plane; all elements have unit magnitude. As the elements are scalars and the group product is multiplication, all elements commute and this is therefore an abelian group.

As the $U(1)$ group is a Lie group, it can be written on the form of equation (A.1). By inspection it can be seen that the generator of the group is a real number. In the Standard Model this generator is denoted Y and is called the hypercharge.

The $SU(2)$ and $SU(3)$ are two of the special unitary groups. A $SU(N)$ group is a group consisting of $N \times N$-matrices. The group is unitary as explained above and special in the sense that all matrices in the group have unit determinant.

The number of generators for a $SU(N)$ group is $N^2 - 1$. The three generators of the $SU(2)$ group are the three Pauli matrices τ_i whose commutation relations are

$$[\tau_i, \tau_j] = i\epsilon_{ijk}\tau_k \tag{A.2}$$

Here ϵ_{ijk} is the Levi-Civita symbol defined as

$$\epsilon_{ijk} = \begin{cases} +1 & \text{if} \quad (i,j,k) \text{ is } (1,2,3),\ (2,3,1) \text{ or } (3,1,2) \\ -1 & \text{if} \quad (i,j,k) \text{ is } (2,1,3),\ (3,2,1) \text{ or } (1,3,2) \\ 0 & \text{if} \quad i=j,\ j=k \text{ or } k=i \end{cases}$$

with indices i, j, k running from 1 to 3. The generators for the $SU(3)$ group are the eight Gell-Mann matrices λ_a. They commute as

$$[\lambda_a, \lambda_b] = 2if_{abc}\lambda_c$$

where f_{abc} is a completely antisymmetric structure constant just as ϵ_{ijk} but with indices a, b, c running from 1 to 8.

A.2 Lagrangian density

The Standard Model is formulated in terms of a Lagrangian density $\mathcal{L}$. The idea of the Lagrangian formulation of mechanics is based on the principle of least action. The Lagrangian density is denoted $\mathcal{L}$ and is defined as

$$\mathcal{L} = T - V$$

where T is the kinetic and V the potential energy of the system. The action of the system is then

$$S = \int \mathcal{L} \mathrm{d}^4 x$$

One should distinguish between the Lagrangian density $\mathcal{L}$ and the Lagrangian L; the action is given by the Lagrangian integrated with respect to time only. In the rest of this thesis though, the term "Lagrangian" will be used to refer to the Lagrangian density.

The equations of motion for a system can be derived by using the Lagrangian and the Euler-Lagrange equation. For a field ϕ_i, with four-gradient $\partial_\mu \phi_i = \frac{\partial}{\partial x^\mu}\phi_i$, the Euler-Dirac equation is

$$\frac{\partial \mathcal{L}}{\partial \phi_i} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial \left(\partial_\mu \phi_i \right)} \right) = 0 \tag{A.3}$$

If the system consists of several fields, we get one Euler-Lagrange equation for each field.

A.3 The Dirac equation

The theory describing particles must not depend on in which frame observations are made. Therefore the Lagrangian must be invariant under Lorentz transformations. Suppose we have a four-vector transforming as $x^\mu \rightarrow x^{\mu\prime} = \Lambda^\mu_\nu x^\nu$ for some 4×4-matrix. A field $\phi(x)$ would then transform as $\phi(x) \rightarrow \phi(x)' = \phi(\Lambda^{-1}x)$. As the product of two Lorentz transforms is another Lorentz transform, they form a group, the Lorentz group.

If we now construct a n-component multiplet of fields, that is a column vector with n rows, each row containing a field, and call it Φ_a will transform as

$$\Phi_a \rightarrow \Phi_b = M_{ab}(\Lambda)\Phi_a(\Lambda^{-1}x)$$

$M(\Lambda)$ is some $n \times n$-matrix. If we perform a series of transforms, say Λ and Λ', we want it to be equivalent to doing all transforms at the same time. Doing this on Φ_a, we want the $M(\Lambda)$-matrices to satisfy

$$M(\Lambda)M(\Lambda')\Phi = M(\Lambda\Lambda')\Phi$$

This means the $M(\Lambda)$-matrices form a representation of the Lorentz group. The generators of this representation are the γ-matrices, which can be chosen in different ways. One set of matrices is

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & -\tau_i \\ \tau_i & 0 \end{pmatrix}$$

where $i = 1, 2, 3$ and τ_i are the three Pauli matrices:

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

This choice of matrices is called the Weyl representation or the chiral representation. We could as well choose complex matrices like

$$\gamma^0 = \begin{pmatrix} 0 & \tau_2 \\ \tau_2 & 0 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} i\tau_1 & 0 \\ 0 & i\tau_1 \end{pmatrix}, \quad \gamma^2 = \begin{pmatrix} 0 & \tau_2 \\ -\tau_2 & 0 \end{pmatrix}, \quad \gamma^3 = \begin{pmatrix} i\tau_3 & 0 \\ 0 & i\tau_3 \end{pmatrix}$$

which then would make the Lorentz group represented in Majorana representation. In the Weyl representation we may construct a four-component field:

$$\psi(x) = \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \\ \psi_3(x) \\ \psi_4(x) \end{pmatrix}$$

This is a Dirac spinor, describing a particle with mass m and its corresponding anti-particle. We now define $\bar{\psi} = \psi^\dagger\gamma^0$. In this way the product $\bar{\psi}\psi$ is a Lorentz scalar. The Lagrangian of these fields will be the Dirac Lagrangian:

$$\mathcal{L}_{Dirac} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi \tag{A.4}$$

If we put this function in equation (A.3) with respect to $\bar{\psi}$, we get

$$i\gamma^\mu\partial_\mu\psi - m\psi = 0$$

which is the Dirac equation, a Lorentz invariant wave equation. Solving equation (A.3) with respect to ψ yields

$$-i\partial_\mu\bar{\psi}\gamma^\mu - m\bar{\psi} = 0$$

which is the Hermitian conjugate form of the equation above. If we would have chosen the Majorana representation when constructing the spinor, it would be a Majorana spinor, describing a particle whose antiparticle is identical, i.e. a particle of no electric charge. It is now useful to define a new matrix $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, which in Dirac representation is

$$\gamma^5 = \begin{pmatrix} -\mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix} \tag{A.5}$$

From this we can form operators P_L and P_R so that

$$P_L = \tfrac{1}{2}(\mathbb{1} - \gamma^5), \quad P_R = \tfrac{1}{2}(\mathbb{1} + \gamma^5)$$

If we write the four-component Dirac spinor in terms of two new spinors, $\psi_L = \begin{pmatrix} \psi_1 & \psi_2 \end{pmatrix}^T$ and $\psi_R = \begin{pmatrix} \psi_3 & \psi_4 \end{pmatrix}^T$ the Dirac spinor can be split up into two so called Weyl spinors with the above operators:

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

so that

$$\psi_L = P_L\psi, \quad \psi_R = P_R\psi \tag{A.6}$$

The L and R stand for left- and right-handed helicity respectively and this distinction will be important later as it turns out that fermions interact differently depending on its helicity. Some important relations of the helicity operators are

$$
\begin{aligned}
P_L^2 &= P_L \\
P_L^R &= P_R \\
P_L + P_R &= 1 \\
P_L P_R &= 0
\end{aligned}
$$

We also have that

$$\bar{\psi}_L = \bar{\psi} P_R, \quad \bar{\psi}_R = \bar{\psi} P_L$$

From this we can draw the conclusion that

$$
\begin{aligned}
\bar{\psi}\psi &= \bar{\psi}\left(P_L^2 + P_R^2\right)\psi \\
&= \bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R
\end{aligned}
\tag{A.7}
$$

This is a useful relation for identifying mass terms of fermions as theyhave the form $m\bar{\psi}\psi$ in the Lagrangian.

A.4 Invariance and the covariant derivative

When constructing a theory, we want it to be invariant under certain symmetry transformations. The equations of motions of a particle should not be dependent on from which angle we observe it, or how someone else chooses to fix their phases.

In quantum physics, observables of a wave function Ψ are dependent of the magnitude $|\Psi|^2$. This means that we should be able to add a phase factor without affecting the observable. This transform is named a global gauge transform and is an example of a $U(1)$ transform:

$$\Psi \to \Psi' = e^{-i\chi}\Psi$$

If the observable is dependent on time and position, it is a local gauge transform:

$$\Psi \to \Psi' = e^{-i\chi(t,\bar{x})}\Psi$$

If a function transforms as a wave function, its derivative should as well. The function Ψ above is gauge invariant, but its derivative $\partial^\mu \Psi$ is not:

$$
\begin{aligned}
\partial^\mu\left(e^{-i\chi}\Psi\right) &= e^{-i\chi}\left(\partial^\mu\Psi\right) - i\left(\partial^\mu\chi\right)e^{-i\chi}\Psi \\
&= e^{-i\chi}\left(\partial^\mu - i\partial^\mu\chi\right)\Psi
\end{aligned}
$$

The solution is to introduce the covariant derivative $D^\mu = \partial^\mu - igA^\mu$ where gA^μ is the field needed to cancel the $\partial^\mu\chi$ term, g being some scaling factor. When doing a transform, we have two covariant derivatives $D^\mu = \partial^\mu - igA^\mu$ and $D^{\mu\prime} = \partial^\mu - igA^{\mu\prime}$ so that, writing the phase factor as U:

$$D^{\mu\prime}(U\Psi) = U(D^\mu\Psi)$$

Writing the covariant derivatives explicitly, we may solve for $A^{\mu\prime}$ and it turns out that it transforms as [15]

$$A^{\mu\prime} = -\frac{i}{g}(\partial^\mu U)U^{-1} + UA^\mu U^{-1}$$

For transformations in $SU(2)$ the factor would be

$$\begin{pmatrix} \psi_1' \\ \psi_2' \end{pmatrix} = e^{i\epsilon_i\cdot\tau_i/2}\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

which represent a rotation in $SU(2)$ space. ϵ_i are three parameters required to specify the rotation and τ_i generates the $SU(2)$ group. To construct a covariant derivative we now need three fields W_i so that

$$D^\mu = \partial^\mu - ig\frac{\tau_i}{2}W_i^\mu$$

In $SU(3)$ space, generated by λ_a, the transformation factor would be $e^{\epsilon_a\lambda_a/2}$ with ϵ_a being the eight parameters needed in this case. There are also eight fields G_a^μ to keep the derivative covariant.

Appendix B Outlines of the Standard Model

The Standard Model includes a big number of particles sorted into different categories depending on their properties. The first distinction to make is the one between bosons and fermions.

The half-integer spin fermions are those who make up matter. The fermions are divided into leptons, which are the particles that do not interact strongly, and quarks, which are the particles that do. The leptons are the electron, the muon and the tau lepton, electrically charged, and the three corresponding uncharged neutrinos.

The integer spin bosons are the force carriers of the Standard Model. Photons are the particle of electromagnetic interaction, gluons of the strong interaction and the weak interaction is mediated by the Z and $W^\pm$ bosons. There is also one spin zero particle, the Higgs particle, that do not mediate any force, but is needed for the mechanism from which mass arise.

The Lagrangian of the Standard Model can be written as

$$\mathcal{L} = \mathcal{L}_{QCD} + \mathcal{L}_{EW} + \mathcal{L}_{Higgs} + \mathcal{L}_{yuk.} \tag{B.8}$$

Below, the meaning and consequences of each term will be gone through.

B.1 Quantum chromodynamics

The theory of Quantum Chromodynamics is based on the group $SU(3)_C$ with C standing for colour. The quarks are divided into six flavours: up, down, charm, strange, top and bottom, and in fundamental representation of $SU(3)$ these are all triplets. The gauge fields of $SU(3)$ are in adjoint representation. As no particle but quarks interact strongly all other particles are colour singlets.

The QCD Lagrangian is

$$\mathcal{L}_{QCD} = -\frac{1}{4}\hat{F}^a_{\mu\nu}\hat{F}^{\mu\nu}_a + \bar{\psi}_i \left(i\gamma^\mu D_\mu - m\right)\psi_i + h.c. \tag{B.9}$$

where $h.c.$ means that one should also add the hermitian conjugate of each term and

$$\hat{F}^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G_{b\mu} G_{c\nu}$$

is the gluon field tensor. G^a_μ are the colour fields and g_S is the strong coupling constant. The covariant derivative of $SU(3)_C$ is

$$D_\mu = \partial_\mu - i g_s \frac{\lambda_a}{2} G^a_\mu$$

In $SU(3)$ the quarks are in fundamental representation while the gluons are in adjoint representation [11, 30, 31].

B.1.1 Quark confinement and asymptotic freedom

The gluons, which are the force carriers of the strong interaction, interact strongly themselves. As a result, the force between two quarks increase when the distance between them increases, as opposed to the electromagnetic force between two charged objects which decreases inversely proportional to the square of the distance between the objects. This means that the quarks are bound together very strongly into hadrons and never appear as free quarks. In processes where the quarks are forced apart the potential energy of the field increases until it contains enough energy to create new quarks. If, for example, the hadron is a quark-antiquark pair, it would be more energetically favourable to create a new $q\bar{q}$ pair in between the two original quarks, so that the new antiquark binds to the old quark and vice versa.

The coupling strength of the QCD lagrangian g_s may also be expressed as

$$\alpha_s = \frac{g_s^2}{4\pi}$$

This coupling strength — and the coupling strength of the other forces — are not constant, but dependent on the energy scale considered. If the coupling strength depend on the (Lorenzt invariant) momentum $-q^2$, we may measure the strength at some reference momentum $-q^2 = -\mu^2$. If the reference strength is denoted $\alpha_s(\mu^2)$ it can be shown, via loop correction calculations, that the strength is given by

$$\alpha_s(-q^2) = \frac{\alpha_s(-\mu^2)}{1 + \frac{\alpha_s(-\mu^2)}{12\pi}(33 - 2N_f)\ln(\frac{-q^2}{-\mu^2})} \tag{B.10}$$

This is called a running coupling, as the coupling strength depend on the momentum of the particles. When the momentum increases, the denominator grows and the coupling strength becomes weaker. This behaviour is known as asymptotic freedom: when the momentum is high enough the quark-quark bonds become looser. On the other side of the spectrum there is a pole where the two terms of the denominator cancel. This pole is named the Landau pole, at $-q^2 = \Lambda^2_{QCD} \approx 200$ MeV. The coupling becomes incredibly strong here (and equation (B.10) is not valid at these energy scales) [14, 11].

B.2 Electroweak theory

The electroweak interactions are based on the group $SU(2)_L \otimes U(1)_Y$. The Lagrangian of the electroweak interaction is

$$\mathcal{L}_{EW} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a - \frac{1}{4}W_{\mu\nu}W^{\mu\nu} + i\bar{\psi}_j\gamma^\mu D_\mu\psi_j + h.c.$$

where $F^a_{\mu\nu}$ is the electromagnetic field tensor

$$F^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\epsilon^{abc}W_{b\mu}W_{c\nu}$$

and $W_{\mu\nu}$ is the weak interaction field tensor

$$W_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

The covariant derivative of $SU(2)_L \otimes U(1)_Y$ is

$$D_\mu = \partial_\mu - ig\frac{Y}{2}B_\mu - ig'\frac{\tau_i}{2}W^i_\mu$$

There is a distinction of the left- and right-handed leptons. Left-handed leptons are doublets in $SU(2) \otimes U(1)$ while right-handed leptons are singlets. This means that the right-handed leptons does not interact weakly. We write

$$L_L = \begin{pmatrix} \nu_e \\ e_L \end{pmatrix} \quad \text{and} \quad e_R$$

for the first family. For the second and third family the e is just exchanged for a μ or τ. As of now the Standard Model does not include any right-handed neutrino. The quarks are also sorted into left-handed doublets and right-handed singlets:

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad \text{and} \quad u_R, \quad d_R$$

As mentioned above, Y is the hypercharge of the particles and $\frac{\tau_i}{2}$ the isospin. The electric charge Q of a particle is related to these quantities as

$$Q = \frac{Y}{2} + T_3$$

where T_3 is the eigenvalue of the matrix $\frac{\tau_3}{2}$ corresponding to the particle in question, i.e. for the singlets $T_3 = 0$, for the upper particle of a doublet $T_3 = 1/2$ and for the lower particle of a doublet $T_3 = -1/2$. The electric charge of the particles are well known: e, μ and τ have $Q = -1$, neutrons have $Q = 0$, upper members of quark doublets, u, c and t, have $Q = 2/3$ and the lower quarks, d, s and b, have $Q = -1/3$. The charge unit in which these quantities are expressed is the elementary charge.

B.3 Yukawa terms

The Yukawa terms of the Standard Model Lagrangian looks like

$$\mathcal{L}_{yuk.} = g_e \bar{L}\phi e_R + g_d \phi \bar{Q} \phi d_R + g_u \bar{Q} \phi_c u_R + h.c. \tag{B.11}$$

where

$$L = \begin{pmatrix} \nu \\ e_L \end{pmatrix}, \quad Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

and $\bar{L}$ and $\bar{Q}$ are the hermitian conjugates of the doublets, g_e, g_u and g_d are some coupling constants and ϕ_c will be explained below. If we start with the first term of equation (B.11) together with its hermitian conjugate and insert the doublet of equation (2.5) the term will be

$$\begin{aligned}
g_e \left(\bar{L}\phi e_R + \phi^\dagger \bar{e}_R L\right) &= g_e \left(\begin{pmatrix} \bar{\nu} & \bar{e}_L \end{pmatrix} \begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix} e_R + \begin{pmatrix} 0 & \frac{v+H}{\sqrt{2}} \end{pmatrix} \bar{e}_R \begin{pmatrix} \nu \\ e_L \end{pmatrix} \right) \\
&= \frac{g_e v}{\sqrt{2}} \left(\bar{e}_L e_R + \bar{e}_R e_L\right) + \frac{g_e}{\sqrt{2}} \left(\bar{e}_L e_R + \bar{e}_R e_L\right) H \\
&= \frac{g_e v}{\sqrt{2}} \bar{e} e + \frac{g_e}{\sqrt{2}} \left(\bar{e}_L e_R + \bar{e}_R e_L\right) H
\end{aligned}$$

where on the last row, the relation of equation (A.7) is used. The electron has gained the mass $M_e = \frac{g_e v}{\sqrt{2}}$ and we have also found the interaction terms of the electrons and the Higgs field. This procedure can be repeated for the other terms of the Lagrangian, but with the distinction that for the third term of equation (2.1) the ϕ_c is used instead:

$$\phi_c = -i\tau_2 \phi^\dagger = \begin{pmatrix} -\phi^{0\dagger} \\ \phi^- \end{pmatrix} \to \begin{pmatrix} -\frac{v+H}{\sqrt{2}} \\ 0 \end{pmatrix}$$

where the arrow marks the electroweak symmetry breaking. ϕ had hypercharge $Y = 1$ and ϕ_c has hypercharge $Y = -1$. Something corresponding to ϕ_c is not used in the lepton case; the mass of the neutron, if it even exist, does not come from the electroweak symmetry breaking.

Appendix C Description of the used software

The calculations has been made using several different programs and packages, which are all tied to the computational software program Mathematica (version 8.0 [32] and 9.0 [18]). Here follows a description of these programs and how they work.

C.1 FeynRules

FeynRules 2.0 [19] is a package for Mathematica, made to calculate Feynman rules in four-dimensional spacetime of a particle physics model. The user writes a model file in which all components of a quantum field theory model are defined: gauge groups, fermion and boson fields, field mixings and coupling constants. Finally the Lagrangian of the model is included. With this information the program expands the Lagrangian and derives the Feynman rules of the model. When the expansion is done, the program can export the results so that they can be used in other packages such as FeynArts. FeynRules supports any Lorentz and gauge invariant Lagrangian and fields with spin 0,1/2,1,3/2 and 2, making it possible to explore models beyond the Standard Model.

C.2 FeynArts

The results from FeynRules is exported to FeynArts 3.9 [20] which generates the Feynman diagrams of the model and calculates the amplitudes of the processes. The user starts by specifying which kind of process to look at: how many particles that should go into the vertex, how many that should leave it, and on which loop level the calculations should be made. FeynArts creates and paints all Feynman diagrams that suits the specifications. When this is done, the user inputs the model file created by FeynRules, and FeynArts determine all combinations of fields allowed in such a process. The amplitudes of these processes are then simplified using FormCalc.

C.3 FormCalc and LoopTools

FormCalc 8.3 [21] is the Mathematica package that calculates the Feynman amplitudes of the verteces determined by FeynArts at tree level and one loop level. It starts by performing some simplifications of the algebraic expressions recieved from FeynArts, such as contracting indices and abbrevating constant expressions, and rewrites it into linear combinations of loop integrals with model parameters and kinematic variables as prefactors. All this rewriting is purely algebraic and analytical, but it does not perform any numerical evaluation. The final step of the program is to automaticaly generate Fortran and C source code for numerical evaluation of the amplitudes. When executing these routines, a fourth software is called, LoopTools 2.10 [22]. This package evaluates the one loop integrals as they are provided by FormCalc.

The results from these calculations are text files containing information about the width of the different particles involved in the processes studied and cross sections of the process for different center-of-momenta. There is a possibillity to calculate cross sections both at a parton level, where only the momentum of each involved particle is considered, but also to calculate it at hadron level, when it is taken into account that a particle making up a composite particle only carries a fraction of the total momentum of the composite particle. When hadron level calculations are done, they are done using a program implementing a parton density function. The program used for this task is LHAPDF 5.9 [33], using the PDF data set MSTW 2008 CPdeut NLO (68 % C.L.) [34].