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Vector Meson Masses in AdS/QCD

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Abstract

We study the anti-de Sitter/conformal field theory correspondence (AdS/CFT correspondence) and investigate in a scalar model how n point functions can be calculated through functional derivatives and how they can be obtained with the use of Witten diagrams instead.

We also study a previous anti-de Sitter/quantum chromodynamics (AdS/QCD) model where the mass of the ϕ meson has not been considered. It turns out to be equivalent to the mass of the ρ meson. A fact that is not supported experimentally. In an attempt to obtain better results for the ϕ meson mass we do a slight modification to the existing model. However our modifications led to computational difficulties and although some results could be obtained none agreed well with experimental data.

Contents

Chapter 1

Introduction

The foundation for this model was laid by Maldacena when he conjectured the AdS/CFT correspondence in 1997 [1]. The main aspect of this AdS/CFT correspondence is that it connects the calculations done in a four-dimensional gauge theory with those of a higher dimensional string theory [2].

Since the original conjecture the research has taken two directions. The first one is the attempts made to formally prove the conjecture, this is beyond the focus of this thesis. The second is the attempts to expand the conjecture beyond concerning the highly symmetric conformal field theories to more realistic gauge theories like QCD [2]. It is with this extended correspondence, AdS/QCD, that we will concern ourselves.

We perform calculations on a classical level in a five dimensional theory and use the duality to relate the results to observables on the quantum level of a four dimensional theory. It is done within a already mostly laid framework much of which is present in an older bachelor thesis by Sven Möller [3]. The reasons for why this formalism might be preferable to the standard perturbation calculations of QCD is both that these calculations may be a simpler way to reach the same results, and that one might obtain results that lie in an energy region where the results are hard to obtain directly in QCD.

In the previous model the ρ meson and the ϕ meson get the same masses. This is however not supported by experimental data. In an attempt to obtain better agreement with experiments we added two terms to the action. It seems though that calculating better values for the masses this way is difficult.

In Chapter 2 the AdS/CFT correspondence is presented. It begins with an overview of the AdS space followed by a section on how the correspondence can be stated. It concludes with a rather lengthy part on how calculations can be done in a scalar theory using Witten diagrams. The reason behind this part being so extensive is that the original plan was to use Witten diagrams to calculate four point functions in the actual model.

In Chapter 3 we present the model. The first part of the chapter presents the previously used action. We also describe how the calculations are performed and the results that have been obtained previously. the second part focuses on the terms we add to the action and what consequences those terms have on the calculations.

In Chapter 4 we present our results. We have results from only fitting the new free parameters to the ϕ and K^* masses and results from trying to refit all the free parameters of the model with a number of observables.

In Chapter 5 we present our conclusions and discuss our results.

Chapter 2

The Correspondence

2.1 Anti-de Sitter Space

The metric of the $AdS_5 \times S^5$ space is given by [2]

$$ds^2 = \frac{u^2}{L^2}((dx^0)^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2) - \frac{L^2}{u^2}du^2 - L^2 d\Omega_5. \tag{2.1}$$

Where the x^0, x^1, x^2 and x^3 are the standard four-dimensional spacetime coordinates, u is the fifth coordinate for the AdS_5 and $d\Omega_5$ is the five-dimensional solid angle on the corresponding hypersphere and L is the curvature radius.

The calculations we shall perform takes place in AdS_5 space and we will ignore the $L^2 d\Omega_5$ part of the metric in the remainder of the report.

With the coordinate transformation $u = 1/z$ the AdS_5 metric can be shown to be conformally equivalent to the flat five-dimensional Minkowski spacetime [2,3]. The metric becomes

$$ds^2 = \frac{L^2}{z^2}((dx^0)^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2 - dz^2). \tag{2.2}$$

The papers concerning the anti-de Sitter space have large differences in notation. Therefore we briefly state a few definitions used throughout the text. As usual greek letters (i.e. μ, ν,...) as indices will run through 0, 1, 2 and 3. Set $z = x^5$ and let capital latin letters (i.e. M, N,...) as indices run through the usual four and the additional fifth. I.e. we can write the metric as

$$ds^2 = g_{MN} dx^M dx^N, \tag{2.3}$$

were we have introduced the metric tensor of the anti-de Sitter space

$$g_{MN} = \frac{L^2}{z^2}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix} = \frac{L^2}{z^2}\eta_{MN}. \tag{2.4}$$

g_{MN} is required to have this form for equations (2.2) and (2.3) to be equivalent. This tensor lowers indices as usual and to its covariant counterpart that raises indices can be obtained through the identity $g_{MN}g^{NL} = \delta_M^L$. Where the Kronecker delta has the form $\delta_M^L = \text{diag}(1,1,1,1,1)$. We have also introduced the shorthand notation $\eta_{MN} = \eta^{MN} = \text{diag}(1,-1,-1,-1,-1)$, which however is not a proper tensor. Another convention is the determinant of the metric tensor

$$g = \det(g_{MN}) = \frac{L^{10}}{z^{10}}. \tag{2.5}$$

However at many times it is preferable to separate the standard coordinates and the z-coordinate. In those cases, as stated above, we use the standard conventions with Greek letters i.e. write the metric as

$$\mathrm{d}s^2 = \frac{L^2}{z^2}(\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu - \mathrm{d}z^2), \tag{2.6}$$

with the standard flat Minkowski metric $\eta_{\mu\nu} = \text{diag}(1,-1,-1,-1)$.

In this model the z-coordinate represents an inverse energy scale [2]. Low z corresponds to high energy and a high z to low energy. In particular the boundary $z = 0$ corresponds to infinite energies. This naturally produces some divergences in the calculations and a ultraviolet (UV) cut-off L_0 has to be introduced. The expressions containing L_0 are implied to be taken in the limit where L_0 goes to zero.

We are doing these calculations in the so called hard-wall model, in which an Infra-red (IR) cut-off is introduced in addition to the UV cut-off. This cut-off, L_1, corresponds to the IR cut-off in QCD, Λ_{QCD}, and simulates confinement [4].

2.2 Formulation of the Correspondence

Now it is time to shed light on how the correspondence is applied. To make use of it we need to have an explicit mathematical formulation. Instead of the previous broad statements about how the string theory side is related to the gauge theory side, here we will look closely at exactly which quantities are related.

In QCD we have sought after quantities, e.g. masses, decay constants, form factors. Within the theory these quantities are given as expectation values of different operators. However via the correspondence these operators can be related to fields in the the AdS_5 space. The calculations can then be performed in 5 dimensions with the fields and the results are then translated back to the language of QCD.

To make this correspondence explicit we have to assume a field theory operator $\mathcal{O}(x^\mu)$ and its related field $\phi(x^\mu, z)$ in AdS_5 space. The field is the bulk field, which is related to the boundary field through

$$\phi(x^\mu, 0) = z^{4-\Delta}\phi_0(x^\mu). \tag{2.7}$$

Where Δ is the conformal dimension of the field.

Now if we call the string action of the bulk field $\mathcal{S}[\phi(x^\mu, z)]$ and define the functional

$$Z = \exp(\mathcal{S}[\phi(x^\mu, 0)]), \tag{2.8}$$

then we can state the correspondence as [2]

$$Z = \left\langle T \exp \int \mathrm{d}^4 x \phi_0(x^\mu) \mathcal{O}(x^\mu) \right\rangle_{\text{field theory}} . \tag{2.9}$$

I.e. from knowing the string action for the fields coupled to the operators we arrive at a generating functional for the field theory side. We can see from the right hand side that the boundary fields, $\phi(x^\mu)$, is the acting as the source for the operators, $\mathcal{O}(x^\mu)$.

To achieve something useful from this, an expectation value that can be related to a measurable quantity, we take the repeated functional derivative with respect to the source field, $\phi_0(x^\mu)$. The n-point correlator is given by the n:th functional derivative of Z. We have

$$\frac{\delta^n Z}{\delta\phi_0(x_1^\mu)...\delta\phi_0(x_n^\mu)} = \langle T\mathcal{O}(x_1^\mu)...\mathcal{O}(x_n^\mu)\rangle_{\text{field theory}} . \tag{2.10}$$

To achieve realistic results that are related to QCD we will naturally have to include more than a single scalar field in our theory. Although first we will look at the example with a single scalar field to show how to calculate physical observables from the 5 dimensional theory.

2.3 A Scalar Example

We consider the action for a massive scalar field on AdS_5. It is given by [2] (with an overall constant 1/2 instead of $1/g_s^2$ and $L = 1$ for simplicity.)

$$\begin{aligned} \mathcal{S} &= \int \mathrm{d}^5 x \sqrt{g} \left[\frac{g^{MN} \partial_M \phi(z,x) \partial_L \phi(z,x)}{2} - \frac{m^2}{2} (\phi(z,x))^2 \right] \\ &= \int \mathrm{d}^5 x \frac{1}{z^3} \left[\frac{\eta^{ML} \partial_M \phi(z,x) \partial_L \phi(z,x)}{2} - \frac{m^2}{2z^2} (\phi(z,x))^2 \right]. \end{aligned} \tag{2.11}$$

Were we use the shorthand notation $\phi(z, x^\mu) = \phi(z, x)$.

We are interested in the 2-point function in momentum space on the four dimensional boundary theory. To achieve this we do a expansion in z-independent Fourier modes in accordance to

$$\phi(z,x) = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} e^{-ik\cdot x} f_k(z) \phi_0(k), \tag{2.12}$$

where $\phi_0(k)$ is the Fourier transform of the boundary field. We take the functional derivative with respect to this boundary field in order to compute n-point functions as shown in equation (2.10).

The resulting action is

$$
\begin{aligned}
\mathcal{S} &= \frac{1}{2}\int \mathrm{d}^5x \int \frac{\mathrm{d}^4k}{(2\pi)^4}\int \frac{\mathrm{d}^4k'}{(2\pi)^4}\frac{1}{z^3}\bigg[- k\cdot k' f_k(z) f_{k'}(z) \\
&\quad - \partial_z f_k(z)\partial_z f_{k'}(z) - \frac{m^2}{z^2} f_k(z) f_{k'}(z)\bigg]\phi_0(k)\phi_0(k')e^{-ix\cdot(k+k')} \\
&= \frac{1}{2}\int_{L_0}^{L_1} \mathrm{d}z \int \frac{\mathrm{d}^4k}{(2\pi)^4}\int \frac{\mathrm{d}^4k'}{(2\pi)^4}\frac{1}{z^3}\bigg[k^2 f_k(z) f_{k'}(z) - \partial_z f_k(z)\partial_z f_{k'}(z) \\
&\quad - \frac{m^2}{z^2} f_k(z) f_{k'}(z)\bigg]\phi_0(k)\phi_0(k')(2\pi)^4\delta^{(4)}(k+k').
\end{aligned}
\tag{2.13}
$$

Before we go any further we should look into the equation of motion (EOM) which will give us an expression for $f_k(z)$. The EOM can be found from the original action, equation (2.11), to be

$$
\left[\eta^{ML}\partial_L\left(\frac{1}{z^3}\partial_M\right) + \frac{m^2}{z^5}\right]\phi(z,x) = 0. \tag{2.14}
$$

By dividing M into indices z and μ we see that the equation reads

$$
\left[\frac{\partial_\mu\partial^\mu}{z^3} - \partial_z\left(\frac{1}{z^3}\partial_z\right) + \frac{m^2}{z^5}\right]\phi(z,x) = 0 \tag{2.15}
$$

and by applying the four dimensional Fourier transform we arrive at

$$
\left[\frac{-k^2}{z^3} - \partial_z\left(\frac{1}{z^3}\partial_z\right) + \frac{m^2}{z^5}\right] f_k(z)\phi_0(k) = 0. \tag{2.16}
$$

Since $\phi_0(k)$ is independent of z we find that $f_k(z)$ must solve this equation regardless of the value of $\phi_0(k)$ and we find an equation for $f_k(z)$

$$
f_k''(z) - \frac{3}{z} f_k'(z) + \left(k^2 - \frac{m^2}{z^2}\right) f_k(z) = 0. \tag{2.17}
$$

To proceed we integrate the term containing derivatives with respect to z by parts, which gives us

$$
\begin{aligned}
\mathcal{S} &= \left[\frac{1}{2}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\int \mathrm{d}^4k'\delta^{(4)}(k+k')\phi_0(k)\phi_0(k')\frac{f_{k'}(z)\partial_z f_k(z)}{z^3}\right]_{L_0}^{L_1} \\
&\quad + \frac{1}{2}\int_{L_0}^{L_1}\mathrm{d}z \int \frac{\mathrm{d}^4k}{(2\pi)^4}\int \mathrm{d}^4k'\frac{1}{z^3}\bigg[\partial_z\partial_z f_k(z) - \frac{3}{z}\partial_z f_k(z) \\
&\quad + k^2 f_k(z) - \frac{m^2}{z^2} f_k(z)\bigg] f_{k'}(z)\phi_0(k)\phi_0(k')\delta^{(4)}(k+k').
\end{aligned}
\tag{2.18}
$$

The expression within the square brackets in the second term is 0 by the EOM and thus the whole term evaluates to 0 and we are left with the boundary term.

To compute the 2-point function we take two functional derivatives of the generating functional, Z, with respect to the boundary field $\phi_0(x)$. However from the chain rule we can see that we just get Z times the functional derivative of the action. This expression is then evaluated at $\phi_0 = 0$ giving $Z = 1$ and it is sufficient to take the functional derivative of the action. Since we are interested in the 2-point function in momentum space we also have to Fourier transform the expression.

$$\begin{aligned} \langle \mathcal{O}(p)\mathcal{O}(p')\rangle &= \int \mathrm{d}^4x \int \mathrm{d}^4x' e^{ix\cdot p} e^{ix'\cdot p'} \langle \mathcal{O}(x)\mathcal{O}(x')\rangle \\ &= \int \mathrm{d}^4x \int \mathrm{d}^4x' e^{ix\cdot p} e^{ix'\cdot p'} \frac{\delta^2 \mathcal{S}[\phi_0(x'')]}{\delta\phi_0(x)\delta\phi_0(x')}. \end{aligned} \tag{2.19}$$

When we expanded $\phi_0(x'')$ in Fourier modes the explicit dependence of $\phi_0(x'')$ was eliminated from the action. However we can treat action as a functional of the the Fourier transform $\phi_0(k)$ which in turn we treat as a functional of $\phi_0(x'')$. Then we apply the chain rule for functional derivatives and get

$$\frac{\delta^2 \mathcal{S}[\phi_0(k)[\phi_0(x'')]]}{\delta\phi_0(x)\delta\phi_0(x')} = \frac{\delta}{\delta\phi_0(x)}\left(\int \mathrm{d}^4q' \frac{\delta\mathcal{S}[\phi_0(k)]}{\delta\phi_0(q')} \frac{\delta\phi_0(q')[\phi_0(x'')]}{\delta\phi_0(x')} \right), \tag{2.20}$$

where the last functional derivative evaluates as

$$\frac{\delta\phi_0(q')[\phi_0(x'')]}{\delta\phi_0(x')} = \frac{\delta}{\delta\phi_0(x')} \int \mathrm{d}^4x'' e^{-iq'\cdot x''} \phi_0(x'') = e^{-iq'\cdot x'}. \tag{2.21}$$

Using this we get

$$\begin{aligned} \frac{\delta^2 \mathcal{S}[\phi_0(k)[\phi_0(x'')]]}{\delta\phi_0(x)\delta\phi_0(x')} &= \int \mathrm{d}^4q' e^{-iq'\cdot x'} \frac{\delta}{\delta\phi_0(q')}\left(\frac{\delta\mathcal{S}[\phi_0(k)[\phi_0(x'')]]}{\delta\phi_0(x)} \right) \\ &= \int \mathrm{d}^4q' e^{-iq'\cdot x'} \frac{\delta}{\delta\phi_0(q')}\left(\int \mathrm{d}^4q \frac{\delta\mathcal{S}[\phi_0(k)]}{\delta\phi_0(q)} \frac{\delta\phi_0(q)[\phi_0(x'')]}{\delta\phi_0(x)} \right) \\ &= \int \mathrm{d}^4q \int \mathrm{d}^4q' e^{-iq'\cdot x'} e^{-iq\cdot x} \left(\frac{\delta^2\mathcal{S}[\phi_0(k)]}{\delta\phi_0(q)\delta\phi_0(q')} \right). \end{aligned} \tag{2.22}$$

Where the functional derivative evaluates as

$$\begin{aligned} \frac{\delta^2\mathcal{S}[\phi_0(k)]}{\delta\phi_0(q)\delta\phi_0(q')} &= \frac{\delta^2}{\delta\phi_0(q)\delta\phi_0(q')} \left[\frac{1}{2} \int \frac{\mathrm{d}^4k}{(2\pi)^4} \int \mathrm{d}^4k' \delta^{(4)}(k+k')\phi_0(k)\phi_0(k') \frac{f_{k'}(z)\partial_z f_k(z)}{z^3} \right]_{L_0}^{L_1} \\ &= \left[\frac{1}{(2\pi)^4} \delta^{(4)}(q+q') \frac{f_{q'}(z)\partial_z f_q(z)}{z^3} \right]_{L_0}^{L_1}, \end{aligned} \tag{2.23}$$

giving

$$\frac{\delta^2 \mathcal{S}[\phi_0(x'')]}{\delta\phi_0(x)\delta\phi_0(x')} = \int \mathrm{d}^4 q \int \mathrm{d}^4 q' e^{-iq'\cdot x'} e^{-iq\cdot x} \left[\frac{1}{(2\pi)^4} \delta^{(4)}(q+q') \frac{f_{q'}(z)\partial_z f_q(z)}{z^3} \right]_{L_0}^{L_1}. \tag{2.24}$$

Which we can insert in the expression for the two point correlator to give

$$\begin{aligned}
\langle \mathcal{O}(p)\mathcal{O}(p')\rangle &= \int \mathrm{d}^4 x \int \mathrm{d}^4 x' \int \mathrm{d}^4 q \int \mathrm{d}^4 q' e^{ix\cdot q} e^{ix'\cdot q'} e^{ix\cdot p} e^{ix'\cdot p'} \\
&\quad \times \left[\frac{1}{(2\pi)^4} \delta^{(4)}(q+q') \frac{f_{q'}(z)\partial_z f_q(z)}{z^3} \right]_{L_0}^{L_1} \\
&= \int \mathrm{d}^4 q \int \mathrm{d}^4 q' \delta^{(4)}(p+q)\delta^{(4)}(p'+q')(2\pi)^8 \left[\frac{1}{(2\pi)^4} \delta^{(4)}(q+q') \frac{f_{q'(z)}\partial_z f_q(z)}{z^3} \right]_{L_0}^{L_1} \\
&= \left[(2\pi)^4 \delta^{(4)}(p+p') \frac{f_{p'}(z)\partial_z f_p(z)}{z^3} \right]_{L_0}^{L_1}.
\end{aligned} \tag{2.25}$$

Since we can see from the EOM that f_p only depends on p^2 and thus $f_{-p} = f_p$. At the IR boundary we either have $L_1 = \infty$ and $f_p(L_1) = \partial_z f_p(L_1) = 0$ or we have a finite L_1 with the boundary condition chosen as $\partial_z f_p(L_1) = 0$ to reduce the interference of the boundary.

either way the expression also vanishes at the IR boundary so we get

$$\langle \mathcal{O}(p)\mathcal{O}(p')\rangle = -(2\pi)^4 \delta^{(4)}(p+p') \frac{f_{p'}(z)\partial_z f_p(z)}{z^3}\bigg|_{L_0}. \tag{2.26}$$

2.4 Witten Diagrams

The previous example shows how all n-point functions in principle can be derived. It is however a long and tedious procedure, especially when going to higher n. Even with some shortcuts that were not taken in the explicit example above the calculations become gruesome. Luckily the calculations can be simplified with the use of Witten diagrams. However to understand the underlying principles we must perform a couple more explicit calculations where we include an interaction. For simplicity only a ϕ^3 term is included.

If we would have used the same procedure as presented here in a four dimensional theory we would have obtained all tree level Feynman diagrams in the end.

2.4.1 Iterative Solution

Consider now the action where an interaction term has been added

$$\mathcal{S} = \int \mathrm{d}^5 x \sqrt{g} \left[\frac{g^{MN}\partial_M \phi(z,x)\partial_L \phi(z,x)}{2} - \frac{m^2(\phi(z,x))^2}{2} - \frac{b}{6}(\phi(z,x))^3 \right]. \tag{2.27}$$

The EOM becomes

$$\frac{1}{\sqrt{g}}\partial_M(g^{ML}\sqrt{g}\partial_L\phi(z,x)) + m^2\phi(z,x) = -\frac{1}{2}b(\phi(z,x))^2, \tag{2.28}$$

where we want $\phi(L_0,x) = \phi_0(x)$.

We can identify the Laplace operator $\frac{1}{\sqrt{g}}\partial_M(g^{ML}\sqrt{g}\partial_L) = \nabla^2$ and write the equation more conveniently

$$(\nabla^2 + m^2)\phi(z,x) = -\frac{1}{2}b(\phi(z,x))^2. \tag{2.29}$$

This equation has no simple solution and we will solve it iteratively, as done in [10]. We start with solving

$$(\nabla^2 + m^2)\phi'(z,x) = 0. \tag{2.30}$$

with boundary condition $\phi'(L_0,x) = \phi_0(x)$. We do so by defining a *bulk to boundary propagator* $K(z,x,x')$ which satisfies

$$(\nabla^2 + m^2)K(z,x,x') = 0, \tag{2.31}$$

$$K(L_0,x,x') = \delta^{(4)}(x-x') \tag{2.32}$$

and write the solution as

$$\phi'(z,x) = \int d^4x'\, K(z,x,x')\phi_0(x'). \tag{2.33}$$

then we insert this solution on the right hand side of the EOM and solve for $\phi''(z,x)$

$$(\nabla^2 + m^2)\phi''(z,x) = -\frac{1}{2}b(\phi'(z,x))^2, \tag{2.34}$$

with boundary condition $\phi''(L_0,x) = 0$. To solve this we define a *bulk to bulk propagator* $G(z,z',x,x')$ that satisfies

$$(\nabla^2 + m^2)G(z,z',x,x') = \frac{\delta(z-z')\delta^{(4)}(x-x')}{\sqrt{g}}, \tag{2.35}$$

$$G(L_0,z',x,x') = 0. \tag{2.36}$$

With which we can write the solution as

$$\begin{aligned}
&\phi''(z,x)\\
&= -\frac{b}{2}\int \mathrm{d}^5x'\sqrt{g}G(z,z',x,x')(\phi'(z',x'))^2\\
&= -\frac{b}{2}\int \mathrm{d}^5x'\sqrt{g}G(z,z',x,x')\int d^4x''\, K(z',x',x'')\phi_0(x'')\int d^4x'''\, K(z',x',x''')\phi_0(x''').
\end{aligned} \tag{2.37}$$

Now the solution up to $\mathcal{O}(\phi_0^2)$ can be written as $\phi_{(2)}(z,x) = \phi'(z,x) + \phi''(z,x)$

In the next iteration we get $\phi'''(z,x)$ which solves

$$(\nabla^2 + m^2)\phi'''(z,x) = -\frac{b}{2}(\phi'(z,x) + \phi''(z,x))^2, \tag{2.38}$$

with $\phi'''(L_0,x) = 0$. We can immediately write down the solution as

$$\begin{aligned}
&\phi'''(z,x) = -\frac{b}{2}\int \mathrm{d}^5x'\sqrt{g}G(z,z',x,x')(\phi'(z,x)+\phi''(z,x))^2 \qquad (2.39)\\
&= -\frac{b}{2}\int \mathrm{d}^5x'\sqrt{g}G(z,z',x,x')\int d^4x''\, K(z',x',x'')\phi_0(x'')\int d^4x'''\, K(z',x',x''')\phi_0(x''')\\
&\quad + b^2\int \mathrm{d}^5x'\sqrt{g}G(z,z',x,x')\int d^4x''\, K(z',x',x'')\phi_0(x'')\\
&\quad \times \int \mathrm{d}^5x'''\sqrt{g}G(z',z''',x',x''')\int d^4x^{(4)}\, K(z''',x''',x^{(4)})\phi_0(x^{(4)})\int d^4x^{(5)}\, K(z''',x''',x^{(5)})\phi_0(x^{(5)})\\
&\quad + \mathcal{O}(\phi_0^4).
\end{aligned}$$

Where we stop at order three since $\phi'''(z,x)$ contains some but not all contributions at the fourth order. the solution up to third order can now be written $\phi_{(3)}(z,x) = \phi'(z,x) + \phi'''(z,x)$. In general the solution to n:th order is $\phi_{(n)}(z,x) = \phi'(z,x) + \phi^{(n)}(z,x)$ where $\phi^{(n)}(z,x)$ solves

$$(\nabla^2 + m^2)\phi^{(n)}(z,x) = -\frac{b}{2}(\phi'(z,x) + \phi^{(n-1)}(z,x))^2, \tag{2.40}$$

with $\phi^{(n)}(L_0,x) = 0$.

2.4.2 Relationship Between G and K

To find a relationship between the bulk to bulk propagator and the bulk to boundary propagator we apply Green's second identity

$$\begin{aligned}
&\int \mathrm{d}^5x\sqrt{g}(G(z,z',x,x')(\nabla^2+m^2)K(z,x,x'') - K(z,x,x'')(\nabla^2+m^2)G(z,z',x,x'))\\
&= \int \mathrm{d}^5x\sqrt{g}(G(z,z',x,x')\nabla^2K(z,x,x'') - K(z,x,x'')\nabla^2G(z,z',x,x'))\\
&= \int \mathrm{d}^4x\sqrt{\gamma}\times\Big[(G(z,z',x,x')n^M\partial_MK(z,x,x'') - K(z,x,x'')n^M\partial_MG(z,z',x,x'))\Big]_{z=L_0}^{z=L_1}
\end{aligned} \tag{2.41}$$

where γ is the determinant of the boundary metric and n^M is the unit vector normal to the boundary and directed outwards. Using the defining equations of the propagators the left hand side evaluates to

$$LHS = \int \mathrm{d}^5x\sqrt{g}\left(G(z,z',x,x')\times 0 - K(z,x,x'')\frac{\delta^{(4)}(x-x')\delta(z-z')}{\sqrt{g}}\right) = -K(z',x',x''). \tag{2.42}$$

The right hand side vanishes at the IR boundary, L_1, and using the defining boundary values for the propagators it evaluates to

$$\begin{aligned} RHS &= -\int \mathrm{d}^4x\sqrt{\gamma} \times (0 \times n^M\partial_M K(z,x,x'') - \delta^{(4)}(x-x'')n^M\partial_M G(z,z',x,x')|_{z=L_0}) \\ &= \sqrt{\gamma}n^M\partial_M G(z,z',x'',x')|_{z=L_0}. \end{aligned} \quad (2.43)$$

Combining this gives the relation

$$K(z',x',x'') = -\sqrt{\gamma}n^M\partial_M G(z,z',x'',x')|_{z=L_0}. \quad (2.44)$$

2.4.3 3 point function

Now we can reinsert the solution into the action to calculate the 3-point function. Before doing that we shall integrate it by parts to obtain

$$\begin{aligned} \mathcal{S} &= \int \mathrm{d}^4x\sqrt{\gamma}\left[\frac{\phi(z,x)n^M\partial_M\phi(z,x)}{2}\right]_{z=L_0}^{z=L_1} + \int \mathrm{d}^4x\int \mathrm{d}z\sqrt{g}\left[-\frac{\nabla^2\phi(z,x)}{2} - \frac{m^2\phi^2}{2} - \frac{b}{6}\phi^3\right] \\ &= \int \mathrm{d}^4x\sqrt{\gamma}\left[\frac{\phi(z,x)n^M\partial_M\phi(z,x)}{2}\right]_{z=L_0}^{z=L_1} + b\int \mathrm{d}^4x\int \mathrm{d}z\sqrt{g}\left(\frac{1}{4}-\frac{1}{6}\right)(\phi(z,x))^3. \end{aligned} \quad (2.45)$$

We only need to expand the action to third order in ϕ_0 since we want the three point function. The only way to achieve this in the bulk term is to expand each of the three fields to first order

$$\begin{aligned} \mathcal{S}^{(3)}_{Bulk} &= b\int \mathrm{d}^4x\int \mathrm{d}z\sqrt{g}\left(\frac{1}{4}-\frac{1}{6}\right) \\ &\quad\times\int \mathrm{d}^4x'\phi_0(x')K(z,x,x')\int \mathrm{d}^4x''\phi_0(x'')K(z,x,x'')\int \mathrm{d}^4x'''\phi_0(x''')K(z,x,x'''). \end{aligned} \quad (2.46)$$

The boundary term is however not quite so straightforward. In the IR boundary it vanishes but we must still evaluate it in the UV boundary. Using $\phi(L_0,x) = \phi_0(x)$ we find that the other factor involving $\phi(z,x)$ must be expanded to second order in ϕ_0 to give a total order of three. Using our expansion for $\phi(z,x)$, equation (2.37), gives us

$$\begin{aligned} &-\sqrt{\gamma}n^M\partial_M\phi^{(2)}(z,x)|_{z=L_0} \\ &= -\frac{b}{2}\int \mathrm{d}^4x'\int \mathrm{d}z'\sqrt{g}(-\sqrt{\gamma}n^M\partial_M(z,z',x,x'))|_{z=L_0} \\ &\quad\times\int \mathrm{d}^4x''\phi_0(x'')K(z',x',x'')\int \mathrm{d}^4x'''\phi_0(x''')K(z',x',x''') \\ &= -\frac{b}{2}\int \mathrm{d}^4x'\int \mathrm{d}z'K(z',x,x')\int \mathrm{d}^4x''\phi_0(x'')K(z',x',x'')\int \mathrm{d}^4x'''\phi_0(x''')K(z',x',x'''), \end{aligned} \quad (2.47)$$

where we used the relation between $K(z', x, x')$ and $G(z, z', x, x')$. If we insert this in the boundary term and relabel the variables $x' \leftrightarrow x$ and $z' \leftrightarrow z$ we get the boundary term of order three in ϕ_0

$$\begin{aligned} \mathcal{S}^{(3)}_{boundary} &= -\frac{b}{4}\int \mathrm{d}^4x \int \mathrm{d}z\sqrt{g}\int \mathrm{d}^4x'\phi_0(x')K(x,x',z) \\ &\times \int \mathrm{d}^4x''\phi_0(x'')K(x,x'',z)\int \mathrm{d}^4x'''\phi_0(x''')K(x,x''',z). \end{aligned} \tag{2.48}$$

Adding this up with the bulk term gives the total third order contribution

$$\begin{aligned} \mathcal{S}^{(3)} &= \mathcal{S}^{(3)}_{Bulk} + \mathcal{S}^{(3)}_{boundary} \\ &= -\frac{b}{6}\int \mathrm{d}^4x \int \mathrm{d}z\sqrt{g}\int \mathrm{d}^4x'\int \mathrm{d}^4x''\int \mathrm{d}^4x'''K(x,x',z) \\ &\times K(x,x'',z)K(x,x''',z)\phi_0(x')\phi_0(x'')\phi_0(x'''). \end{aligned} \tag{2.49}$$

The three point function is now obtained by taking three functional derivatives of the action with respect to ϕ_0

$$\begin{aligned} \langle T(\mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3))\rangle &= \frac{\delta^3\mathcal{S}}{\delta\phi_0(x_1)\delta\phi_0(x_2)\delta\phi_0(x_3)} \\ &= -b\int \mathrm{d}^4x\int \mathrm{d}z\sqrt{g}K(x,x_1,z)K(x,x_2,z)K(x,x_3,z), \end{aligned} \tag{2.50}$$

where the factor 1/6 goes away since there are 6 ways to match up x', x'', x''' with x_1, x_2, x_3. This result can however be directly obtained from the diagram in figure 2.1.

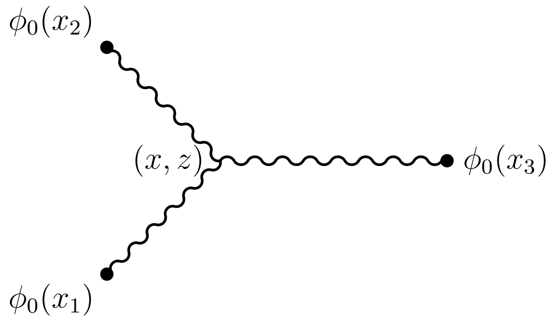

Figure 2.1: Witten diagram for the three point function

by

1. multiply a bulk to boundary propagator for each line that ends at the boundary (with a dot).

2. multiply a bulk to bulk propagator for each line with both ends in the bulk (none present here).

3. add a factor $-b$ for each vertex.

4. integrate over vertex positions with $\int \mathrm{d}^5x\sqrt{g}$.

We are however often interested in the momentum space n point function, by which we mean momentum space for the first four coordinates and position space for z. To obtain these there are analogous rules for the diagrams in momentum space. To show an example we Fourier transform the three point function

$$\begin{aligned}\langle \mathcal{O}(p_1)\mathcal{O}(p_2)\mathcal{O}(p_3)\rangle &= \int \mathrm{d}^4x_1 \int \mathrm{d}^4x_2 \int \mathrm{d}^4x_3 e^{ip_1\cdot x_1}e^{ip_2\cdot x_2}e^{ip_3\cdot x_3}\langle T(\mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3))\rangle \\ &= -b\int \mathrm{d}^4x \int \mathrm{d}z\sqrt{g}\int \mathrm{d}^4x_1 \int \mathrm{d}^4x_2 \int \mathrm{d}^4x_3 e^{ip_1\cdot x_1}e^{ip_2\cdot x_2}e^{ip_3\cdot x_3} \\ &\quad \times K(x,x_1,z)K(x,x_2,z)K(x,x_3,z).\end{aligned} \tag{2.51}$$

This may not seem straightforward at first to evaluate, but the bulk to boundary propagators only depend on the distance between its two x arguments [10] so with the right variable substitution the integrals are easily evaluated.

$$\begin{aligned}\langle \mathcal{O}(p_1)\mathcal{O}(p_2)\mathcal{O}(p_3)\rangle &= -b\int \mathrm{d}^4x \int \mathrm{d}z\sqrt{g}\int \mathrm{d}^4x_1 \int \mathrm{d}^4x_2 \int \mathrm{d}^4x_3 e^{ip_1\cdot x_1}e^{ip_2\cdot x_2}e^{ip_3\cdot x_3} \\ &\quad \times K(x_1-x,z)K(x_2-x,z)K(x_3-x,z) \\ &= \left\{\begin{matrix} u = x_1 - x \\ v = x_2 - x \\ w = x_3 - x \end{matrix}\right\} \\ &= -b\int \mathrm{d}^4x \int \mathrm{d}z \int \mathrm{d}^4u \int \mathrm{d}^4v \int \mathrm{d}^4w e^{ix\cdot(p_1+p_2+p_3)}e^{ip_1\cdot u}e^{ip_2\cdot v}e^{ip_3\cdot w} \\ &\quad \times \sqrt{g}K(u,z)K(v,z)K(w,z) \\ &= -b(2\pi)^4\delta^{(4)}(p_1+p_2+p_3)\int \mathrm{d}z\sqrt{g}K_{p_1}(z)K_{p_2}(z)K_{p_3}(z).\end{aligned} \tag{2.52}$$

If we define the Fourier transform of $K(u,z)$ as

$$K_p(z) = \int \mathrm{d}^4u e^{ip\cdot u}K(u,z). \tag{2.53}$$

This expression can be found from the diagram in figure 2.2
with the rules

1. multiply a momentum space bulk to boundary propagator for each line that ends at the boundary (with a dot).

2. multiply a momentum space bulk to bulk propagator for each line with both ends in the bulk (none present here).

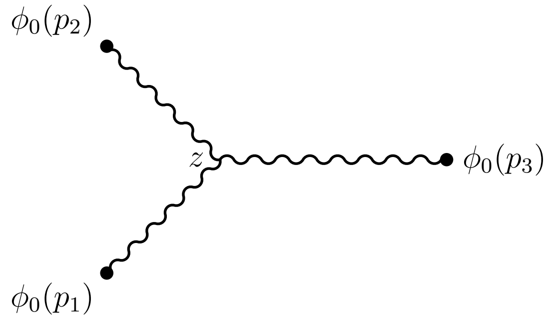

Figure 2.2: Witten diagram for the three point function in momentum space

3. add a factor $-b$ for each vertex.
4. add a factor $(2\pi)^4\delta^{(4)}(\sum_i p_i)$
5. integrate over vertex positions in the z-direction with $\int \mathrm{d}z\sqrt{g}$.

To evaluate this expression further we need to find the momentum space bulk to boundary propagators. It can of course be done by taking the long route and finding the expression in position space and Fourier transforming. However it is easier to transform the equation it satisfies and solve directly in momentum space. Since the bulk to bulk propagator satisfies almost the same equation we will also find its equation in momentum space with only a little more effort.

Note that equation (2.31) can be written as

$$[-z^2\partial_z\partial_z + 3z\partial_z + z^2\partial_\mu\partial^\mu + m^2]K(w,z) = 0. \tag{2.54}$$

Fourier transforming the left hand side gives us

$$\begin{aligned} LHS &= \int \mathrm{d}^4w e^{ip\cdot w}[-z^2\partial_z\partial_z + 3z\partial_z + z^2\partial_\mu\partial^\mu + m^2]K(w,z) \\ &= [-z^2\partial_z\partial_z + 3z\partial_z - z^2p^2 + m^2]K_p(z), \end{aligned} \tag{2.55}$$

while the right hand side is still 0 after a transform. The right hand side for equation (2.35) is however nonzero. by taking the transform we get

$$RHS = \int \mathrm{d}^4w e^{ip\cdot w} z^5\delta^{(4)}(w)\delta(z-z') = z^5\delta(z-z'). \tag{2.56}$$

giving us the two equations in momentum space as

$$\left[-\partial_z\partial_z + \frac{3}{z}\partial_z - p^2 + \frac{1}{z^2}m^2\right]K_p(z) = 0 \tag{2.57}$$

and

$$\left[-\partial_z\partial_z + \frac{3}{z}\partial_z - p^2 + \frac{1}{z^2}m^2\right]G_p(z,z') = z^3\delta(z-z'), \tag{2.58}$$

where $G_p(z, z')$ is the Fourier transform of $G(w, z, z')$. The boundary conditions can also be transformed, which gives

$$K_p(L_0) = 1 \tag{2.59}$$

and

$$G_p(L_0, z') = 0. \tag{2.60}$$

Chapter 3

The Model

The action we are going to work with here is defined to correlate to a three flavoured version of QCD. First we will present the action used in [3,4] and some results obtained from it. Then we will add a couple of terms and work out the consequences in an attempt to improve the results.

To build up this theory we must incorporate some relevant operators. we start with defining a left/right-handed vector containing the three lightest quark flavours: up, down and strange.

$$q_{L,R} = \begin{pmatrix} u \\ d \\ s \end{pmatrix}_{L,R} . \tag{3.1}$$

This together with the matrices T^a related to the Gell-Mann matrices λ^a through $T^a = \lambda^a/2$ allows us to express the relevant operators. These matrices naturally share the following two properties with the Gell-Mann matrices

$$\mathrm{Tr}(T^a T^b) = \frac{1}{2}\delta^{ab} \tag{3.2}$$

and

$$[T^a, T^b] = i f^{abc} T^c. \tag{3.3}$$

We are interested in the current operators $J^a_{L\mu} = \bar{q}_L \gamma_\mu T^a q_L$ and $J^a_{R\mu} = \bar{q}_R \gamma_\mu T^a q_R$. We also want the quark bilinear $\bar{q}_L q_R$. To make use of the correspondence we need the fields in AdS$_5$ space related to these operators. These are found to be the following [3,5,12,13]

$$J^a_{L\mu} \longleftrightarrow L^a_M(x^\mu, z), \tag{3.4}$$

$$J^a_{R\mu} \longleftrightarrow R^a_M(x^\mu, z) \tag{3.5}$$

and

$$\bar{q}_L q_R \longleftrightarrow \frac{2}{z} X(x^\mu, z). \tag{3.6}$$

With these fields we can write down the action that was used in [3,4]. They used the following action with a 5 dimensional $\mathrm{SU}(3)_L \otimes \mathrm{SU}(3)_R$ local flavour symmetry

$$\mathcal{S} = \int \mathrm{d}^5x\sqrt{g}\,\mathrm{Tr}\left[(\mathrm{D}_M X)^\dagger(\mathrm{D}^M X) + \frac{3}{L^2}X^\dagger X - \frac{1}{4g_5^2}(F^{(L)}_{MN}F^{MN}_{(L)} + F^{(R)}_{MN}F^{MN}_{(R)})\right] \quad (3.7)$$

where we have to keep in mind that the curvature radius, L, is 1 in [4].

To explain the action we start with the definitions for the field strengths

$$F^{(L)}_{MN} = \partial_M L_N - \partial_N L_M - i[L_M, L_N] \quad (3.8)$$

and similarly

$$F^{(R)}_{MN} = \partial_M R_N - \partial_N R_M - i[R_M, R_N]. \quad (3.9)$$

where L_M and R_M are related to the fields dual to the operators through

$$L_M = T^a L^a_M \quad (3.10)$$

and

$$R_M = T^a R^a_M. \quad (3.11)$$

It is also important for the future to state the relations between left/right-handed and the vector, V_M, and axial, A_M, fields. We will follow [3,4] and use

$$L_M = V_M + A_M \quad (3.12)$$

and

$$R_M = V_M - A_M. \quad (3.13)$$

They are not of great importance right now, but will come in handy when discussing the equations of motion. The change from left/right-handed to vector and axial makes it possible to separate the equations of motion in an axial part and a vector part. Something that is not possible for the left and right handed fields. These are also the combinations that will give the mass eigenstates.

To completely understand the action we must also have the expression for the covariant derivative D_M which is the source of the interactions between the scalar field X and the gauge fields L_M and R_M. It is given by

$$\mathrm{D}_M X = \partial_M X - iL_M X + iXR_M. \quad (3.14)$$

With this foundation laid down it is time to investigate the equations of motions.

3.1 The Vacuum Solution

The X field can be expanded as [4]

$$X(x^\mu, z) = e^{i\pi^a(x^\mu,z)T^a}X_0(z)e^{i\pi^a(x^\mu,z)T^a}, \quad (3.15)$$

where we have introduced the pion field $\pi = \pi^a T^a$. In a flavour symmetric world the $X_0(z)$ is a multiple of the unit matrix. Hence it would commute with the exponential functions and X could be written as

$$X(x^\mu, z) = e^{2i\pi^a(x^\mu,z)T^a} X_0(z). \tag{3.16}$$

This form has sometimes been used anyway [6,7], but we will follow [3,4] and not let X commute with the exponential. We will however keep isospin symmetry i.e. the up and down quark masses are interchangeable.

The vacuum expectation value is the solution to the EOMs with all fields but $X_0(z)$ set to zero. By setting all fields but $X_0(z)$ in the action, equation (3.7), we arrive at

$$\mathcal{S} = \int \mathrm{d}^5x\sqrt{g}\,\mathrm{Tr}\left[(\partial_z X_0)^\dagger(\partial^z X_0) + \frac{3}{L^2}X_0^\dagger X_0\right]. \tag{3.17}$$

This can then be divided into separate equations of motions for the for the different elements of $X_0(z)$ referred to as $X_{0ij}(z)$. The EOMs can be solved yielding [3,4]

$$2X_{0ij}(z) = v_{ij}(z) = \zeta M_{ij} z + \frac{1}{\zeta}\Sigma_{ij} z^3, \tag{3.18}$$

where we like [3,4] have introduced the rescaling parameter $\zeta = \sqrt{N_c}/2\pi$ as advocated by [8,12]. In the previous expression we also introduced

$$M = \begin{pmatrix} m_q & 0 & 0 \\ 0 & m_q & 0 \\ 0 & 0 & m_s \end{pmatrix} \tag{3.19}$$

and

$$\Sigma = \begin{pmatrix} \sigma_q & 0 & 0 \\ 0 & \sigma_q & 0 \\ 0 & 0 & \sigma_s \end{pmatrix}. \tag{3.20}$$

M is the quark mass matrix with the up and down quark mass m_q and the strange quark mass m_s. Σ is related to the quark condensate.

If we define

$$v_q(z) = \zeta m_q z + \frac{1}{\zeta}\sigma_q z^3 \tag{3.21}$$

and

$$v_s(z) = \zeta m_s z + \frac{1}{\zeta}\sigma_s z^3. \tag{3.22}$$

then the vacuum solution can be written as

$$2X_0(z) = v(z) = \begin{pmatrix} v_q & 0 & 0 \\ 0 & v_q & 0 \\ 0 & 0 & v_s \end{pmatrix}. \tag{3.23}$$

3.2 The Equations of Motion

With that done we can start looking at the EOMs for the A, V and π fields. However to have a chance of obtaining any equations we have to expand the exponential factors and then limit ourselves to second order in the fields, which will be sufficient to obtain the masses and wave functions to the order we work in.

We will also explicitly write the summation over the index in T^a. By doing so we find ourselves with expressions of the sort $\mathrm{Tr}([T^a, X_0][T^b, X_0])$ and $\mathrm{Tr}(\{T^a, X_0\}\{T^b, X_0\})$. Since these evaluate to 0 if $a \neq b$ we define

$$\frac{1}{2}M_V^{a\,2}\delta^{ab} = -\mathrm{Tr}([T^a, X_0][T^b, X_0]) \tag{3.24}$$

and

$$\frac{1}{2}M_A^{a\,2}\delta^{ab} = \mathrm{Tr}(\{T^a, X_0\}\{T^b, X_0\}). \tag{3.25}$$

Writing X_0 explicitly in v_q and v_s these evaluate to

$$M_V^{a\,2} = \begin{cases} 0 & a = 1,2,3 \\ \frac{1}{4}(v_s - v_q)^2 & a = 4,5,6,7 \\ 0 & a = 8 \end{cases} \tag{3.26}$$

and

$$M_A^{a\,2} = \begin{cases} v_q^2 & a = 1,2,3 \\ \frac{1}{4}(v_s + v_q)^2 & a = 4,5,6,7 \\ \frac{1}{3}(v_q + 2v_s)^2 & a = 8 \end{cases}. \tag{3.27}$$

To be able to calculate m_ϕ we must split the eighth component, I.e T^8, in two. For convenience we call them T^9 and T^{10} and define them, for later use, as

$$T^9 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{3.28}$$

and

$$T^{10} = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{3.29}$$

T^9 corresponds to the ϕ meson and T^{10} to the ω^0 meson. For these the associated ${M_{V,A}^a}^2$ are

$$M_V^{a\,2} = \begin{cases} 0 & a = 9 \\ 0 & a = 10 \end{cases} \tag{3.30}$$

and

$$M_A^{a\,2} = \begin{cases} v_s^2 & a = 9 \\ v_q^2 & a = 10 \end{cases}. \tag{3.31}$$

With these definitions we find that the resulting action is [3,4]

$$\begin{aligned}\mathcal{S} = \int \mathrm{d}^5x \sum_a \bigg(&- \frac{L}{4g_5^2 z}(\partial_M V_N^a - \partial_N V_M^a)^2 + \frac{M_V^{a\,2} L^3}{2z^3} V_M^{a\,2} \\ &- \frac{L}{4g_5^2 z}(\partial_M A_N^a - \partial_N A_M^a)^2 + \frac{M_V^{a\,2} L^3}{2z^3}(\partial_M \pi^a - A_M^a)^2 \bigg),\end{aligned} \tag{3.32}$$

where the square of a field means

$$V_M^{a\,2} = \eta^{MM'} V_M^a V_{M'}^a = \frac{L^2}{z^2} g^{MM'} V_M^a V_{M'}^a = \frac{L^2}{z^2} V_M^a V^{Ma}. \tag{3.33}$$

To simplify even further we will for also define

$$\alpha^a(z) = \frac{g_5^2 M_V^{a\,2} L^2}{z^2} \tag{3.34}$$

and

$$\beta^a(z) = \frac{g_5^2 M_A^{a\,2} L^2}{z^2}. \tag{3.35}$$

3.3 The Vector Sector

With these recent definitions we can write the part of the action that is quadratic in the vector field as

$$\mathcal{S}_V = \int \mathrm{d}^5x \frac{L}{2g_5^2} \sum_a \bigg(- \frac{1}{2z}(\partial_M V_N^a - \partial_N V_M^a)^2 + \frac{\alpha^a(z)}{z} V_M^{a\,2} \bigg). \tag{3.36}$$

From which we can find the EOMs for the vector field [3,4]

$$\eta^{ML} \partial_M \bigg(\frac{1}{z}(\partial_L V_N^a - \partial_N V_L^a) \bigg) + \frac{\alpha^a(z)}{z} V_N^a = 0. \tag{3.37}$$

The first four components of the vector field, $V_\mu(z, x^\nu)$, can be separated into a transversal part, $V_{\mu\perp}(z, x^\nu)$, and a longitudinal part , $V_{\mu\|}(z, x^\nu)$, by

$$V_\mu(z, x^\nu) = V_{\mu\perp}(z, x^\nu) + V_{\mu\|}(z, x^\nu). \tag{3.38}$$

Where we note that the transversal part satisfies $\partial^\mu V_{\mu\perp}(z, x^\nu) = 0$. We also transform the equation in the first four components by the Fourier transform defined as $\hat{f}(z, k^\nu) = \int \mathrm{d}^4 x e^{i\eta_{\nu\mu} k^\nu x^\mu} f(z, x^\mu)$ on a general function $f(z, x^\mu)$. With this done we can find the EOMs for the Fourier transformed transverse part of V_μ^a as [3,4]

$$\partial_z \bigg(\frac{1}{z} \partial_z + \frac{k^2 - \alpha^a(z)}{z} \bigg) \hat{V}_{\mu\perp}^a(z, k^\nu) = 0. \tag{3.39}$$

The field $\hat{V}^a_{\mu\perp}(z,k^\nu)$ can be written as the product of its boundary value at the UV boundary $\hat{V}^{0a}_{\mu\perp}(k^\nu)$ and a bulk to boundary propagator, $\mathcal{V}^a(z,k^2)$, i.e.

$$\hat{V}^a_{\mu\perp}(z,k^\nu) = \hat{V}^{0a}_{\mu\perp}(k^\nu)\mathcal{V}^a(z,k^2), \tag{3.40}$$

where the boundary value acts as the Fourier transform of the source of the vector current operator. The bulk to boundary propagator is defined with the boundary value $\mathcal{V}^a(L_0,k^2) = 1$ at the UV boundary. Since the boundary value is independent of z it is clear that the bulk to boundary propagator obeys the same EOM as the field $\hat{V}^a_{\mu\perp}(z,k^\nu)$, i.e.

$$\partial_z\left(\frac{1}{z}\partial_z + \frac{k^2-\alpha^a(z)}{z}\right)\mathcal{V}^a(z,k^2) = 0. \tag{3.41}$$

To find a unique solution to this equation we do however need another boundary condition. The choice will that the derivative of the function vanishes at the IR boundary, $\partial_z\mathcal{V}^a(L_1,k^2) = 0$, in accordance to [3,4]. In the case where $a = 1,2,3,8,9,10$ we have $\alpha^a(z) = 0$ and an analytical solution can be found in terms of Bessel functions [3]

$$\mathcal{V}^a(z,k^2) = \frac{z}{L_0}\frac{J_1(kz)Y_0(kL_1) - Y_1(kz)J_0(kL_1)}{J_1(kL_0)Y_0(kL_1) - Y_1(kL_0)J_0(kL_1)}. \tag{3.42}$$

However for $a = 4,5,6,7$ the function $\alpha^a(z)$ is generally an even polynomial of degree 4 and no analytical solution exists. We have to rely on numerical solutions.

3.4 The Axial Sector

For the axial sector we have the action

$$\mathcal{S}_A = \int \mathrm{d}^5x\frac{L}{2g_5^2}\sum_a\left(-\frac{1}{2z}(\partial_M A^a_N - \partial_N A^a_M)^2 + \frac{\beta^a(z)}{z}(\partial_M\pi^a - A^a_M)^2\right), \tag{3.43}$$

which results in the following EOMs

$$\eta^{ML}\partial_M\left(\frac{1}{z}(\partial_L A^a_N - \partial_N A^a_L)\right) + \frac{\beta^a(z)}{z}(\partial_N\pi^a - A^a_N) = 0. \tag{3.44}$$

The axial part is then similarly to the vector part decomposed in a transverse and longitudinal part and Fourier transformed. This gives for the transverse part an equation of motion analogous to the transverse part of the vector sector, but with $\beta^a(z)$ exchanged for $\alpha^a(z)$, i.e.

$$\partial_z\left(\frac{1}{z}\partial_z + \frac{k^2-\beta^a(z)}{z}\right)\hat{A}^a_{\mu\perp}(z,k^\nu) = 0. \tag{3.45}$$

However in the axial sector we are mainly interested in the longitudinal part. Defining $A^a_{\mu\|}(z,x^\nu) = \partial_\mu\phi^a(z,x^\nu)$ gives us the EOMs for the Fourier transforms of $\phi^a(z,x^\nu)$ and $\pi^a(z,x^\nu)$ [3,4]

$$\partial_z\left(\frac{1}{z}\partial_z\hat{\phi}^a(z,k^2)\right) - \frac{\beta^a(z)}{z}(\hat{\phi}^a(z,k^2) - \hat{\pi}^a(z,k^2)) = 0 \tag{3.46}$$

and

$$k^2 \partial_z \hat{\phi}^a(z, k^2) - \beta^a(z) \partial_z \hat{\pi}^a(z, k^2) = 0. \tag{3.47}$$

The boundary conditions for these equations are $\hat{\phi}^a(L_0, k^2) = 0$ and $\hat{\pi}^a(L_0, k^2) = -1$ at the UV boundary and $\partial_z \hat{\phi}^a(L_1, k^2) = 0$ and $\partial_z \hat{\pi}^a(L_1, k^2) = 0$ at the IR boundary. These two equations can also be combined into one equation with the definition $y^a(k^2, z) = \frac{1}{z} \partial_z \hat{\phi}^a(L_0, k^2)$ resulting in

$$\partial_z \left(\frac{z}{\beta^a(z)} \partial_z y^a(z, k^2) \right) + z \left(\frac{k^2}{\beta^a(z)} - 1 \right) y^a(z, k^2) = 0. \tag{3.48}$$

To find the boundary conditions we use the ones we have for equations (3.46) and (3.47). using $\partial_z \hat{\phi}^a(L_1, k^2) = 0$ gives us directly that $y^a(L_1, k^2) = 0$. To find another boundary condition we insert $\hat{\phi}^a(L_0, k^2) = 0$ and $\hat{\pi}^a(L_0, k^2) = -1$ into equation (3.46) which gives us $\partial_z y^a(L_0, k^2) = \beta^a(L_0)/L_0$. Note also that from equation (3.47) we can find that $y^a(z, k^2) = \frac{\beta^a(z)}{k^2 z} \hat{\pi}^a(z, k^2)$.

3.5 Normalizable Solutions

The normalizable modes that solve the EOMs correspond to hadrons [3,4,9]. The modes must vanish at the UV boundary to keep the action finite. At the IR boundary we keep the Neumann boundary condition which guarantees that the boundary terms vanish. These modes can for example be found through the following steps

1. Define a new boundary condition by setting derivative at the UV boundary to a chosen constant.

2. Solve the EOMs with the two UV boundary conditions as a function of z and k^2.

3. Find the values of k^2 for which the boundary condition at the IR boundary holds, these are generally an infinite number of discrete values.

4. Normalize the solution for these k^2 and thus remove the arbitrariness introduced in step 1.

What is described here is essentially the *shooting method*. It was implemented in Mathematica for the numerical calculations performed for this thesis.

The values $k^2 = m_n^{a\,2}$, where m_n^a is the mass of the relevant hadron. Higher n correspond to radial excitations, but in the hard wall model these scale as $m_n^{a\,2} \sim n^2$ which is not consistent with the measured scaling behaviour $m_n^{a\,2} \sim n$. Physical values for the ground states can be obtained though.

3.5.1 The Vector Sector

The normalizable modes of the transverse part of the vector sector corresponds to vector mesons. Lets define the normalizable modes as $\psi_n^a(z)$. The first three are shown in figure 3.1. They obey the orthogonality and normalization condition [3,4]

$$\int_{L_0}^{L_1} \frac{\mathrm{d}z}{z} \psi_n^a(z) \psi_m^a(z) = \delta_{mn} \tag{3.49}$$

and the boundary conditions $\psi_n^a(L_0) = 0$ and $\partial_z \psi_n^a(L_1) = 0$ We also want and identification of the nonet, $a = 1, 2, ..., 10$ of vector mesons. The first three, $a = 1, 2, 3$ corresponds to ρ mesons, (ρ^+, ρ^-, ρ^0), the next four, $a = 4, 5, 6, 7$, correspond to K^* mesons, $(K^{*+}, K^{*-}, K^{*0}, \bar{K}^{*0})$ and $a = 8$ corresponds to a combination of the ω^0 meson and the ϕ meson or, if divided, $a = 9$ corresponds to the ϕ meson and $a = 10$ to the ω^0 meson.

Already here we can see a problem with the masses for ρ, ϕ and ω^0. Since $M_V^a = 0$ for $a = 1, 2, 3, 9, 10$ the equations become identical for these cases and thus the masses for ρ, ϕ and ω^0 will also be identical.

One can also find that the bulk to boundary propagator can be written as a sum over the normalizable modes [3,4]

$$\mathcal{V}^a(z, k^2) = \sum_n \frac{-g_5 F_n^a \psi_n^a(z)}{k^2 - m_n^{a\,2}}, \tag{3.50}$$

where the factor F_n^a is defined as

$$F_n^a = \frac{\partial_z \psi_n^a(L_0)}{g_5 L_0}, \tag{3.51}$$

which can be identified as the decay constant of the corresponding meson.

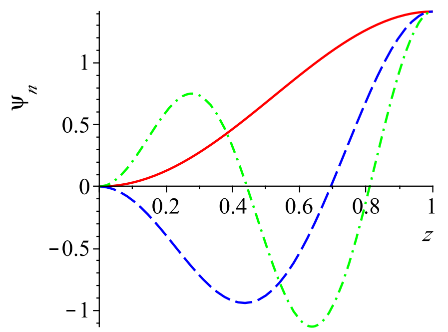

Figure 3.1: first three normalizable modes of ψ. ψ_1 (red curve), ψ_2 (dashed blue curve) and ψ_3 (dash-dot green curve). The z-axis are in units of L_1 Taken from [4].

3.5.2 The Axial Sector

In the axial sector we study the longitudinal part. The transverse part can be dealt with in the same way as in the vector case. Here the normalizable modes correspond to pseudoscalar mesons. The first three of our octet, $a = 1, 2, 3$ are the pions, (π^+, π^-, π^0), and the following four are kaons, (K^+, K^-, K^0, $\bar{K}^0$).

One can go about this defining the normalizable modes for equations (3.46) and (3.47) as $\hat{\phi}^a_n(z)$ and $\hat{\pi}^a_n(z)$ or one for equation (3.48) as $y^a_n(z) = \frac{1}{z}\partial_z\hat{\phi}^a_n(z)$. The first two modes for $\hat{\phi}^a_n(z)$ and $\hat{\pi}^a_n(z)$ are shown in figure 3.2. The boundary conditions here are $\hat{\phi}^a_n(L_0) = 0$ and $\hat{\pi}^a_n(L_0) = 0$ at the UV boundary and $\partial_z\hat{\phi}^a_n(L_1) = 0$ and $\partial_z\hat{\pi}^a_n(L_1) = 0$ at the IR boundary. The equivalent boundary conditions for $y^a_n(z)$ are $y^a_n(L_1) = 0$ and $\partial_z y^a_n(L_0) = 0$.

Since the normalization and orthogonality relation here is given by [3,4]

$$\int_{L_0}^{L_1} \mathrm{d}z \frac{z}{\beta^a(z)} y^a_n(z) y^a_m(z) = \frac{\delta_{mn}}{m^{a\,2}_n}, \tag{3.52}$$

one can find a solution for y^a_n with an extra arbitrary boundary condition and then normalize the solution with the previous relation. The solution can be used to set normalized boundary conditions for the $\hat{\phi}^a_n(z)$ and $\hat{\pi}^a_n(z)$ instead of arbitrary ones to find solutions which do not require normalizations.

However if we only are interested in the masses we can use equations (3.46) and (3.47) directly since the masses are independent of the normalization.

Similarity to the vector sector the general solutions can be expressed as sums over the normalizable modes. For $y^a(z, k^2)$ we have [3,4]

$$y^a(z, k^2) = \sum_n \frac{m^{a\,2}_n y^a_n(L_0) y^a_n(z)}{k^2 - m^{a\,2}_n} \tag{3.53}$$

and for $\hat{\phi}^a(z, k^2)$ and $\hat{\pi}^a(z, k^2)$ we get

$$\hat{\phi}^a(z, k^2) = \sum_n \frac{-g_5 m^{a\,2}_n f^a_n \hat{\phi}^a_n(z)}{k^2 - m^{a\,2}_n} \tag{3.54}$$

and

$$\hat{\pi}^a(z, k^2) = \sum_n \frac{-g_5 m^{a\,2}_n f^a_n \hat{\pi}^a_n(z)}{k^2 - m^{a\,2}_n}. \tag{3.55}$$

Where we have used

$$f^a_n = -\frac{\partial_z\hat{\phi}^a_n(L_0)}{g_5 L_0}, \tag{3.56}$$

which too can be identified as the decay constant of the corresponding meson.

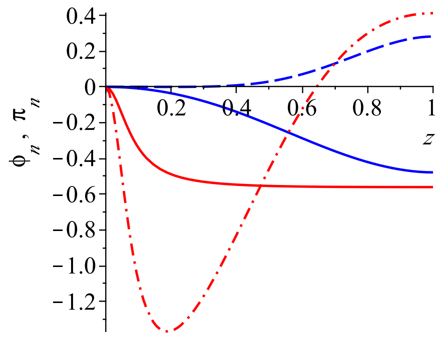

Figure 3.2: first two normalizable modes of ϕ_n^a and π_n^a for $a = 1, 2, 3$. ϕ_1^a (upper solid curve, in blue), ϕ_2^a (dashed blue curve), π_1^a (lower solid curve, in red) and π_2^a (dash-dot red curve). The z-axis are in units of L_1 and ϕ_n^a and π_n^a are in units of L_1^{-1}. Taken from [4]

Parameter	Value
L_1	$(322.47\,\mathrm{MeV})^{-1}$
m_q	$8.291\,\mathrm{MeV}$
m_s	$188.48\,\mathrm{MeV}$
σ_q	$(213.66\,\mathrm{MeV})^3$
σ_s	$(213.66\,\mathrm{MeV})^3$

Table 3.1: Parameter values used in model A1 in [3]

3.6 Previous Results

With the parameter values in table 3.1 Sven Möller [3] obtained the results in table 3.2 in his thesis.

We are however also concerned with the ϕ meson mass. If it is calculated in the same model as table 3.2 we find, as mentioned before, that the masses for the ρ, ϕ and ω^0 mesons are identical. We can also see in the table that $m_\rho \neq m_{K^*}$. Experimentally though one finds the approximate relation

$$m_\phi - m_{K^*} \simeq m_{K^*} - m_\rho. \tag{3.57}$$

Which does not hold under these circumstances.

In an attempt to obtain better agreement with these experimental results, while still conserving the promising results from others we shall add a couple of terms to the action.

observable	sector	a	n	Model[MeV]	Measured[MeV]
m_π	pseudoscalar	1,2,3	1	(fit)	139.57
f_π	pseudoscalar	1,2,3	1	(fit)	92.4 ± 0.35
m_K	pseudoscalar	4,5,6,7	1	(fit)	495.7
f_K	pseudoscalar	4,5,6,7	1	103.8	113 ± 1.4
$m_{K_0^*}$	scalar	4,5,6,7	1	791.0	672
$f_{K_0^*}$	scalar	4,5,6,7	1	27.6	
m_ρ	vector	1,2,3	1	(fit)	775.49 ± 0.34
$\sqrt{f_\rho}$	vector	1,2,3	1	329.3	345 ± 8[4]
m_{K^*}	vector	4,5,6,7	1	791.0	893.8
$\sqrt{f_{K^*}}$	vector	4,5,6,7	1	329.7	
m_{a1}	pseudovector	1,2,3	1	1366.2	1230 ± 40
$\sqrt{f_{a1}}$	pseudovector	1,2,3	1	488.8	433 ± 13[4]
m_{a1}	pseudovector	4,5,6,7	1	1458.1	1272 ± 7
$\sqrt{f_{a1}}$	pseudovector	4,5,6,7	1	511.1	

Table 3.2: Results in model A1 in [3]. The measured values are also taken from [3], but Originally they come from [11] and [4](where stated). We have also reproduced these results for the Vector/axial vector and pseudoscalar sections, including the wave functions.

3.7 Additional Terms

The addition we do to the previous action is

$$\mathcal{S}_{add} = \int \mathrm{d}^5x\sqrt{g}\,\mathrm{Tr}\Big[-\frac{d_1}{4g_5^2}XX^\dagger(F^{(L)}_{MN}F^{MN}_{(L)} + F^{(R)}_{MN}F^{MN}_{(R)}) - \frac{d_2}{2g_5^2}XF^{(R)}_{MN}X^\dagger F^{MN}_{(L)}\Big], \quad (3.58)$$

keeping the $\mathrm{SU}(3)_L \otimes \mathrm{SU}(3)_R$ symmetry. Here we have introduced the two new free parameters d_1 and d_2. These shall be used to make a better fit to m_ϕ. We can also immediately conclude that neither of these terms has any effect on the vacuum solution.

When evaluating the terms added to the action to second order in fields we will come across the traces $\mathrm{Tr}(X_0X_0T^aT^b)$ and $\mathrm{Tr}(X_0T^aX_0T^b)$. These both evaluate to 0 when $a \neq b$ and we can define

$$\frac{1}{2}\gamma_1^a(z)\delta^{ab} = \mathrm{Tr}(X_0X_0T^aT^b) \quad (3.59)$$

and

$$\frac{1}{2}\gamma_2^a(z)\delta^{ab} = \mathrm{Tr}(X_0T^aX_0T^b). \quad (3.60)$$

Writing X_0 explicitly in v_q and v_s these evaluate to

$$\gamma_1^a(z) = \begin{cases} \frac{1}{4}v_q^2 & a = 1,2,3 \\ \frac{1}{8}(v_q^2 + v_s^2) & a = 4,5,6,7 \\ \frac{1}{4}v_s^2 & a = 9 \\ \frac{1}{4}v_q^2 & a = 10 \end{cases} \quad (3.61)$$

and

$$\gamma_2^a(z) = \begin{cases} \frac{1}{4}v_q^2 & a = 1,2,3 \\ \frac{1}{4}v_q v_s & a = 4,5,6,7 \\ \frac{1}{4}v_s^2 & a = 9 \\ \frac{1}{4}v_q^2 & a = 10 \end{cases}. \tag{3.62}$$

With these definitions we find the addtional terms to second order as

$$\begin{aligned} \mathcal{S}_{add} = \int \frac{\mathrm{d}^5 x}{4g_5^2} L \sum_a \Bigg(& \frac{-d_1\gamma_1^a(z) - d_2\gamma_2^a(z)}{z}(\partial_M V_N^a - \partial_N V_M^a)^2 \\ & + \frac{-d_1\gamma_1^a(z) + d_2\gamma_2^a(z)}{z}(\partial_M A_N^a - \partial_N A_M^a)^2 \Bigg). \end{aligned} \tag{3.63}$$

A derivation of this can be found in appendix A.

We now define

$$\gamma_V^a(z) = 1 + d_1\gamma_1^a(z) + d_2\gamma_2^a(z) \tag{3.64}$$

and

$$\gamma_A^a(z) = 1 + d_1\gamma_1^a(z) - d_2\gamma_2^a(z). \tag{3.65}$$

To be able to write the full action up to second order in fields as

$$\begin{aligned} \mathcal{S} &= \int \mathrm{d}^5 x \sum_a \Bigg(- \frac{L\gamma_V^a(z)}{4g_5^2 z}(\partial_M V_N^a - \partial_N V_M^a)^2 + \frac{M_V^{a\,2} L^3}{2z^3} V_M^{a\,2} \\ &\qquad - \frac{L\gamma_A^a(z)}{4g_5^2 z}(\partial_M A_N^a - \partial_N A_M^a)^2 + \frac{M_A^{a\,2} L^3}{2z^3}(\partial_M \pi^a - A_M^a)^2 \Bigg) \\ &= \int \mathrm{d}^5 x \sum_a \Bigg(- \frac{L\gamma_V^a(z)}{4g_5^2 z}(\partial_M V_N^a - \partial_N V_M^a)^2 + \frac{L\alpha^a(z)}{2g_5^2 z} V_M^{a\,2} \\ &\qquad - \frac{L\gamma_A^a(z)}{4g_5^2 z}(\partial_M A_N^a - \partial_N A_M^a)^2 + \frac{L\beta^a(z)}{2g_5^2 z}(\partial_M \pi^a - A_M^a)^2 \Bigg). \end{aligned} \tag{3.66}$$

From which we can find new slightly different EOMs. Since the boundary terms still vanish with the same boundary conditions they remain unchanged.

3.7.1 New Vector Sector

In the vector sector we find the EOMs analogous to equation (3.37) to be

$$\eta^{ML}\partial_M\left(\frac{\gamma_V^a(z)}{z}(\partial_L V_N^a - \partial_N V_L^a)\right) + \frac{\alpha^a(z)}{z}V_N^a = 0. \tag{3.67}$$

From which it follows that the EOMs corresponding to equation (3.39) is

$$\partial_z\left(\frac{\gamma_V^a(z)}{z}\partial_z \hat{V}_{\mu\perp}^a(z,k^\nu)\right) + \left(\frac{\gamma_V^a(z)k^2 - \alpha^a(z)}{z}\right)\hat{V}_{\mu\perp}^a(z,k^\nu) = 0. \tag{3.68}$$

The derivation can be found in appendix B

3.7.2 New Axial Sector

Similarly to the new EOMs for the vector sector we find the EOMs analogous to (3.44) to be

$$\eta^{ML}\partial_M\left(\frac{\gamma_A^a(z)}{z}(\partial_L A_N^a - \partial_N A_L^a)\right) + \frac{\beta^a(z)}{z}(\partial_N \pi^a - A_N^a) = 0. \tag{3.69}$$

Not surprisingly we find the EOMs corresponding to equation (3.45) to be

$$\partial_z\left(\frac{\gamma_A^a(z)}{z}\partial_z \hat{A}_{\mu\perp}^a(z,k^\nu)\right) + \left(\frac{\gamma_A^a(z)k^2 - \beta^a(z)}{z}\right)\hat{A}_{\mu\perp}^a(z,k^\nu) = 0. \tag{3.70}$$

As for the transversal the new EOMs in place of (3.46) and (3.47) is

$$\partial_z\left(\frac{\gamma_A^a(z)}{z}\partial_z\hat{\phi}^a(z,k^2)\right) - \frac{\beta^a(z)}{z}(\hat{\phi}^a(z,k^2) - \hat{\pi}^a(z,k^2)) = 0 \tag{3.71}$$

and

$$\gamma_A^a(z)k^2\partial_z\hat{\phi}^a(z,k^2) - \beta^a(z)\partial_z\hat{\pi}^a(z,k^2) = 0. \tag{3.72}$$

If we define $y^a(z,k^2) = \frac{\gamma_A^a(z)}{z}\partial_z\hat{\phi}^a(z,k^2)$ we can find the EOMs corresponding to (3.48) as

$$\partial_z\left(\frac{z}{\beta^a(z)}\partial_z(y^a(z,k^2))\right) + z\left(\frac{k^2}{\beta^a(z)} - \frac{1}{\gamma_A^a(z)}\right)y^a(z,k^2) = 0. \tag{3.73}$$

The derivation of these EOMs can be found in appendix B.

Chapter 4

Results

4.1 Refitting With d_1 and d_2

Keeping the parameters as in table 3.1 and fitting the calculated masses of the ϕ and K^* mesons to the measured masses by varying d_1 and d_2 gave us the results in table 4.1. Note that increasing m_ϕ also increases m_ρ.

observable	sector	a	n	Model[MeV]	Measured[MeV]
m_π	pseudoscalar	1,2,3	1	135.2	139.57
f_π	pseudoscalar	1,2,3	1	95.35	92.4 ± 0.35
m_K	pseudoscalar	4,5,6,7	1	477.0	495.7
f_K	pseudoscalar	4,5,6,7	1	106.7	113 ± 1.4
m_ρ	vector	1,2,3	1	861.8	775.49 ± 0.34
m_{K^*}	vector	4,5,6,7	1	896.6	893.8
m_ϕ	vector	9	1	906.2	1019.455

Table 4.1: Results when fitting the calculated masses of the ϕ and K^* mesons to the measured masses by varying d_1 and d_2. The values obtained for the parameters were $d_1 = 3.817$ and $d_2 = -6.494$.

4.2 Refitting With All Parameters

In an attempt to obtain better results than the ones in the previous section we did a fit by varying all the free parameters to minimize the square of the relative errors in all the computed observables. The parameters obtained can be found in table 4.2 and the results in table 4.3.

Parameter	Value
L_1	$3.0978 \times 10^{-3}\,\mathrm{MeV}^{-1}$
m_q	$9.0395\,\mathrm{MeV}$
m_s	$218.08\,\mathrm{MeV}$
σ_q	$(211.05\,\mathrm{MeV})^3$
σ_s	$(211.05\,\mathrm{MeV})^3$
d_1	7.01974
d_2	-9.7301

Table 4.2: Parameter values when fitting with all free parameters to obtain a the least square in the relative errors of the calculated observables.

observable	sector	a	n	Model[MeV]	Measured[MeV]
m_π	pseudoscalar	1,2,3	1	139.0	139.57
f_π	pseudoscalar	1,2,3	1	95.21	92.4 ± 0.35
m_K	pseudoscalar	4,5,6,7	1	504.2	495.7
f_K	pseudoscalar	4,5,6,7	1	108.5	113 ± 1.4
m_ρ	vector	1,2,3	1	856.3	775.49 ± 0.34
m_{K^*}	vector	4,5,6,7	1	895.4	893.8
m_ϕ	vector	9	1	906.8	1019.455

Table 4.3: Results when fitting with all free parameters to obtain a the least square in the relative errors of the calculated observables.

Chapter 5

Conclusions

We have shown how the calculations of n point functions can be performed with the AdS/CFT correspondence in a scalar field theory. Both by explicitly taking functional derivatives and by relating the n point functions to Witten diagrams.

Additionally we tried to improve an existing AdS/QCD model to produce better results for the mass of the ϕ meson. This was done by introducing two new free parameters through adding two terms to the action. Although we saw some improvement in the mass of the ϕ and K^* mesons it was at the expense of the mass of the ρ meson. We also encountered some numerical difficulties which only allowed us to obtain masses for ϕ in a region well below its experimentally measured mass.

In the first fit we expected to get good values for the ϕ and K^* meson masses. Since it is in general possible to fit two quantities with the use of two free parameters. The reason why we did not get the correct ϕ mass stems from the fact that for certain values of d_1 and d_2 the functions $\gamma^a_{V,A}(z)$ changes sign in the z range we are working with. This is a problem because a sign change in $\gamma^a_{V,A}(z)$ leads to singularities in the solutions to the EOMs and we found no way to handle this numerically.

The values we found and presented are such that the sign change in $\gamma^a_{V,A}(z)$ falls just outside the z range. Other values for the parameters would either produce a lower mass for the ϕ meson or move the sign change of $\gamma^a_{V,A}(z)$ inside the z range. However they could be varied quite drastically without affecting the masses much and we might have been to narrow in our search.

In the second fit we tried refit our calculated quantities by varying all free parameters. That the first fit did not work as we hoped does not imply that an overall fit is unable produce better values for the observables. The behaviour of $\gamma^a_{V,A}(z)$ changes with m_s, m_q and $\sigma_{m,s}$ and we also change the range in which a sign change is not allowed.

Our attempt at an overall fit did however reproduce quite similar values for the original parameters and the masses. Although the original parameters are slightly different we can see by comparing to the first fit how little the masses changes with d_1 and d_2. We have not investigated this thoroughly enough to rule out the possibility that a better fit can be produced with the parameters in this model. However our results indicate that the additional terms in the action are not sufficient to produce the ϕ mass while still maintaining good values for the other masses.

Since the original plan of the thesis was different much work has been done that is not presented in the report. For example the form factor $f_+(q)$ for K_{l3} [4] was rederived, but no further numerics were done with it. It is also the reason why the investigation with the masses is not more extensive.

Appendix A

The New Terms to Second Order

In this appendix it is implied that "=" means equal up to second order in fields.

The purpose of the appendix is to explicitly write the steps that show

$$\int \mathrm{d}^5x\sqrt{g}\left[\frac{(-d_1)}{4g_5^2}\,\mathrm{Tr}(XX^\dagger(F^{(L)}_{MN}F^{MN}_{(L)} + F^{(R)}_{MN}F^{MN}_{(R)})) + \frac{(-d_2)}{2g_5^2}\,\mathrm{Tr}(XF^{(R)}_{MN}X^\dagger F^{MN}_{(L)})\right]$$

$$= \int \frac{\mathrm{d}^5x}{4g_5^2}\frac{z}{L}\sum_a[(-d_1\gamma_1^a - d_2\gamma_2^a)(\partial_M V_N^a - \partial_N V_M^a)^2 + (-d_1\gamma_1^a + d_2\gamma_2^a)(\partial_M A_N^a - \partial_N A_M^a)^2] \tag{A.1}$$

This is done independently for each term. What is left for the reader is to combine the results of the two sections below.

A.1 The first term

Rewriting the first term up to second order in fields goes as follows

$$
\begin{aligned}
&\int \mathrm{d}^5x \frac{(-d_1)}{4g_5^2}\sqrt{g}\,\mathrm{Tr}(XX^\dagger(F^{(L)}_{MN}F^{MN}_{(L)} + F^{(R)}_{MN}F^{MN}_{(R)}))\\
&= \int \mathrm{d}^5x \frac{(-d_1)}{4g_5^2}\sqrt{g}\sum_a\sum_b \mathrm{Tr}(X_0X_0T^aT^b)\\
&\quad\times[(\partial_M(V^a_N + A^a_N) - \partial_N(V^a_M + A^a_M))(\partial^M(V^{Nb} + A^{Nb}) - \partial^N(V^{Mb} + A^{Mb}))\\
&\quad+(\partial_M(V^a_N - A^a_N) - \partial_N(V^a_M - A^a_M))(\partial^M(V^{Nb} - A^{Nb}) - \partial^N(V^{Mb} - A^{Mb}))]\\
&= \int \mathrm{d}^5x \frac{(-d_1)}{4g_5^2}\sqrt{g}\sum_a\sum_b \delta^{ab}\frac{\gamma_1^a}{2}\\
&\quad\times[(\partial_M(V^a_N + A^a_N) - \partial_N(V^a_M + A^a_M))(\partial^M(V^{Nb} + A^{Nb}) - \partial^N(V^{Mb} + A^{Mb}))\\
&\quad+(\partial_M(V^a_N - A^a_N) - \partial_N(V^a_M - A^a_M))(\partial^M(V^{Nb} - A^{Nb}) - \partial^N(V^{Mb} - A^{Mb}))]\\
&= \int \mathrm{d}^5x \frac{(-d_1)}{4g_5^2}\sqrt{g}\sum_a \frac{\gamma_1^a}{2}\\
&\quad\times[2\partial_M(V^a_N + A^a_N)\partial^M(V^{Na} + A^{Na}) - 2\partial_N(V^a_M + A^a_M)\partial^M(V^{Na} + A^{Na})\\
&\quad+2\partial_M(V^a_N - A^a_N)\partial^M(V^{Na} - A^{Na}) - 2\partial_N(V^a_M - A^a_M)\partial^M(V^{Na} - A^{Na})]\\
&= \int \mathrm{d}^5x \frac{(-d_1)}{4g_5^2}\sqrt{g}\sum_a \frac{\gamma_1^a}{2}\\
&\quad\times[4(\partial_M V^a_N\partial^M V^{Na} - \partial_N V^a_M\partial^M V^{Na}) + 4(\partial_M A^a_N\partial^M A^{Na} - \partial_N A^a_M\partial^M A^{Na})]\\
&= \int \mathrm{d}^5x \frac{(-d_1)}{4g_5^2}\sqrt{g}\sum_a \frac{\gamma_1^a}{2}\\
&\quad\times[2(\partial_M V^a_N\partial^M V^{Na} - \partial_N V^a_M\partial^M V^{Na}) + 2(\partial_N V^a_M\partial^N V^{Ma} - \partial_M V^a_N\partial^N V^{Ma})\\
&\quad+2(\partial_M A^a_N\partial^M A^{Na} - \partial_N A^a_M\partial^M A^{Na}) + 2(\partial_N A^a_M\partial^N A^{Ma} - \partial_M A^a_N\partial^N A^{Ma})]\\
&= \int \mathrm{d}^5x \frac{(-d_1)}{4g_5^2}\sqrt{g}\sum_a \frac{\gamma_1^a}{2}\\
&\quad\times[2(\partial_M V^a_N - \partial_N V^a_M)(\partial^M V^{Na} - \partial^N V^{Ma}) + 2(\partial_M A^a_N - \partial_N A^a_M)(\partial^M A^{Na} - \partial^N A^{Ma})]\\
&= \int \mathrm{d}^5x \frac{(-d_1)}{4g_5^2}\frac{L^5}{z^5}\sum_a \gamma_1^a\\
&\quad\times\left[\frac{z^4}{L^4}(\partial_M V^a_N - \partial_N V^a_M)^2 + \frac{z^4}{L^4}(\partial_M A^a_N - \partial_N A^a_M)^2\right]\\
&= \int \mathrm{d}^5x \frac{(-d_1)}{4g_5^2}\frac{L}{z}\sum_a \gamma_1^a[(\partial_M V^a_N - \partial_N V^a_M)^2 + (\partial_M A^a_N - \partial_N A^a_M)^2]
\end{aligned}
\tag{A.2}
$$

A.2 The Second term

Rewriting the second term term up to second order in fields goes as follows

$$
\begin{aligned}
&\int \mathrm{d}^5x \frac{(-d_2)}{2g_5^2}\sqrt{g}\,\mathrm{Tr}(XF^{(R)}_{MN}X^\dagger F^{MN}_{(L)}) = \int \mathrm{d}^5x \frac{(-d_2)}{2g_5^2}\sqrt{g}\sum_a\sum_b \mathrm{Tr}(X_0T^aX_0T^b) \\
&\times(\partial_M(V_N^a - A_N^a) - \partial_N(V_M^a - A_M^a))(\partial^M(V^{Nb}+A^{Nb}) - \partial^N(V^{Mb}+A^{Mb})) \\
=&\int \mathrm{d}^5x \frac{(-d_2)}{2g_5^2}\sqrt{g}\sum_a\sum_b \frac{\gamma_2^a}{2}\delta^{ab} \\
&\times(\partial_M(V_N^a - A_N^a) - \partial_N(V_M^a - A_M^a))(\partial^M(V^{Nb}+A^{Nb}) - \partial^N(V^{Mb}+A^{Mb})) = \\
&\int \mathrm{d}^5x \frac{(-d_2)}{2g_5^2}\sqrt{g}\sum_a \frac{\gamma_2^a}{2} \\
&\times[\underbrace{\partial_M V_N^a\partial^M V^{Na}}_{1} + \underbrace{\partial_M V_N^a\partial^M A^{Na}}_{3} - \underbrace{\partial_M V_N^a\partial^N V^{Ma}}_{1} - \underbrace{\partial_M V_N^a\partial^N A^{Ma}}_{6} \\
&-\underbrace{\partial_M A_N^a\partial^M V^{Na}}_{3} - \underbrace{\partial_M A_N^a\partial^M A^{Na}}_{2} + \underbrace{\partial_M A_N^a\partial^N V^{Ma}}_{4} + \underbrace{\partial_M A_N^a\partial^N A^{Ma}}_{2} \\
&-\underbrace{\partial_N V_M^a\partial^M V^{Na}}_{1} - \underbrace{\partial_N V_M^a\partial^M A^{Na}}_{4} + \underbrace{\partial_N V_M^a\partial^N V^{Ma}}_{1} + \underbrace{\partial_N V_M^a\partial^N A^{Ma}}_{5} \\
&+\underbrace{\partial_M A_M^a\partial^M V^{Na}}_{6} + \underbrace{\partial_N A_M^a\partial^M A^{Na}}_{2} - \underbrace{\partial_N A_M^a\partial^N V^{Ma}}_{5} + \underbrace{\partial_N A_M^a\partial^N A^{Ma}}_{2}] = \\
&\int \mathrm{d}^5x \frac{(-d_2)}{2g_5^2}\sqrt{g}\sum_a \frac{\gamma_2^a}{2} \\
&\times[\underbrace{(\partial_M V_N^a - \partial_N V_M^a)(\partial^M V^{Na} - \partial^N V^{MA})}_{1} - \underbrace{(\partial_M A_N^a - \partial_N A_M^a)(\partial^M A^{Na} - \partial^N A^{MA})}_{2} \\
&+\underbrace{(\partial_M V_N^a\partial^M A^{Na} - \partial_M V_N^a\partial^M A^{Na})}_{3} + \underbrace{(\partial_M A_N^a\partial^N V^{Ma} - \partial_M A_N^a\partial^N V^{Ma})}_{4} \\
&+\underbrace{(\partial_N V_M^a\partial^N A^{Ma} - \partial_N V_M^a\partial^N A^{Ma})}_{5} + \underbrace{(\partial_N A_M^a\partial^M V^{Na} - \partial_N A_M^a\partial^M V^{Na})}_{6}] = \\
&\int \mathrm{d}^5x \frac{(-d_2)}{2g_5^2}\sqrt{g}\sum_a \frac{\gamma_2^a}{2} \\
&\times[(\partial_M V_N^a - \partial_N V_M^a)(\partial^M V^{Na} - \partial^N V^{MA}) - (\partial_M A_N^a - \partial_N A_M^a)(\partial^M A^{Na} - \partial^N A^{MA})] = \\
&\int \mathrm{d}^5x \frac{(-d_2)}{2g_5^2}\frac{L^5}{z^5}\sum_a \frac{\gamma_2^a}{2}\left[\frac{z^4}{L^4}(\partial_M V_N^a - \partial_N V_M^a)^2 - \frac{z^4}{L^4}(\partial_M A_N^a - \partial_N A_M^a)^2\right] = \\
&\int \mathrm{d}^5x \frac{(-d_2)}{4g_5^2}\frac{L}{z}\sum_a \gamma_2^a[(\partial_M V_N^a - \partial_N V_M^a)^2 - (\partial_M A_N^a - \partial_N A_M^a)^2]
\end{aligned}
\tag{A.3}
$$

Appendix B

The New Equations of Motion

In this appendix we explicitly write the steps needed to go from the equations (3.67) and (3.69) to equations (3.68), (3.70), (3.71), (3.72) and (3.73).

We suppress the arguments for simplicity.

B.1 The Vector Sector

From the action we get the EOMs

$$\eta^{ML}\partial_M\left(\frac{\gamma_V^a(z)}{z}(\partial_L V_N^a - \partial_N V_L^a)\right) + \frac{\alpha^a(z)}{z}V_N^a = 0. \tag{B.1}$$

By writing N explicitly as ν and z we get

$$\eta^{\mu\lambda}\partial_\mu\left(\frac{\gamma_V^a(z)}{z}(\partial_\lambda V_\nu^a - \partial_\nu V_\lambda^a)\right) - \partial_z\left(\frac{\gamma_V^a(z)}{z}(\partial_z V_\nu^a - \partial_\nu V_z^a)\right) + \frac{\alpha^a(z)}{z}V_\nu^a = 0 \tag{B.2}$$

and

$$\eta^{\mu\lambda}\partial_\mu\left(\frac{\gamma_V^a(z)}{z}(\partial_\lambda V_z - \partial_z V_\lambda)\right) + \frac{\alpha^a(z)}{z}V_z = 0. \tag{B.3}$$

Decomposing V_α^a into its longitudinal and transversal part $V_\alpha^a = V_{\alpha\perp}^a + V_{\alpha\parallel}^a$ and using $\partial^\alpha V_{\alpha\perp}^a = 0$ gives

$$\begin{aligned}&\frac{\gamma_V^a(z)}{z}\eta^{\mu\lambda}\partial_\mu\partial_\lambda V_{\nu\perp}^a - \partial_z\left(\frac{\gamma_V^a(z)}{z}\partial_z V_{\nu\perp}^a\right) + \frac{\alpha^a(z)}{z}V_{\nu\perp}^a \\ &- \partial_z\left(\frac{\gamma_V^a(z)}{z}\partial_z V_{\nu\parallel}^a\right) + \frac{\alpha^a(z)}{z}V_{\nu\parallel}^a + \partial_z\left(\frac{\gamma_V^a(z)}{z}\partial_\nu V_z^a\right) = 0\end{aligned} \tag{B.4}$$

and

$$\frac{\gamma_V^a(z)}{z}\eta^{\mu\lambda}\partial_\mu\partial_\lambda V_z^a + \frac{\alpha^a(z)}{z}V_z^a - \frac{\gamma_V^a(z)}{z}\eta^{\mu\lambda}\partial_\mu\partial_z V_{\lambda\parallel}^a = 0. \tag{B.5}$$

Fourier transforming the equations gives

$$\begin{aligned}
&-\frac{\gamma_V^a(z)}{z}k^2\hat{V}^a_{\nu\perp} - \partial_z\left(\frac{\gamma_V^a(z)}{z}\partial_z\hat{V}^a_{\nu\perp}\right) + \frac{\alpha^a(z)}{z}\hat{V}^a_{\nu\perp} \\
&- \partial_z\left(\frac{\gamma_V^a(z)}{z}\partial_z\hat{V}^a_{\nu\parallel}\right) + \frac{\alpha^a(z)}{z}\hat{V}^a_{\nu\parallel} - ik_\nu\partial_z\left(\frac{\gamma_V^a(z)}{z}\hat{V}^a_z\right) = 0
\end{aligned} \tag{B.6}$$

and

$$-\frac{\gamma_V^a(z)}{z}k^2\hat{V}^a_z + \frac{\alpha^a(z)}{z}\hat{V}^a_z + i\frac{\gamma_V^a(z)}{z}\eta^{\mu\lambda}k_\mu\partial_z\hat{V}^a_{\lambda\parallel} = 0. \tag{B.7}$$

Multiplying the first equation with $-ik_\nu$ makes the transverse parts vanish. The result is two equations that show that the longitudinal part vanishes independently of the transverse part. Because of this we know that the opposite also must be true, i.e. the transverse part vanishes independently of the longitudinal part. Using this we find the EOMs for the transverse part as the transverse parts in (B.6), i.e.

$$\partial_z\left(\frac{\gamma_V^a(z)}{z}\partial_z\hat{V}^a_{\nu\perp}\right) + \frac{\gamma_V^a(z)k^2 - \alpha^a(z)}{z}\hat{V}^a_{\nu\perp} = 0. \tag{B.8}$$

Which is equation (3.68)

B.2 The Axial Sector

From the action we get the EOMs

$$\eta^{ML}\partial_M\left(\frac{\gamma_A^a(z)}{z}(\partial_L A^a_N - \partial_N A^a_L)\right) + \frac{\beta^a(z)}{z}(A^a_N - \partial_N\pi^a) = 0. \tag{B.9}$$

By writing N explicitly as ν and z we get

$$\eta^{\mu\lambda}\partial_\mu\left(\frac{\gamma_A^a(z)}{z}(\partial_\lambda A^a_\nu - \partial_\nu A^a_\lambda)\right) - \partial_z\left(\frac{\gamma_A^a(z)}{z}(\partial_z A^a_\nu - \partial_\nu A^a_z)\right) + \frac{\beta^a(z)}{z}(A^a_\nu - \partial_\nu\pi^a) = 0 \tag{B.10}$$

and

$$\eta^{\mu\lambda}\partial_\mu\left(\frac{\gamma_A^a(z)}{z}(\partial_\lambda A_z - \partial_z A_\lambda)\right) + \frac{\beta^a(z)}{z}(A_z - \partial_z\pi^a) = 0. \tag{B.11}$$

By the gauge invariance

$$\begin{aligned}
A^a_M &\to A'^a_M = A^a_M - \partial_M\lambda^a \\
\pi^a &\to \pi'^a_M = \pi^a_M - \lambda^a
\end{aligned} \tag{B.12}$$

allows us to set any λ^a. We set $\partial_z\lambda^a = A_z$ and thus effectively setting A_z to zero. We also decompose the A^a_α into its longitudinal and transversal part $A^a_\alpha = A^a_{\alpha\perp} + A^a_{\alpha\parallel}$ and define

$A^a_{\alpha\|} = \partial_\alpha \phi^a$. Using this together with $\partial^\alpha A^a_{\alpha\perp} = 0$ we get

$$\begin{aligned}&\frac{\gamma^a_A(z)}{z}\eta^{\mu\lambda}\partial_\mu\partial_\lambda A^a_{\nu\perp} - \partial_z\left(\frac{\gamma^a_A(z)}{z}(\partial_z A_{\nu\perp})\right) + \frac{\beta^a(z)}{z}A_{\nu\perp}\\ &- \partial_z\left(\frac{\gamma^a_A(z)}{z}(\partial_z\partial_\nu\phi^a)\right) + \frac{\beta^a(z)}{z}(\partial_\nu\phi^a - \partial_\nu\pi^a) = 0\end{aligned} \tag{B.13}$$

and

$$-\frac{\gamma^a_A(z)}{z}\eta^{\mu\lambda}\partial_\mu\partial_\lambda\partial_z\phi^a - \frac{\beta^a(z)}{z}\partial_z\pi^a = 0. \tag{B.14}$$

Fourier transforming these equations gives

$$\begin{aligned}&-\frac{\gamma^a_A(z)}{z}k^2\hat{A}^a_{\nu\perp} - \partial_z\left(\frac{\gamma^a_A(z)}{z}(\partial_z\hat{A}_{\nu\perp})\right) + \frac{\beta^a(z)}{z}\hat{A}_{\nu\perp}\\ &+ ik_\nu\partial_z\left(\frac{\gamma^a_A(z)}{z}(\partial_z\hat{\phi}^a)\right) - ik_\nu\frac{\beta^a(z)}{z}(\hat{\phi}^a - \hat{\pi}^a) = 0\end{aligned} \tag{B.15}$$

and

$$\frac{\gamma^a_A(z)}{z}k^2\partial_z\hat{\phi}^a - \frac{\beta^a(z)}{z}\partial_z\hat{\pi}^a = 0. \tag{B.16}$$

Multiplying the first equation with $-ik_\nu/k^2$ makes all the transversal parts vanish. If we also multiply the second equation with z we get

$$\partial_z\left(\frac{\gamma^a_A(z)}{z}(\partial_z\hat{\phi}^a)\right) - \frac{\beta^a(z)}{z}(\hat{\phi}^a - \hat{\pi}^a) = 0 \tag{B.17}$$

and

$$\gamma^a_A(z)k^2\partial_z\hat{\phi}^a - \beta^a(z)\partial_z\hat{\pi}^a = 0. \tag{B.18}$$

Which is equations (3.71) and (3.72).

Defining $y^a = \frac{\gamma^a_z(z)}{z}\partial_z\phi^a$ gives

$$\partial_z y^a - \frac{\beta^a(z)}{z}(\hat{\phi}^a - \hat{\pi}^a) = 0 \tag{B.19}$$

and

$$zk^2y^a - \beta^a(z)\partial_z\hat{\pi}^a = 0. \tag{B.20}$$

Multiplying the first equation with $z/\beta^a(z)$ and taking the derivative with respect to z gives

$$\partial_z\left(\frac{z}{\beta^a(z)}\partial_z y^a\right) - \partial_z(\hat{\phi}^a - \hat{\pi}^a) = 0 \tag{B.21}$$

and

$$zk^2y^a - \beta^a(z)\partial_z\hat{\pi}^a = 0. \tag{B.22}$$

Relating $\hat{\phi}^a$ with y^a through the definition and $\hat{\pi}^a$ with y^a through the second equation gives

$$\partial_z\left(\frac{z}{\beta^a(z)}\partial_z y^a\right) + z\left(\frac{k^2}{\beta^a(z)} - \frac{1}{\gamma_A^a(z)}\right)y^a = 0 \tag{B.23}$$

which is equation (3.73).

The longitudinal and z part vanishes independently and thus so must the transverse part. With this we easily find the EOMs from equation (B.15) as

$$\partial_z\left(\frac{\gamma_A^a(z)}{z}(\partial_z \hat{A}_{\nu\perp})\right) + \frac{\gamma_A^a(z)k^2 - \beta^a(z)}{z}\hat{A}_{\nu\perp} = 0 \tag{B.24}$$

which is equation (3.70).

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