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Quantum Gravity effect on neutrino oscillations in a strong gravitational field

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In the framework of quantum field theory, a graviton interacts locally with a quantum state having definite mass, i.e. the gravitational mass eigenstate, while a weak boson interacts with a state having definite flavor, i.e. the flavor eigenstate. At the quantum level an interaction of a neutrino with an energetic graviton may trigger the collapse of the neutrino to a definite mass eigenstate at a short time scale with a probability expressed in terms of the corresponding PMNS mixing matrix element. Thus, gravitons induce quantum decoherence of a coherent neutrino flavor state similarly to how weak bosons induce quantum decoherence of a neutrino in a definite mass state.We demonstrate that such an essentially quantum gravity effect may have strong consequences for neutrino oscillation phenomena in astrophysics due to the relatively large scattering cross section of relativistic neutrinos off massive sources of strong gravitational fields (the quasi-classical case of gravitational Bethe-Heitler scattering). This graviton-induced decoherence is compared to standard sources of decoherence in the presence of the Earth MSW effect. Based on this study, we propose a new technique for the indirect detection of energetic gravitons by measuring the flavor composition of astrophysical neutrinos.

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I. INTRODUCTION

A theoretical extrapolation of the fundamental Quantum Mechanics concepts to Einstein's gravity suffers from major difficulties with quantization of space-time, ultraviolet behavior and non-renormalizability of the resulting theory (for more details, see Ref. [1, 2] and references therein). A wealth of theoretical studies have been presented in the literature and many different quantum gravity models have been developed. However, no conclusive statement about the true quantum nature of gravity has been made. Only a real experiment can ultimately settle the longstanding confusion between the different approaches and provide guidance in developing the correct underlying theory.

Typically, in the standard quantum field theory framework which unifies three of four basic forces of Nature, the quantum gravity effects are disregarded as being phenomenologically irrelevant at energy scales much smaller than the Planck scale, $M_{Pl} \sim 10^{19}$ GeV. Moreover, due to enormous suppression, quantum gravity effects are often referred to as nearly unobservable [3, 4]. While observing a single graviton directly may be impossible, it is not impossible to find an indirect evidence for quantum gravity. For an overview of potential phenomenological opportunities for indirect signatures of quantum gravity, see Refs. [5–8]. Nevertheless, our understanding of the quantum nature of gravity suffers from the lack of accessible sources of information.

In this paper, we propose a new approach for indirect experimental studies of (local) quantum gravity interactions based upon an effect of the large-angle energetic gravitational Bremsstrahlung (or Gravi-strahlung, in short) off an astrophysical neutrino passing through an external classical gravitational potential on neutrino oscillation observables. This process, known as the gravitational Bethe-Heitler (GBH) process, can be considered a quasi-classical approximation for a large angle and/or large energy graviton emission i.e. the Born approximation is sufficient. Such a process may happen with a rather high probability, such as in the case of an astrophysical neutrino scattering off a massive source of classical gravitational field (like a star or a black hole). In Quantum Mechanics, the latter process may serve as a direct *quantum measurement* of the microscopic properties of the gravitational field at astrophysical scales.

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II. DECOHERENCE OF NEUTRINO STATE

Generically, weakly-interacting neutrinos can be considered as an efficient carrier of information across the Universe as they are not absorbed or scattered by interstellar mediums. In practice, this unique property of neutrinos enables us to utilize them for large-scale astrophysical "experiments", such as searching for possible tiny signatures of Lorentz invariance violation [9], testing General Relativity [10] and Quantum Mechanics [11–13], testing the equivalence principle [14, 15], testing minimal length models [16, 17], etc. Ultimately, it is possible to identify an *extraterrestrial large-scale quantum experiment* in natural conditions where neutrinos "change" their quantum state due to a local quantum gravity process (in terms of local graviton coupling to a fundamental matter particle) and further convey information about such a process unchanged through the cosmological medium to the Earth.

A. Propagation decoherence

The traditional source of decoherence typically referred to in astrophysical neutrino oscillations studies can be called *propagation decoherence*. This is when the distance that a neutrino travels exceeds the neutrino oscillation length. In this case, the neutrino mass states have separated so that they no longer interfere at large distances from the production point. This source of decoherence depends on the energy resolution of the detection process, the energy of the neutrino, the masses of the neutrino mass states, and other details of the production and detection processes. In neutrino experiments, the time between neutrino production and detection is normally not measured. In a real experiment, this means that beyond the neutrino oscillation length the propagating neutrino mass states no longer interfere during the interaction process in a detector [18, 19]. In some cases this decoherence effect is irrelevant [20].

B. Classical Diósi-Penrose decoherence

The role of classical Einstein's gravity effects in Quantum Mechanics is under extensive consideration in the literature, and may be sizeable under certain conditions. As was claimed in Ref. [21], the gravity-induced quantum state reduction can be tested by observing the neutrino flavor oscillations at cosmological distances, while in Ref. [22] it was regarded as practically undetectable. This classical gravity effect on real-time evolution of a quantum state composed of several mass eigenstates was initially considered by Diósi [23] and Penrose [24]. In the classical gravity limit, the latter can be approximated by a change in the phase of the flavor wave function which appears mainly due to a non-degeneracy of neutrino mass eigenstates, i.e. $\Delta m^2_{ij} \equiv m^2_j - m^2_i \neq 0$, where m_j is the mass of the mass eigenstate j. This is caused by different mass states traveling along different geodesics in curved space-time and the whole effect gradually accumulates over large cosmological distances [28]. This is the essence of *classical decoherence* of a neutrino flavor state which is typically regarded as a probe for neutrino wave function collapse models and, more generally, alternatives to conventional (linear) Quantum Mechanics [25]. Instead, we consider another possible decoherence mechanism of a neutrino flavor state triggered *at the quantum level* by a single local graviton-neutrino interaction. Let us discuss this phenomenon in detail.

C. Quantum decoherence

In the limit of weak gravity, the quasi-classical approximation to quantum gravity is a valid framework. In this case, the graviton field is a correction determined on the flat Minkowskian background and the metric operator in the Heisenberg representation is given by $\hat{g}_{\mu\nu} = \eta_{\mu\nu} + \hat{h}_{\mu\nu}$. Here, the c-number part $\eta_{\mu\nu}$ is the Minkowski metric and $\hat{h}_{\mu\nu}$ is the graviton arising after the quantization procedure. The Einstein-Hilbert action provides the mechanism for *virtual gravitons* to propagate in the flat spacetime and to interact with one another in the quantum case as an analog of the standard QED picture of the Coulomb field around an electric charge. These virtual gravitons should be distinguished from *real gravitons* which are radiated off an accelerated massive body and their coherent wave packets correspond to gravitational waves in the classical limit. A "cloud" of virtual gravitons around a static massive body can be reinterpreted geometrically in terms of a deviation from the flat metric (or curvature) in Einstein's classical relativity [29][1].

Obviously, a graviton couples to the full energy-momentum tensor. From the quantum-mechanical point of view, we work in the mass eigenstate basis where the Hamiltonian of local quantum-gravitational interactions has a diagonal form and identify the particle mass eigenstates with gravitational eigenstates (due to equivalence of gravitational and inertial mass). In this approach, higher Fock states are created by the graviton creation operator acting on a particle mass eigenstate.

Also, we expect elementary particles in the mass basis to be gravitational eigenstates of the Hamiltonian of quantum-gravitational interactions in the same way as leptons and quarks are weak eigenstates in the flavor and CKM basis, respectively. The advantage of the neutrino which we exploit here is that they interact via the weak force, neutrino mass and flavor eigenstates are not

[1] The background must be chosen to be flat since only in this case is it possible to use the Casimir operators of the Poincaré group and show that the quanta have spin two and rest mass zero, thus being identified as gravitons.

the same, and that they propagate at cosmological distances/times. For particles whose flavor and mass eigenstates are identical this technique would not work to identify that a graviton induced quantum mechanical interaction had happened, which means that the neutrino is a unique carrier of astrophysical quantum gravity interactions.

Consider first a relativistic neutrino state propagating in the gravitational potential of a static black hole, a massive star or a binary system which not only are sources of strong gravitational fields but could also be significant sources of astrophysical neutrinos. Suppose now that at the quantum level a graviton interacts only with a definite mass state (or gravitational mass eigenstate) $a = 1, 2$ or 3. This is equivalent to saying that definite mass eigenstates (the propagating states) are conserved by the quantum gravity hamiltonian while superpositions, such as the flavor eigenstates, are not. Note, the astrophysical neutrinos are initially produced in electro-weak processes (e.g. in SNe core) in a definitive flavor state, $f = e, \mu$ or τ, which are coherent superpositions of mass eigenstates. In an astrophysical environment, a high-energy graviton can interact only with a definite mass component of the neutrino wave function thus causing *quantum decoherence* of the neutrino which is in a superposition of mass states, effectively "converting" it into a definitive mass eigenstate. This neutrino is quantum mechanically observed as being in a definite mass state. This means that between the production in an AGN or SuperNova or other astrophysics source and the detection in an Earth based detector, the neutrino exists in a definite mass state. This is independent of whether it undergoes propagational decoherence or not.

The neutrino is "converted" to mass state with a probability $P_{\nu_f\to\nu_a} = |\Psi_{\nu_f\to\nu_a}|^2$, given in terms of the corresponding wave function $\Psi_{\nu_f\to\nu_a}$ which projects out a flavor state ν_f onto a mass state ν_a and is typically expressed in terms of the corresponding PMNS mixing matrix element, $\Psi_{\nu_f\to\nu_a} \equiv V_{af}^2$. The considering effect is different from other known classical decoherence sources emerging due to a mere propagation (without a hard radiation) in classical gravitational potential and/or neutrino propagation in flat space-time. The effect under consideration is a straightforward consequence of fundamental time-energy uncertainty relation for the real hard Gravi-strahlung and should be taken into account in astrophysical neutrino oscillations.

The amplitudes of typical quantum gravity scattering processes which may lead to the quantum decoherence effect under certain conditions can be represented as follows:

$$
\begin{aligned}
A^{(G),1}_{\nu_f\to\nu_a} &= \Psi_{\nu_f\to\nu_a} A^{(G)}(\nu_a + G \to \nu_a + G)\,.\\
A^{(G),2}_{\nu_f\to\nu_a} &= \Psi_{\nu_f\to\nu_a} A^{(G)}(\nu_a + M \to \nu_a + G + M)\,.
\end{aligned}
$$

Here, M is a source of strong classical gravitational fields, such as a massive star or a black hole. The first amplitude corresponds to the gravitational Compton scattering of a neutrino mass state off a real graviton in the medium, the second amplitude represents the GBH scattering of a neutrino in gravitational mass state off a classical heavy source M. Clearly, a mass eigenstate ν_a "produced" in this interaction due to decoherence does not undergo oscillation until it interacts weakly with normal matter (e.g. in an Earth detector) by means of W, Z-exchange. Therefore, quantum decoherence may have an non-negligible effect on neutrino oscillation observables, along with other existing sources of classical decoherence and medium MSW effects [32, 33]. Explicitly, oscillation characteristics of neutrinos coming from e.g. a vicinity of the Galactic Center may differ from vacuum oscillations. The latter case could be where a source of neutrinos is "nearby" but where there is no massive objects between the source and the Earth (nor significant variations in dark matter density). Such neutrinos, if identified, could be used as a control sample.

In a sense, the quantum gravity-induced decoherence of a definite flavor state described above is in close analogy to the weak-induced decoherence of a definite mass state. For example, W, Z bosons interact only with a coherent flavor state inducing a "conversion" of a definite mass state into a definite flavor state. Namely, a neutrino in a mass eigenstate ν_a turns into a flavor eigenstate ν_f through an interaction with the deeply virtual Z, W-bosons propagating in the t-channel, i.e. four different reactions are possible

$$
\begin{aligned}
A^{(\mathrm{w}),1}_{\nu_a\to\nu_f} &= \Psi_{\nu_a\to\nu_f} A^{(\mathrm{w})}(\nu_f + l'_f \to \nu'_f + l_f)\,,\\
A^{(\mathrm{w}),2}_{\nu_a\to\nu_f} &= \Psi_{\nu_a\to\nu_f} A^{(\mathrm{w})}(\nu_f + l'_f \to \nu_f + l'_f)\,,\\
A^{(\mathrm{w}),3}_{\nu_a\to\nu_f} &= \Psi_{\nu_a\to\nu_f} A^{(\mathrm{w})}(\nu_f + N \to \nu_f + X)\,,\\
A^{(\mathrm{w}),4}_{\nu_a\to\nu_f} &= \Psi_{\nu_a\to\nu_f} A^{(\mathrm{w})}(\nu_f + N \to l_f + X)\,,
\end{aligned}
$$

such that $\Psi_{\nu_a\to\nu_f} = \Psi^*_{\nu_f\to\nu_a}$. Here, a definitive mass state which may exist due to previous hard neutrino-graviton interaction or due to the resonance MSW effect [32, 33] is "converted" back into a flavor state which may undergo oscillation. It is important to note that because the neutrino is not likely to interact weakly between the source and the Earth, the neutrino is likely to be in a definitive mass state induced by the hard neutrino-graviton scattering event when it arrives at the Earth. The distance between a hard neutrino-graviton scattering event and detection event does not matter in this case.

In the case of vacuum neutrino oscillations, the traveling neutrino is not in a definitive mass eigenstate but is rather in a superposition of mass eigenstates which evolves when the neutrino travels in space-time. Then, with respect to the weak interactions, the non-diagonal $\Psi_{\nu_f\to\nu_{f'}}$ transition amplitude between two flavor states f and f' is given by [34]

$$
\Psi_{\nu_f\to\nu_{f'}} = \sum_j V_{f'j} e^{-i\frac{m_j^2}{2E_\nu}L} V^*_{fj}\,, \tag{2.1}
$$

here L is the distance from where the neutrino was created in a definite flavor eigenstate ν_f, and E_ν is the energy of the neutrino. Analogically, for neutrino-graviton interactions the $\Psi_{\nu_f \to \nu_a}$ transition amplitude between a flavor state f and a mass state a can be written as

$$\Psi_{\nu_f \to \nu_a} = e^{-i\frac{m_a^2}{2E_\nu}L}\, V_{af} \tag{2.2}$$

which means that the probability for a given flavor neutrino state f to decohere by transforming into a mass state a due to a hard graviton-neutrino interaction, given by $P^{(G)}_{\nu_f \to \nu_a} \sim |A^{(G)}_{\nu_f \to \nu_a}|^2 = |\Psi_{\nu_f \to \nu_a}|^2 |A^{(G)}|^2$, is independent of the neutrino mass, m_a, the mass splitting, Δm_{ab}, and the distance from the neutrino source, L. The dependence on the relativistic neutrino energy, $E_\nu \gg m_a$, for a given scattering comes from the neutrino mass state scattering amplitude squared, $|A^{(G)}|^2$ (for more details, see the next Section).

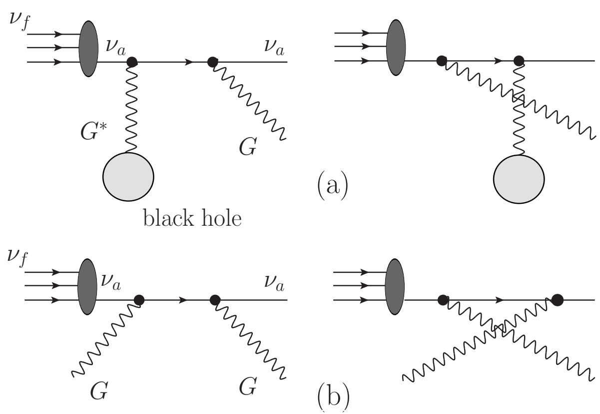

FIG. 1: The quantum gravity processes which destroy the coherence of the neutrino flavor eigenstate ($f = e, \mu, \tau$) at the quantum level effectively turning it to a mass eigenstate ($a = 1, 2, 3$) – the gravitational Bethe-Heitler-type scattering of neutrino off a massive object, e.g. a black hole (a), and the gravitational Compton scattering (b). The dark ellipse is a projection to a fixed mass state and the shaded circle is a classical source of the gravitational field.

Contrary to the Penrose-Diósi effect of classical decoherence [23, 24], the quantum decoherence of a neutrino flavor state happens at small space-time scales, Δl_{dec}, which are much smaller than the neutrino wave length scale: $\Delta l_{dec} \ll L_\nu$, due to the quantum nature of the (real or virtual) tree-level graviton-neutrino interaction. An additional significant difference, the quantum decoherence effect is not sensitive to the mass differences of the mass eigenstates, or to Δm^2_{ij}, while they are crucial for and determine classical decoherence of the neutrino flavor state at large separations, $\Delta l_{dec} \gg L_\nu$. Most importantly, quantum decoherence provides us with a key for phenomenological verification of quantum gravity models through measurement of neutrino oscillation characteristics.

The proposed effect is also different from the standard propagation decoherence. In propagation decoherence, the neutrino mass states are separated in time and/or space and so the local weak interaction (the detection process) *observes* an incoherent sum of the propagating mass states in a given space-time point. In quantum decoherence, the neutrino exists only within a given mass eigenstate after being "observed" by the hard graviton (e.g. in the quantum processes of hard GBH or Compton scattering, see below). This difference is important. Indeed, while a flux of neutrinos which have undergone the quantum decoherence is *observed* by a weak interaction in an Earth-based detector as an incoherent sum of the mass states, they do not experience a change of potential induced by matter (the MSW effect) as an incoherent sum of mass states. Namely, the neutrino which has not undergone the quantum decoherence experiences matter coherently, while the neutrino which has undergone the quantum decoherence would not experience matter coherently. Also, it is possible that a neutrino passing through densities which change non-adiabatically might demonstrate interference phenomena as presented in Ref. [20]. As we will explicitly demonstrate below, such a difference between the quantum and propagational decoherences in the presence of the Earth MSW effect may be observable and is important in studies of astrophysical neutrino oscillations.

III. GRAVITON-NEUTRINO SCATTERING

Now consider which quantum gravity processes the neutrino could possibly experience so as to experience the quantum decoherence effect in the astrophysical medium.

As is known the Coulomb field is measured by inserting a charged probe into it. From the quantum electrodynamics (QED) point of view, an electromagnetic scattering of a charged particle off the Coulomb field is due to an exchange of virtual photons (with small negative momentum transfer squared $-q^2 = Q^2 > 0$ in the t-channel) between the probe and the source. Analogically, it is correct to discuss multiple exchange of virtual t-channel gravitons in a scattering event as a signature of non-zeroth curvature itself (for more detailed discussions of the principles, see e.g. Ref. [36]).

Generically, in quantum electrodynamics (QED) the virtual photons may become real (produced on-mass-shell) if one disturbs the field pumping energy into it. This is the physical reason for photon Bremsshtrahlung in QED. Specifically, the standard Bethe-Heitler scattering in electrodynamics demonstrates that only an accelerated charge emits real photons (corresponding to electromagnetic wave in the classical limit of multiple soft photon radiation). Likewise, in the quasi-classical gravity framework, the virtual graviton, as a quantum of the gravitational field of a static massive object, may turn into the real one (corresponding to gravitational wave in the classical limit of multiple soft graviton radiation) if the source of the gravitational field is accelerated or, in general, when the energy-momentum tensor experiences disturbances. Possible sources of real gravitons in the Universe include: active galactic nuclei (AGN), binary systems, supernovae explosions (SNe), primordial black holes collisions, compact star/black holes binaries, quantum bremsstrahlung of gravitons of particles scattering off a massive object, black hole (BH) evaporation, relic isotropic gravitational background from the early universe, inflation, phase transitions in the primordial plasma, the decay or interaction of topological defects (e.g. cosmic strings), etc. For details and references, see Ref. [37].

Consequently, in the cosmological medium a neutrino can scatter either off a classical gravitational potential with accompanying radiation of an energetic real graviton off the scattered neutrino (e.g. Bethe-Heitler-type scattering) or off real graviton in the astrophysical medium (e.g. Compton-type scattering). Let us consider both cases and conditions for initiation of the quantum neutrino decoherence in more detail.

A. Gravitational Bethe-Heitler scattering

In fact, all elementary particles, including neutrinos, when traveling in the vicinity of massive objects (sources of classical gravitational field) can emit real gravitons with a certain energy spectrum. This process has a straightforward QED analog of a photon mission in relativistic electron scattering off the Coulomb field of a heavy nucleus mentioned above, the Bethe-Heitler process at the Born level. Even though the energy spectrum of radiated real gravitons is indeed peaked in the forward direction and in the infrared limit (corresponding to forward radiation of classical gravitational waves), there is a non-negligible probability to radiate *hard or energetic gravitons*, namely, with energies comparable to the incoming relativistic neutrino energy. The quantum-mechanical uncertainty tells us that the latter process can trigger a dramatic decoherence of an incoming neutrino flavor state at the quantum level during a very short time scale inversely proportional to the energy of the radiated hard graviton.

Indeed, the neutrino decoherence at the quantum level can only be initiated by hard energetic interactions with relatively hard gravitons whose energies exceed the mass difference between different mass states $E_G \gtrsim \Delta m_{ij}$ and therefore requires a hard real graviton emission. In this case, the hard graviton probe has a small wave length and thus can resolve separate mass states in a coherent neutrino flavor state in a quantum-mechanical sense[2].

Of course, a soft graviton with a large wave length exceeding the space-time spreading of a neutrino flavor state as a coherent superposition of mass eigenstates will not resolve individual mass eigenstates in this superposition and will instead couple to the whole energy-momentum tensor of the flavor state, non-locally, which is the classical General Relativity limit. In the latter case, the quantum decoherence is not triggered.

The Born-level calculation is good enough in the case of off-forward hard graviton emissions at large angles relevant for the quantum decoherence effect – this is the reason why one can disregard higher-order radiative corrections which are highly suppressed in this case (by extra powers of the Planck mass) as long as one cuts off the problematic but uninteresting infrared/collinear parts of the phase space. Indeed, as was previously shown in Ref. [31], the radiative corrections can only be relevant in the deep infrared limit of soft real gravitons $E_G \to 0$ emitted in the forward direction where they will cancel the soft/collinear divergences. The latter classical limit represents classical gravitational waves emitted off a neutrino state without or very small impact on it.

In the considering GBH case, shown in Fig. 1(a), one deals with the graviton exchange with negative momentum transfer squared $t = -q^2 < 0$ in the t-channel with the propagator stretched between the relativistic neutrino of mass m_ν and energy $E_\nu \gg m_\nu$ and a massive classical gravitational field source with mass $M \gg E_\nu$. The wave function, $\Psi_{\nu_f \to \nu_a}$, describes a projection of a given flavor state f onto a fixed mass state a is denoted as a dark ellipse, while the heavy classical source of the gravitational field is shown by a shaded circle.

The GBH cross section has initially been calculated for the gravitational scattering of scalar particles with $M \gg m$ in Ref. [31]. In the soft graviton limit, the graviton-neutrino coupling is not sensitive to the spin of an incident relativistic particle to leading order, while the classical non-relativistic source can be considered to be spinless in this first discussion for simplicity (in principle, a helicity dependence of hard graviton-neutrino interactions can be a relevant topic for further studies). We therefore use their formula as a sufficiently good approximation to estimate the neutrino-black hole cross section numerically. In this case, as an order-of-magnitude estimate the GBH cross section at the Born level behaves as

$$\sigma_{\rm GBH} \sim \frac{M^2 E_\nu^2}{M_{Pl}^6}\,, \quad M \gg E_\nu \gg m_\nu\,, \tag{3.1}$$

and thus may not always be very small since the Planck scale suppression can be largely eliminated by having a huge mass M of a heavy classical source in numerator. In particular, for a Solar-mass object $M \sim 10^{57}$ GeV we have $M^2/M_{Pl}^6 \sim 1$ GeV^{-4}, so there is no significant suppression of the cross section for relativistic neutrinos.

In Fig. 2 we have presented the differential (in radiated graviton energy E_G and neutrino angle θ_ν) and integrated cross sections of the GBH process for typical MeV-scale astrophysical neutrinos and a solar mass scale source of the gravitational field. As expected, the main

[2] Likewise a hard enough photon can resolve an internal substructure of the proton wave function and interacts with separate quarks it is composed of while a soft one "sees" a proton as a whole only.

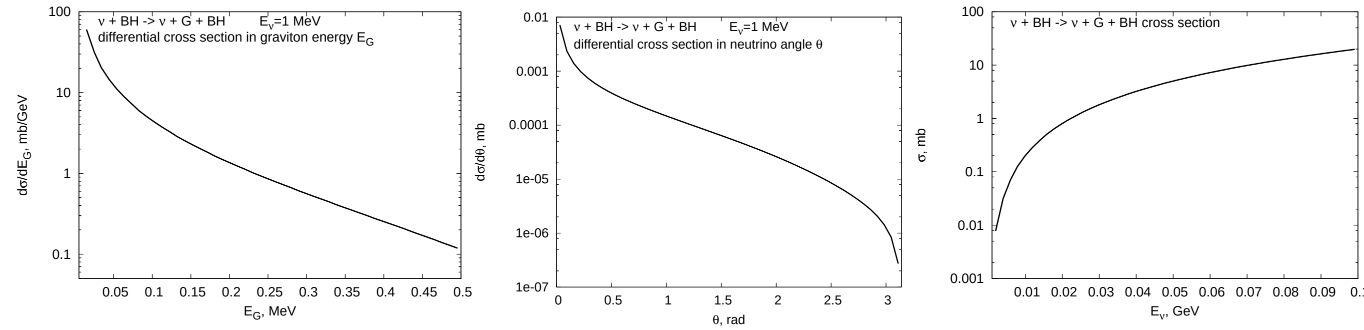

FIG. 2: Differential cross section of the gravitational Bethe-Heitler scattering of neutrino off a massive object e.g. a black hole (BH) in radiated graviton energy E_G (left), in polar angle of the final-state neutrino θ_ν, and the integrated cross section as a function of incoming neutrino energy E_ν typical for astrophysical sources, e.g. see Ref. [38] (right).

bulk of the cross section comes from the soft gravitons (gravitational waves) emission in the forward limit. It is remarkably important, however, that there is a long non-negligible tail in the differential distributions of the GBH cross section in the single real graviton energy E_G and emission angle θ. It turns out that such a tail to harder/off-forward gravitons is not very strongly suppressed – typical GBH scattering cross sections for SNe neutrino energies of $E_\nu \sim 10 - 100$ MeV and a Solar-mass classical source are found to be around $\sigma \sim 0.1 - 10$ millibarns, which are some $16 - 18$ orders of magnitude larger than typical neutrino-electron scattering cross sections which are less than an attobarn at the same energies.

This observation strongly suggesting the importance of the quantum decoherence initiated by interactions with such energetic real gravitons. The latter source of decoherence does not have a classical interpretation. As we have already mentioned, due to a quantum-mechanical nature of a *single hard graviton* emission at energies $E_G \gtrsim \Delta m_{ij}$ (with local coupling to a gravitational mass eigenstate) and universal quantum-mechanical time-energy uncertainty arguments the considered effect of neutrino flavor decoherence as a purely *quantum effect.* The hard Gravi-strahlung effect is thus relevant for a broad range of neutrino energies, and one can utilize the SNe neutrinos as a clear sample since (1) fluxes of SNe neutrinos are the largest among astrophysical neutrinos and (2) SNe neutrino emission mechanisms are the best understood among other possible astrophysical sources.

Of course, the t-channel gravitons are extremely soft and form classical gravitational potential of a classical massive source and they do not trigger a decoherence of the neutrino state – only the hard real graviton emissions are relevant here.

B. Probability for quantum gravitational decoherence

The cross section of the considering GBH process can be strongly enhanced for e.g. a supermassive black hole in the Galactic Center whose mass is about $\sim 10^6 - 10^9$ Solar masses, as well as for ultra-relativistic neutrinos which are potentially detectable at neutrino observatories such as IceCube and Super-K. As is our main result, we notice that the GBH scattering may cause the quantum decoherence of astrophysical neutrinos and this effect can be measured via neutrino flavor composition measurements. A massive classical source of the gravitational field may not necessarily be a black hole, but any compact star or, in general, any bound gravitational potential induced by continuous matter distribution in the Galactic disk and Halo.

Due to rather large cross sections it is likely that most of the astrophysical neutrinos which are observed at the Earth from a given direction and have passed in close vicinity of a massive object would have experienced the quantum decoherence due to a graviton-induced scattering. In other words, the probability for a given neutrino in a superposition of mass states f to decohere, being "transformed" into one of the mass states $a = 1, 2$ or 3 in the GBH process, $P_G \sim |A^{(G)}|^2$, is expected to be large for typical (massive) astrophysical sources of classical gravitational fields, i.e.

$$P_G \equiv \frac{N_{\mathrm{G}\nu}}{N_{\mathrm{init}}} \sim 1 \qquad (3.2)$$

where $N_{\mathrm{G}\nu}$ is the number of neutrinos which have been scattered off a massive object via at least one graviton exchange[3], and N_{init} is the total number of neutrinos which have been emitted off an astrophysical source. As we will demonstrate below, the P_G value can be measured via neutrino flavor composition observations leading to a promising opportunity for experimental tests of quantum gravity induced interactions.

[3] On the way to the Earth, a produced mass state may experience more graviton-mediated rescatterings which do not affect the coherence of the neutrino state any longer, but may cause an additional energy loss of the propagating neutrino into gravitational radiation.

A precise theoretical calculation for P_G is influenced by many potentially relevant aspects. First, it depends on a quantum gravity model through model-dependent local neutrino-graviton couplings thus offering a good opportunity for experimental tests of quantum gravity. Second, it may be influenced by yet unknown higher-order corrections and by multiple rescatterings of a neutrino off a massive source, multiple massive sources, or a diffuse source such as the dark matter halo which the neutrino passes through on its path to the Earth (in this case, the eikonal approximation for neutrino-graviton rescattering can be used [39]). Thus, the actual cross sections may significantly vary depending on environment a neutrino propagates in. Thirdly, the astrophysical neutrino flavor composition may depend on production processfes which may currently be unknown (unless we deal with a well-known "standard candle" emitter like a SNe). Also, an energy loss of the neutrino due to the hard graviton Bremsstrahlung in each scattering event should be taken into consideration, together with other effects which change the coherence of the neutrino state. Finally, including possible dense astrophysical media might be important as the neutrino may have additional weak rescatterings off normal matter acting on the neutrino leaving leaving the neutrino in a superposition of mass eigenstates when it arrives at the earth. Therefore, additional astrophysical information is desired to constrain these uncertainties. All of the above aspects are the major unknowns in making predictions for the P_G quantity which require a further effort of the quantum gravity, neutrino and astrophysics communities.

C. Gravitational Compton scattering

Another possibility for quantum gravity induced interactions with neutrino participation is shown in Fig. 1(b). This is the (tree level) gravitational Compton scattering of a neutrino off a real graviton (or a gravitational wave) in cosmological medium. The latter process has been previously studied in Ref. [35] and in many other papers. The cross section in this case is always extremely small $\sigma \sim E_\nu^2/M_{Pl}^2$ for a MeV neutrino, and real graviton fluxes are not expected to compensate for such a huge suppression. This process seems less interesting when applied to astrophysical neutrino flavor composition. Hypothetically, this effect could be considered in exotic cases of ultra-relativistic neutrinos and/or in the very early Universe where the graviton fluxes might have been rather intense.

D. Quantum Gravity measurement proposal

As presented above, the neutrino in a mass eigenstate does not oscillate unless it scatters off ordinary matter via a weak channel which brings it back to a flavor eigenstate or to a lepton. It is likely that the Z, W-mediated scattering happens only in the Earth-based detector enabling us to access information about the graviton-neutrino scattering which might have happened far away from the solar system. In the considered situation, the neutrino plays an analogical role of an electric charge in a quantum measurement of the microscopic Coulomb field properties. From the quantum mechanical point of view, a black hole vicinity can then be viewed as a macroscopic "detector" of gravitons. The neutrino scattering off a black hole by means of the graviton-neutrino coupling would be considered as an elementary act of quantum mechanical measurement, and the neutrino conveys the quantum information about the act of graviton measurement to the Earth. The neutrino does not undergo oscillation or demonstrate properties consistent with being a superposition of mass eigenstates since it is in a definitive mass state during the propagation and graviton interaction. That the neutrino does not interact weakly as it travels is considered to be a good approximation due to extremely weak interactions of neutrinos with ordinary matter. Then an Earth-based detector will "read off" the results of the "graviton measurement" which has taken place at a black hole or massive object. In experimentalist jargon, the vicinity around a black hole is the "detector", and the neutrino is the "cabling", and the neutrino detector at the Earth is a "data acquisition system" or "classical observer" − the analogy is rather close.

Previously, in Ref. [4], it has been claimed that it is not possible to detect a single graviton with a planet-scale detector. Our proposal is to measure the described graviton-neutrino scattering effect (specifically, the gravitational Bethe-Heitler scattering of neutrino off a black hole or off any other massive object) experimentally, which is the best possibility for indirect graviton detection proposed. Remarkably, we consider a black hole-scale or a star-scale "detector" of gravitons, with neutrinos serving as the most efficient carrier of the information about such a measurement to the Earth.

IV. QUANTUM GRAVITATIONAL DECOHERENCE EFFECT ON NEUTRINO OSCILLATIONS

Here we consider a very massive source of strong gravitational fields like the *Supermassive Black Hole* (SMBH) in the center of our Galaxy as a good example of the closest and one of the most efficient graviton "detectors". This Section provides predictions for such an extreme large-scale quantum-gravity measurement.

As we have demonstrated above, the probability of an individual (elementary) act of the "quantum gravity measurement" defined by the graviton-neutrino cross section can be rather large due to a large GBH cross section and cannot be neglected. Especially, utilizing the SMBH in the Galactic Center as our "graviton detector" in the above sense, one could expect that a significant fraction of neutrinos passing by the SMBH would have experienced the GBH scattering. Then since most of the neutrinos are now in a mass eigenstate, they will no longer undergo flavor oscillation. The process is irreversible with respect to gravitational interactions, so the resulting mass eigenstate will not "transform" any further due to subsequent graviton-neutrino re-scattering. Depending on the astrophysical process, one might favor relatively low energies of neutrinos where the neutrino oscillation may not be suppressed due to matter effects where the neutrino exists in a single mass eigenstate, so that the graviton-induced effect would be cleaner. We suggest that this effect could be tested in neutrino telescopes and observatories by looking at the Galactic Center neutrino flavor composition and comparing it to the composition expected without quantum gravitational decoherence. It might be possible that close, "standard candle", neutrino emitters in other parts of the sky provide a flux of neutrinos which have not undergone quantum gravitational decoherence[4].

The general formula for the number of electron type neutrinos observed from an electron type source in the vacuum is:

$$\frac{N_{\mathrm{e,det}}}{N_{\mathrm{e,init}}} \propto P^{\mathrm{vac}}_{\mathrm{ee},\infty}(1-P_G) + P_G \sum_{i=1,2,3} V_{ei}V^*_{ie}V_{ei}V^*_{ie} \,. \quad (4.1)$$

Here $P^{\mathrm{vac}}_{\mathrm{ee},\infty}$ is the standard vacuum oscillation probability ([34]) far away from the neutrino source and P_G is the probability for neutrino in a flavor state to interact with at least one graviton (3.2) which will depend on the graviton-neutrino scattering cross section. Naively, every mass eigenstate of the (relativistic) neutrino shares the same energy so P_G takes the same value.

If all neutrinos have interacted with at least one graviton, i.e. fixing $P_G = 1$, than the expression for the total $\nu_e \to \nu_e$ transition probability becomes

$$P^G_{\mathrm{ee}} = \cos^4\theta_{12}\cos^4\theta_{13} + \cos^4\theta_{13}\sin^4\theta_{12} + \sin^4\theta_{13} \quad (4.2)$$

where θ_{12}, θ_{13}, and θ_{23} are the standard neutrino vacuum mixing angles. Unlike for *propagation decoherence* this prediction is independent of (non-adiabatic) matter effects such as the earth MSW effect [20]. This basic formula is our main prediction for the "maximal decoherence" scenario valid for $P_G \simeq 1$. In the standard Large Mixing Angle (LMA) global fit with $\sin^2\theta_{13} = 0.025$, $\sin^2\theta_{12} = 0.31$, and $\sin^2\theta_{23} = 0.60$ [43], the value for the transition probability in this case is $P^G_{\mathrm{ee}} = 0.544$. The difference between the predictions for $P_G = 1$ and $P_G = 0$ is that for the $P_G = 0$ case the neutrino will not behave decoherently when traveling through the earth giving an approximately 3% change in the survival probability at 10 MeV. This can be seen in Fig. 3, here the simulation for neutrino propagation in matter and vacuum was based on [47].

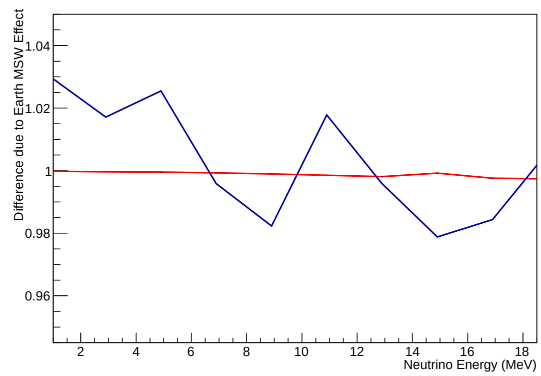

FIG. 3: Here is plotted the difference due to the Earth MSW effect in the observed number of electrons (assuming an initial flux that only contains electron neutrinos). Shown are two curves, blue for $P_G = 0$ (propagation decoherence case) and red for the $P_G = 1$ (quantum decoherence case). This calculation was done with a uniform distribution of electron neutrinos. Shown is an expected energy resolutions of 2 MeV and the expected MSW effect on the order of 3%. In the case of $P_G = 1$, the flavor composition of the neutrino flux will not be affected by the earth.

In general, the P_G value can be considered as an observable and extracted from the flavor composition data and further compared to theoretical calculations. Possible sources of neutrinos in extreme astrophysical environments include the aforementioned SMBH, but also SNe, GRB, AGN, and other galactic or extragalactic sources. Quantum gravity models should aim at predicting the P_G in these extreme environments so that favored models can be constrained by the neutrino flavor data.

The neutrino flux spectrum from astrophysical sources is still being modelled [40–42]. By comparing observed neutrino flavor composition for neutrinos passing through the earth to that of neutrinos which have not passed through the earth the flux can be divided out and a 3% effect (See Fig. 3) may be observed (for the above LMA global fit). This could be visible in flavor data at large statistics. However, for other fits larger differences may be possible possible.

This is our basic prediction for the quantum gravity-induced effect on the detected flavor composition in the "maximal quantum decoherence" scenario. Since numerically the effect is at the percentage level, one would certainly need to have a good understanding of all the other statistical and systematical uncertainties to a better than percent level for the predicted signal to be significant. Current generation neutrino observatories can observe ten thousand total events from nearby ($\sim$ 10 kpc) SNe, however, next generation neutrino observatories (such as Hyper-K) are needed to provide enough statistics to reduce systematical uncertainties [44]. Ad-

[4] Note, a very similar effect should take place in flavor oscillations in the neutral kaons K_l, K_s system as well.

ditionally, improvements are needed in current neutrino flavor reconstruction technologies, which can reconstruct the neutrino flavor to the level of $\sim 3\%$ [45]. It has been pointed out that sensitive neutrino detectors should be placed in North America (Sudbury), Japan (Hyper-K), Chile (ANDES), and Antarctica (Beyond DeepCore) to best observe SNe neutrinos and the effect that the Earth has on SNe neutrinos [48].

V. CONCLUSION

In conclusion, we have considered a quantum gravity process, the gravitational Bethe-Heitler scattering of a neutrino off a massive object, which can have a rather large cross section proportional to the mass squared of the classical source and enhanced at small angles. Due to gravitons interacting with a neutrino mass eigenstate only, opposite to weak bosons which interact with a flavor eigenstate only, the considered process is a measurement of the incoming neutrino at the quantum level, causing decoherence in a different manner than propagation decoherence or other sources of decoherence. This quantum decoherence affects astrophysical neutrino behavior. Namely, quantum decoherene can be considered as a specific quantum measurement of the neutrino propagating state, which changes the behaviour of the neutrino in the presence of a potential (such as the Earth MSW Effect), which can be observed in the neutrino flavor composition in an Earth based detector.

This enables the utilization of neutrinos traveling across the Galaxy as a source of information about the graviton-induced interactions they might have experienced on their journey to Earth. Specifically, the measured probability to find a given flavor component in the neutrino flux coming from a vicinity of a black hole or another compact massive object will be different from the corresponding probability measured from a source of neutrinos where the neutrinos never pass near a massive system. In the case where no astrophysical neutrinos can be identified which have not interacted with a gravitational potential, the flavor composition can be compared to the expectation for the earth MSW effect which can be determined using reactor, atmospheric, and accelerator neutrinos. We have explicitly demonstrated that the maximal difference corresponding to an assumption that all of the detected neutrinos have experienced an interaction with a graviton, i.e. $P_G = 1$, is at the percentage level and can be measurable at high statistics. This would provide a first measurement of quantum gravity. Further discrimination of quantum gravity models would require more statistics and detailed calculations using these models.

Thus, the probability for a neutrino state to interact with at least one graviton, P_G, is considered to be a new observable containing information about the quantum gravity scattering process. An estimate of the P_G value from neutrino flavor composition data with good angular resolution would provide an important experimental test for quantum gravity models. This is the major proposal we make in our paper. A realistic theoretical estimate for P_G depends on many factors and is not well-constrained yet.

Having all that in mind, as a natural starting point in this very first and short paper we would like to present the basic concept/idea of quantum decoherence due to large angle neutrino-graviton interactions (gravistrahlung) in strong gravitational fields and its possible effect on neutrino flavor observables. In this paper we report on our preliminary study of such a graviton-induced effect on neutrino oscillations and motivate future studies in this direction. We will improve our simulation with fluxes and the astrophysical medium in a future study. The possibility that P_G is not zero in the vicinity of the Sun should be considered as well. Explicitly the length and energy dependence of neutrino flavor oscillation will depend on the relative strengths of the graviton-neutrino interactions, the matter properties, and the vacuum oscillations. Inclusion of these possibilities in the global neutrino oscillation parameter fit will be left for a later paper. Additionally, extragalactic neutrinos should be considered with additional care as GBH scattering of the neutrino off the diffuse dark matter Halo may play a role. Finally, the issue of coherent production of neutrinos is not considered in this study and should be considered in a future study.

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