

# Hard Pion Chiral Perturbation Theory: What is it and is it relevant for $\eta'$ decays?<sup>1</sup>

Johan Bijnens

Department of Astronomy and Theoretical Physics,  
Lund University, Sölvegatan 14A, SE 223 62 Lund, Sweden

## Abstract

In this talk I give a short introduction to hard pion Chiral Perturbation Theory and an overview of the available applications  $K \rightarrow \pi\pi$ ,  $B, D \rightarrow D, \pi, K, \eta$  semileptonic decays and  $\chi_{c0,2} \rightarrow \pi\pi, KK$ . It is pointed out that the results for the semileptonic decays obey the LEET relation between  $f_+$  and  $f_-$ .

---

<sup>1</sup>Talk presented at the PrimeNet meeting, Meson Physics in Low-Energy QCD, September 26 - 28, 2011, Forschungszentrum Jülich, Germany.

# Hard Pion Chiral Perturbation Theory: What is it and is it relevant for $\eta'$ decays?

JOHAN BIJNENS<sup>†</sup>

## 1 Introduction

In this talk I will try to convince you that we can give predictions from chiral symmetry also for cases where not all pions are soft. This is something I called hard pion Chiral Perturbation Theory (HPChPT) and there have been a few recent papers using this [1, 2, 3, 4, 5].

I will first give a short introduction to effective field theory (EFT) and remind you of the underlying principles of Chiral Perturbation Theory (ChPT). I will remind you of the fact that in ChPT with baryons and other heavy particles a power-counting has been achieved by consistently absorbing the heavy mass dependence into the low-energy-constants (LECs).

The arguments will then be generalized to the case of processes with high energy or hard pions. The arguments also apply to cases where we can treat the strange quark mass as small as well.

After that I will show applications to  $K \rightarrow \pi\pi$ , to semileptonic decays of pseudo-scalar mesons or to more general vector form-factors and to charmonium decays to two pseudo-scalars.

Unfortunately, there seem to be no  $\eta'$  decays where the present method are applicable.

## 2 Effective field theory and ChPT

The underlying idea of EFT is a general theme in science, restrict yourself to the relevant degrees of freedom. So, in cases where there exists an energy or mass gap we keep only the lower degrees of freedom. Lorentz-invariance and quantum mechanics imply that we are restricted to a field theory but we should build the most general one with our chosen degrees of freedom. We have no predictability left since the most general Lagrangian will have an infinite number of parameters. This can be cured if we find an ordering principle, power-counting, for the importance of terms.

ChPT is “exploring the consequences of the chiral symmetry of QCD and its spontaneous breaking using effective field theory techniques” and was introduced as an EFT in [6, 7]. The degrees of freedom are the Goldstone bosons from the spontaneous breakdown of the global chiral  $SU(n)_L \times SU(n)_R$  to the diagonal vector subgroup  $SU(n)_V$ . That the Goldstone boson interactions vanish at zero momentum allows to construct a consistent power-counting [6].

The basic form of ChPT has since been extended to baryons, mesons and baryons containing a heavy quark, vector mesons, structure functions and related quantities as well as beyond the pure strong interaction by including weak and electromagnetic internal interactions. Many models of alternative Higgs sectors also use the same technology.

## 3 Power-counting and one large scale

In purely mesonic ChPT the power-counting is essentially dimensional counting and this works since all the lines in all diagrams have “small” momenta. Already when discussing baryons, this lead to problems because now there is a large scale, the baryon mass. However, by setting the baryon momentum  $p_B = M_B v + k$  with  $v$  the baryon four-velocity, a consistent power-counting can be achieved. This works “obviously” since the heavy line goes through the entire diagram and all momenta apart from  $M_B v$  are soft as indicated by the thick line in Fig. 1(a). The same arguments apply to ChPT for mesons containing a heavy quark. It works because the “soft” stuff can be expanded in “soft/ $M_B$ ” and the remaining  $M_B$  dependence can be absorbed in LECs. For vector meson ChPT there is a problem since they can decay. A typical diagram is shown in

---

<sup>†</sup>Department of Astronomy and Theoretical Physics, Lund University, Sölvegatan 14A, SE 223 62 Lund, Sweden, [bijnens@thep.lu.se](mailto:bijnens@thep.lu.se)

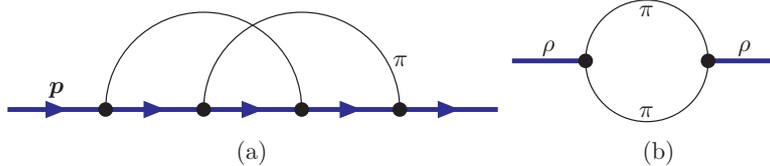


Figure 1: (a) A typical baryon ChPT diagram with the baryon going through the entire diagram. (b) An example of a diagram in vector meson ChPT with no continuous vector meson line.

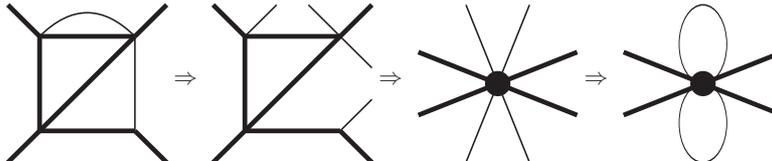


Figure 2: The process of cutting the soft lines and reproducing the non-analytic dependence by the diagram on the right. The hard lines are thick, soft lines are shown thin.

Fig. 1(b). However it was argued that the non-analytic dependence on the light quark mass could still be obtained, see the discussions in [8]. Again, the underlying idea is that the large  $M_V$  allows to expand in “soft/ $M_V$ ” and the remaining  $M_V$  dependence is absorbed in the LECs.

## 4 Several large scales or HPChPT

In [1] the authors applied heavy Kaon ChPT to  $K_{\ell 3}$  at the endpoint, i.e. the pion is soft, this works as in usual ChPT. They also applied it to the region for small  $q^2$  where the pion has a large momentum and gave arguments based on partial integrations why this would give a correct chiral logarithm. The argument was generalized in [2, 3, 4, 5]. The underlying idea is similar to the previous section. The “heavy/fast/hard” dependence on the soft stuff can always be expanded and the remaining dependence goes into the LECs. That this might be possible follows also from current algebra. Non-analyticities in the light masses come from the soft lines and soft pion couplings are restricted by current algebra via  $\lim_{q \rightarrow 0} \langle \pi^k(q) \alpha | O | \beta \rangle = -\frac{i}{F_\pi} \langle \alpha | [Q_5^k, O] | \beta \rangle$ . Nothing prevents hard pions to be in the states  $\alpha$  or  $\beta$ , so by heavily using current algebra one can get the light quark mass non-analytic dependence

A field theoretic argument is: (1) Take a diagram with a given external and internal momentum configuration. (2) Identify the soft lines and cut them. (3) The resulting part is analytic in the soft stuff, so it can be described by an effective Lagrangian with coupling constants dependent on the external given momenta (Weinberg’s folklore theorem [6]). (4) The non-analytic dependence on the soft stuff is reproduced by loops in the latter Lagrangian. The process is depicted in Fig. 2.

The remaining problem is that we have no power-counting. The Lagrangian that reproduces the non-analyticities is fully general. In the HPChPT papers it was shown that for the processes at hand, all higher order terms can be reduced to those with the fewest derivatives thus allowing the light quark mass chiral logarithm to be predicted. The underlying arguments were tested by comparing to a two-loop calculation[4] and by explicitly keeping some higher order terms [2, 3, 4, 5].

## 5 Applications

We have applied the method to  $K \rightarrow \pi\pi$  decays [2] where we treat the Kaon as heavy and look for the dependence on the pion mass  $M^2$ . The result is, up to linear in  $M^2$  and higher order:

$$A_0^{NLO}/A_0^{LO} = 1 + (3/8)\hat{A}, \quad A_2^{NLO}/A_2^{LO} = 1 + (15/8)\hat{A}, \quad \hat{A} = -M^2 \ln(M^2/\mu^2)/(16\pi^2 F^2).$$

The scalar and vector form-factors of the pion are known to two-loops in ChPT [9]. HPChPT predicts at large  $t$  [4] with  $F_V(t, 0)$  and  $F_S(t, 0)$ , the form-factors at large  $t$  in the chiral limit, completely free:

$$F_V(t, M^2) = F_V(t, 0) (1 + \hat{A}), \quad F_S(t, M^2) = F_S(t, 0) (1 + (5/2)\hat{A}).$$

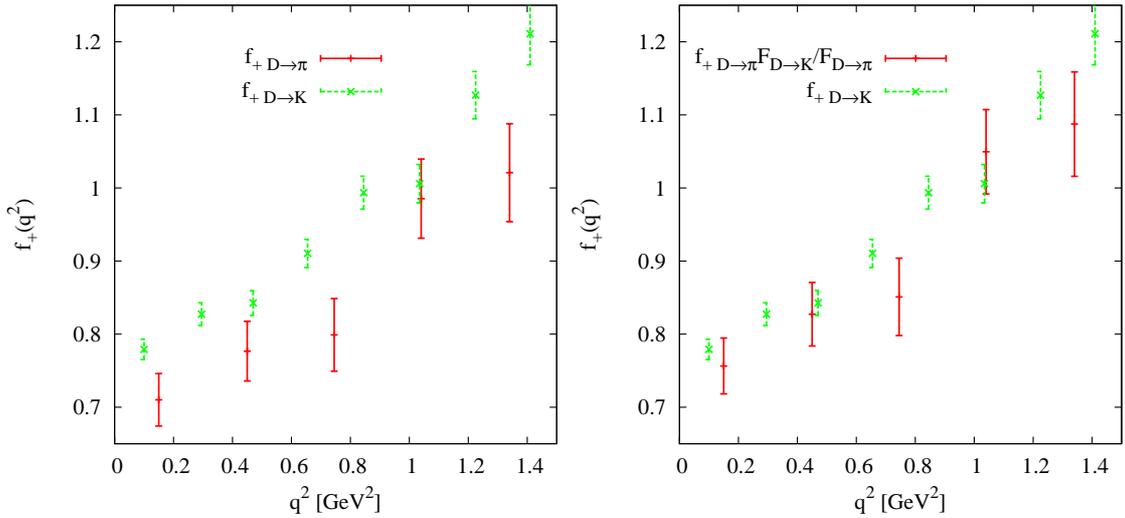


Figure 3: The CLEO data on  $D \rightarrow \pi$  and  $D \rightarrow K$ . Form-factors as measured to the the left and corrected with the chiral logarithms to the right. Note the improved agreement.

The full two-loop result expanded for large  $t$  should have this form and it does with for e.g.  $F_V$

$$F_V(t, 0) = 1 + (t/(16\pi^2 F^2)) (5/18 - 16\pi^2 l_6^r + i\pi/6 - (1/6) \ln(t/\mu^2)) .$$

The first application was semileptonic form-factors in  $K_{\ell 3}$ . We extended this to  $B, D \rightarrow D, \pi, K, \eta$ -decays in [3, 4]. This allowed to test our results experimentally. The form factor  $f_+(t)$  as measured by CLEO [10] in  $D \rightarrow \pi$  and  $D \rightarrow K$  decays are different by about the amount expected from the chiral logarithms as shown in Fig. 3. One puzzling observation [4] was that in the limit of a hard pseudo-scalar in the final state the correction was always the same for  $f_+$  and  $f_-$ . This is in fact due to the LEET relation [11] which shows that there is only one form-factor in this limit and it is nice to see that our calculation respects this without it being used as input.

A last application was to  $\chi_{c0,2} \rightarrow \pi, KK, \eta\eta$  [5]. Here it was found that there were no chiral logarithms to the order considered. Comparing with the known experimental results indeed shows  $SU(3)$  breaking to be somewhat smaller than in e.g.  $F_K/F_\pi$ . Details can be found in [5].

## Acknowledgments

This work is supported in part by the EU, HadronPhysics2 Grant Agreement n. 227431, and the Swedish Research Council, grants 621-2008-4074 and 621-2010-3326.

## References

- [1] J. M. Flynn and C. T. Sachrajda, *Nucl. Phys. B* **812** (2009) 64 [arXiv:0809.1229 [hep-ph]].
- [2] J. Bijnens and A. Celis, *Phys. Lett. B* **680** (2009) 466 [arXiv:0906.0302 [hep-ph]].
- [3] J. Bijnens and I. Jemos, *Nucl. Phys. B* **840** (2010) 54 [arXiv:1006.1197 [hep-ph]].
- [4] J. Bijnens, I. Jemos, *Nucl. Phys. B* **846** (2011) 145 [arXiv:1011.6531 [hep-ph]].
- [5] J. Bijnens, I. Jemos, arXiv:1109.5033 [hep-ph], to be published in *Eur. Phys. J. A*.
- [6] S. Weinberg, *Physica* **A96** (1979) 327.
- [7] J. Gasser and H. Leutwyler, *Annals Phys.* **158** (1984) 142.
- [8] J. Bijnens, P. Gosdzinsky, P. Talavera, *JHEP* **9801** (1998) 014 [hep-ph/9708232].
- [9] J. Bijnens, G. Colangelo and P. Talavera, *JHEP* **9805** (1998) 014 [arXiv:hep-ph/9805389].
- [10] J. Y. Ge *et al.* [CLEO Coll.], *Phys. Rev. D* **79** (2009) 052010 [arXiv:0810.3878 [hep-ex]].
- [11] J. Charles *et al.*, *Phys. Rev. D* **60** (1999) 014001 [hep-ph/9812358].