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Abstract. We apply hard pion Chiral perturbation Theory to charmonium decays to $\pi\pi$, KK and $\eta\eta$. We first discuss why we expect to be able to provide results for the chiral logarithms in χ_{c0} and χ_{c2} decays to two pseudoscalars while for the decays from J/ψ, $\psi(nS)$ and χ_{c1} no simple prediction is possible. The leading chiral logarithm turns out to be absent for $\chi_{c0}, \chi_{c2} \to PP$. This result is true for all fully chiral singlet states of spin zero and two.

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1 Introduction

The study of charmonium decays has seen in the last years a remarkable progress and a renewed interest. New experimental measurements, mainly coming from Belle, BES, CLEO and E835 [1] have improved existing data on several exclusive hadronic decay channels leading to better determinations of the charmonium branching fractions. On the theory side several heavy quarkonium decay observables can be studied using effective field theories of QCD as Non-Relativistic QCD (NRQCD) and its extensions [2]. The latter is also a recent review in general of quarkonium physics.

In this letter we focus on the OZI suppressed decays of χ_{c0}, χ_{c2} into two light pseudoscalar mesons, $\pi\pi$, KK, $\eta\eta$ ($\chi_{c0}, \chi_{c2} \to PP$) and we present the arguments why we have no similar results for $J/\psi, \psi(nS), \chi_{c1}$ decays to the same final state. We explore the possibility to calculate the contributions to the decay amplitudes due to the so-called chiral logarithms. These have the form $m^2 \log(m^2)$, where m is the mass of a light meson[1] and these are potentially the largest contribution from the light quark masses. Surprisingly, we find that for $\chi_{c0}, \chi_{c2} \to PP$ these contributions vanish. This is not necessarily the case for scalar quantities at high q^2 as we show with the case of the scalar formfactor.

Terms of the type $m^2 \log(m^2)$ and other non-analytic dependence on the input parameters can be produced by soft meson loops [3]. The modern version of this method is Chiral Perturbation Theory (ChPT) [4,5,6], see also the lectures [7,8]. All of these applications require the octet of pseudoscalar mesons to have soft momenta which is not the case in charmonium decays. However, it has been argued that even for cases with pseudoscalar mesons at large momenta there are still predictions possible. This was first argued for $K_{\ell 3}$ decays in [9] and later argued to be more general and applied to $K \to \pi\pi$ [10] and $B \to D, \pi, K, \eta$ vector formfactors and the pion and kaon electromagnetic formfactors [11,12]. The underlying arguments were tested at two-loop level for the pion vector and scalar formfactor in [12]. We refer to this method as hard pion ChPT (HPChPT).

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[1] In the remainder of this letter m stands for m_π, m_K and m_η.

The underlying argument in HPChPT is that the chiral logarithms are coming from the soft part of all diagrams when we envisage a theory with hadrons with an effective Lagrangian to all orders. This should be able to describe all effects according to Weinberg's folklore theorem [4]. The hard part of these diagrams around a particular kinematical situation are describable by a tree level Lagrangian that is analytic in the soft physics. Loops using the latter Lagrangian thus reproduce the nonanalytic dependence of the underlying loop diagrams in the full theory. The second part is then proving that we use a sufficiently complete tree level Lagrangian. This has been done in all the previous works [9,10, 11,12] by showing that higher order operators have the same matrix element as the lowest order operator up to terms analytic in the light pseudoscalar masses. This allowed the determination of the chiral logarithms for the processes mentioned earlier. The same arguments are applicable to the present decays.

In Section 2 we define the notation and give the lowest order Lagrangians involving charmonium fields. Section 3 describes the relevant loop calculations. We note that our zero result for the chiral logarithm is in fact valid for all fully chiral singlet states of spin 0 and 2. One consequence of our result is that light quark mass corrections in the $\chi_{c0}, \chi_{c2} \to PP$ decays are not enhanced. We compare this statement with the available experimental results in Section 4.

2 Formalism

First we shortly summarize the formalism of ChPT in its three-flavour version [6]. Introductions to ChPT can be found in [7,8]. Hereafter we will use the same notation as in [13]. In ChPT it is assumed that the spontaneous symmetry breaking of chiral symmetry takes place. In group theory language it has the pattern $SU(3)_R \times SU(3)_L / SU(3)_R \to SU(3)_V$. The oscillations around the vacuum are described by the field $u \in SU(3)$

$$u = \exp\left(\frac{i}{\sqrt{2}F_0}\phi\right), \tag{1}$$

where ϕ is an hermitian matrix containing the pseudo Goldstone bosons, i.e. the light pseudoscalar mesons

$$\phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix} \tag{2}$$

The lowest order Lagrangian describing the strong interactions of the mesons must satisfy the same symmetries of QCD and reads

$$\mathcal{L}^{(2)}_{\pi\pi} = \frac{F_0^2}{4}\left(\langle u_\mu u^\mu\rangle + \langle \chi_+\rangle\right), \tag{3}$$

with

$$\begin{aligned} u_\mu &= i\{u^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - il_\mu)u^\dagger\}, \\ \chi_\pm &= u^\dagger \chi u^\dagger \pm u\chi^\dagger u, \\ \chi &= 2B_0(s + ip). \end{aligned}$$

The fields s, p, $l_\mu = v_\mu - a_\mu$ and $r_\mu = v_\mu + a_\mu$ are the standard external scalar, pseudoscalar, left- and right-handed vector fields introduced by Gasser and Leutwyler [5,6]. s includes the light quark mass matrix.

The field u and u_μ transform under a chiral transformation $g_L \times g_R \in SU(3)_L \times SU(3)_R$ as

$$u \longrightarrow g_R u h^\dagger = h u g_L^\dagger, \qquad u_\mu \longrightarrow h u_\mu h^\dagger. \tag{4}$$

In (4) h depends on u, g_L and g_R and is the so called compensator field. The notation $\langle X\rangle$ stands for trace over up, down, strange.

As anticipated in Section 1 the use of ChPT is limited to those processes where the pseudoscalar mesons are not very energetic. Indeed the power counting is based on an expansion in m^2/Λ^2 and in p^2/Λ^2, where Λ is around 1 GeV and is the scale up to which ChPT is believed to work. However sometimes the dependence on the light pseudoscalar masses outside such energy regime is needed. In such cases the extension of ChPT to hard pion ChPT is necessary. For an extended explanation of the arguments leading to hard pion ChPT we refer the interested reader to [9,10,11]. Here we only stress the key features of such extension. When calculating the amplitudes of processes involving hard pseudoscalar mesons the Feynman diagrams contain both hard and soft lines. The application of hard pion ChPT involves a separation of the first ones from the latter. The hard lines get contracted into a vertex of another effective Lagrangian. The couplings appearing there must therefore depend on the hard quantities. The soft lines instead encode the dependence on the soft parts, like the light masses of the pseudoscalar mesons, and are used to calculate the chiral logarithms. Such calculation is done using a Lagrangian built up in the same fashion as the one of standard ChPT since the hard parts must satisfy the constraints from chiral symmetry. However one must keep in mind that now the expansion that survives is in the small parameter m^2/Λ^2, while the one in momenta must be dropped. This means that adding extra derivatives to the Lagrangian is in principle allowed since they are not suppressed by extra powers of momentum. Fortunately it often turns out that the matrix elements of operators containing these extra derivatives are proportional to those of the lowest order operator up to terms either analytic in m^2 or higher order. The coefficient of $m^2 \log m^2$ is thus predictable. It is important then that this happens in the case under study as well.

The arguments in [9,10] are for a heavy to two light decay and the exact same arguments apply to the present case of charmonium to two light pseudoscalars.

Charmonium states are full chiral singlets, i.e. they do not transform at all under the chiral $SU(3)_L \times SU(3)_R$. Their kinetic terms are thus the standard kinetic terms for fields of the given spin without couplings to pions. The results obtained here are thus valid for all full chiral singlets.

We only study decays to two light pseudoscalar mesons, in principle we could study decays to three light mesons as well but the extension is not totally straightforward since the HPChPT should be applied to each kinematic configuration separately. Also for more particles the operator structure is typically much more involved.

We could as well study $J/\psi, \psi(nS)$ and χ_{c1} decays into two light pseudoscalar mesons. Again the reason why we have not done this is that it would be impossible to compare $J/\psi \to \pi\pi$ with respect to $J/\psi \to KK$ and similar for the others. These two decays are caused by different operators. The first one violates G-parity,[2] it thus proceeds either through the electromagnetic interaction or the quark mass difference $m_d - m_u$. The decays to a pair of kaons have the same problem but instead with U or V-spin rather than isospin. These decays thus proceed electromagnetically or through the mass differences $m_s - m_u$ or $m_s - m_d$. Notice that this is also the reason why the decay of J/ψ into $\pi\pi$ is suppressed compared to the one into KK. It follows that a simple comparison, as the one done in Section 3 for χ_{c0}, χ_{c2}, is not possible.

[2] Or more general, an $L = 1$ state of two pions is in an $I = 1$ state.

We describe the χ_{c0} state with a chiral singlet, scalar field χ_0 and the χ_{c2} with a chiral singlet symmetric traceless tensor field $\chi_{2\mu\nu}$ satisfying $\chi_{2\mu\nu} = \chi_{2\nu\mu}$, $\eta^{\mu\nu}\chi_{2\mu\nu} = 0$ and on-shell $p_\chi^\mu \chi_{2\mu\nu} = 0$.

We remind the reader that we are interested in calculating the dependence of the amplitudes on m^2. To predict this we will stop at the chiral logarithm level so we only calculate contributions like $m^2 \log m^2$. Terms of order m^2 are instead of higher order and thus neglected. This means we can neglect all effects of $\chi_\pm$ except for the contribution to the light meson masses and their effects in loop diagrams. In particular, effects of the light quark masses on the charmonium mass are linear in the light quark masses or higher order.

It turns out that there is only one lowest order operator for each case

$$\mathcal{L}_{\chi_c} = E_1 F_0^2\, \chi_0 \, \langle u^\mu u_\mu \rangle + E_2 F_0^2\, \chi_2^{\mu\nu} \, \langle u_\mu u_\nu \rangle \,. \quad (5)$$

We have added a factor of F_0^2, the chiral limit pion decay constant, to have the lowest order independent of F_0. We have actually added a few more terms for the scalar case as an explicit check on the HPChPT arguments. These are

$$\mathcal{L}^E_{\chi_c} = E_3 F_0^2\, \chi_0^2 \, \langle u_\mu u^\mu \rangle + E_4 F_0^2\, \chi_0 \, \langle \nabla^\mu u^\nu \nabla_\mu u_\nu \rangle \,. \quad (6)$$

3 The Calculation

The tree-level diagrams and the loop-diagrams that contribute to $\chi_{c0}, \chi_{c2} \to PP$ are shown in Figure 1. To these diagrams we also need to add the wavefunction renormalization for the pseudoscalar meson. The wave function renormalization for the charmonium states has no contributions of order $m^2 \log m^2$. The tree level result from the diagram in Figure 1(1) reads

$$\begin{aligned} iA(\chi_{c0} \to PP) &= -(m_\chi^2 - 2m_P^2) \times \\ &\qquad \left(2E_1 + E_4(m_\chi^2 - 2m_P^2)\right) \,, \\ iA(\chi_{c2} \to PP) &= -4E_2 p_1^\mu p_2^\nu T_{\chi\mu\nu} \,. \end{aligned} \quad (7)$$

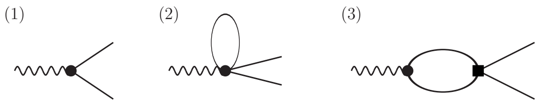

Fig. 1. The tree-level (1) and the loop-diagrams (2,3) contributing to the decays $\chi_{c0}, \chi_{c2} \to PP$ up to the order $m^2 \log m^2$. The wiggly lines correspond to a χ_c while the continuous line are π, K or η. A round vertex correspond to an interaction from $\mathcal{L}_{\chi_c}$ while a box is a vertex from $\mathcal{L}_\pi$. Notice that the diagrams of type (3) do not contribute with chiral logarithms to the amplitudes.

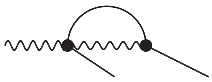

Fig. 2. An example diagram involving a propagator of the heavy particle in the loop. These do not contribute to the chiral logarithms of order $m^2 \log m^2$.

The amplitude is the same for the final states $\pi^+\pi^-, K^+K^-, K^0\overline{K^0}$ and $\eta\eta$ when the meson mass m_P is chosen to be the appropriate one. $T_{\chi\mu\nu}$ is the polarization vector of χ_{c2} and p_1 and p_2 are the fourmomenta of the two mesons in the final state. Here we also see that the example of a higher derivative term containing E_4 indeed gives a result proportional to the lowest term up to corrections of order m^2.

When we go over the powercounting arguments, only soft pseudoscalar propagators where no derivatives act on the pseudoscalar fields associated with that propagator can give contributions of order $m^2 \log m^2$. As a consequence only pseudoscalar meson wave function renormalization and the diagram in Figure 1(2) can contribute terms of order $m^2 \log m^2$ and we find that these two contributions exactly cancel for all decays considered. We indeed find that the diagram in Figure 1(3) does not contribute terms of order $m^2 \log m^2$.

In baryon ChPT, and other extensions with heavy particles, one can also get logarithmic contributions from diagrams involving propagators of the heavy particle. The powercounting argument together with the fact that all interactions terms have derivatives show that these should not be present here. We have explicitly confirmed this with the diagram of Figure 2. Putting in the vertices with E_1 and E_3 we indeed find no contributions of order $m^2 \log m^2$.

We conclude this part with a final remark. The above discussion is valid assuming that other charmonium states are far enough in mass, so that such states cannot appear as virtual particles in the loops, i.e. their contributions are sufficiently hard that they can be described by the same tree level Lagrangian (5).

We were somewhat surprised to find that there were no chiral logarithms of leading order in these decays, and more, since the only assumption that goes in is the HPChPT and the fact that we have a chiral singlet field. The same arguments go through for the energy momentum tensor $T^{\mu\nu}$ which is a spin two chiral singlet. The expressions for the matrix element of the energy momentum tensor between pseudoscalar states $\langle P|T^{\mu\nu}|P\rangle$ are known to one-loop order in ChPT [14]. Our result should predict the $m^2 \log m^2$ parts of their result in the limit $q^2 \gg m^2$. Expanding (29) and (33) in [14] in m^2/q^2 we indeed find that there are no logarithms of order $m^2 \log m^2$ appearing there.

In earlier work we have found many instances of nonzero chiral logarithms in HPChPT. Most of these were for the vector formfactor but we did find a nonzero result for the pion scalar formfactor $\langle \pi|\bar{u}u + \bar{d}d|\pi\rangle$ at large q^2 in two-flavour HPChPT. Note that $\bar{u}u + \bar{d}d$ is a singlet under $SU(2)_V$ but not under the full chiral $SU(2)_L \times SU(2)_R$ so the calculations given above do not restrict the chiral logarithms at large q^2 here.

4 Comparison with experiment for $\chi_{c0}, \chi_{c2} \to PP$

The fact that the chiral logarithms of order $m^2 \log m^2$ vanish means that the leading term in the expansion of m^2 of the amplitudes vanish. One could thus expect that the $SU(3)_V$ breaking in $\chi_{c0}, \chi_{c2} \to PP$ should be somewhat smaller than the "usual" 20% as e.g. in F_K/F_π or m_Λ/m_p. Terms of order m^2 are however not predicted, so a very clear statement that $SU(3)_V$ breaking effects are small is not possible.

Let us however see how well the measured amplitudes live up to the "small $SU(3)_V$ breaking effects." We take the input data from [1]. These are given in Table 1. It should be taken into account that the $\pi\pi$ is the sum of $\pi^+\pi^-$ and $\pi^0\pi^0$ final state. Isospin predicts that the $\pi^0\pi^0$

	χ_{c0}		χ_{c2}	
Mass	3414.75 ± 0.31 MeV		3556.20 ± 0.09 MeV	
Width	10.4 ± 0.6 MeV		1.97 ± 0.11 MeV	
Final state	10^3 BR	$10^{10}\, G_0[\text{MeV}^{-5/2}]$	10^3 BR	$10^{10} G_2[\text{MeV}^{-5/2}]$
$\pi\pi$	8.5 ± 0.4	3.15 ± 0.07	2.42 ± 0.13	3.04 ± 0.08
K^+K^-	6.06 ± 0.35	3.45 ± 0.10	1.09 ± 0.08	2.74 ± 0.10
$K^0_S K^0_S$	3.15 ± 0.18	3.52 ± 0.10	0.58 ± 0.05	2.83 ± 0.12
$\eta\eta$	3.03 ± 0.21	2.48 ± 0.09	0.59 ± 0.05	2.06 ± 0.09
$\eta'\eta'$	2.02 ± 0.22	2.43 ± 0.13	< 0.11	< 1.2

Table 1. The Experimental results for the decays $\chi_{c0}, \chi_{c2} \to PP$ and the resulting factors corrected for the known m^2 effects.

is half the $\pi^+\pi^-$ state and the data that go into the PDG average are compatible with this. We thus need to multiply it by 2/3 to get the $\pi^+\pi^-$ result. Similarly, the $K^0_S K^0_S$ final state is half of the $K^0\overline{K^0}$ final state. Here we need to multiply by two to obtain the full final state.

One question is how we deal with factors of m_P^2 that we know are present. We include the phase space correction and note that for χ_{c0} the amplitude always contains $p_1 \cdot p_2 = (m_\chi^2 - 2m_P^2)/2$. The phase space contains the factor $|\boldsymbol{p}_1| = \sqrt{m_\chi^2 - 4m_P^2}/2$. For the decays of the scalar we thus define

$$G_0 = \sqrt{BR/|\boldsymbol{p}_1|}/(p_1.p_2)\,. \tag{8}$$

The same question arises for the χ_{c2}. Here the amplitude must always contain $\chi_2^{\mu\nu} p_{1\mu} p_{2\nu}$. The field $\chi_2^{\mu\nu}$ must be replaced by its polarization tensor $T_\chi^{\mu\nu}$ and the amplitude squared and averaged over. The formula to perform such a sum is

$$\sum_{\text{polarizations}} T^{\mu\nu} T^{*\alpha\beta} = \frac{1}{2}\left(K^{\mu\alpha}K^{\nu\beta} + K^{\mu\beta}K^{\nu\alpha} - \frac{2}{3} K^{\mu\nu}K^{\alpha\beta} \right) \tag{9}$$

where $K^{\mu\nu} = -g^{\mu\nu} + q^\mu q^\nu / m_\chi^2$ and q^μ is the four-momentum of the χ_{c2} particle. The contribution thus always contains a factor

$$\frac{1}{5}\sum_{pol} T_\chi^{\mu\nu} p_{1\mu} p_{2\nu} T_\chi^{*\alpha\beta} p_{1\alpha} p_{2\beta} = \frac{1}{30}\left(m_\chi^2 - 4m_P^2\right)^2 . \tag{10}$$

Alternatively we can choose the final state configuration with $p_1 = (E_P, 0, 0, |\boldsymbol{p}_1|)$ and $p_2 = (E_P, 0, 0, -|\boldsymbol{p}_1|)$ and choose an explicit set of five orthogonal polarization tensors satisfying $p_\chi^\mu T_{\chi\mu\nu} = 0$, $T^\mu_{\chi\mu} = 0$, $T_{\chi\mu\nu} = T_{\chi\nu\mu}$ and $T^{(a)}_{\chi\mu\nu} T_\chi^{*(b)\mu\nu} = \delta^{ab}$ which shows that the amplitude squared always contains a $|\boldsymbol{p}_1|^4$. We thus define a normalized factor also for the χ_{c2} decays via

$$G_2 = \sqrt{BR/|\boldsymbol{p}_1|}/|\boldsymbol{p}_1|^2\,. \tag{11}$$

Looking at the columns G_0 and G_2 for the decays to pions and kaons we see indeed that $SU(3)_V$ breaking is somewhat smaller than usual, about 10% for both χ_{c0} and χ_{c2} decays. For the decays to $\eta\eta$ we get a decent agreement in both cases but the[3] $\eta'\eta'$ is not so good for the χ_{c2} decay.

5 Conclusions

In this letter we have calculated the chiral logaritms for the decays χ_{c0}, χ_{c2} to two light pseudoscalars. We have found that they vanish and that this is a general result for all such decays of heavy chiral singlet states of spin 0 and 2. We checked our result against the known result for the energy momentum tensor in ChPT and showed using the example of the scalar formfactors that the chiral logarithms do not vanish at high q^2 in all cases.

Our result implies that the $SU(3)_V$ corrections are expected to be "small" in these decays and we have compared our results with the available experimental data and find that the corrections are reasonably small.

[3] The arguments given in this paper are only applicable to the η' in the large N_c limit where this state also becomes a pseudo Goldstone boson.

Acknowledgments

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