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Integral equation for spin dependent unintegrated parton distributions incorporating double $ln^2(1/x)$ effects at low x

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Abstract

In this paper we derive an integral equation for the evolution of unintegrated (longitudinally) polarized quark and gluon parton distributions. The conventional CCFM framework is modified at small x in order to incorporate the QCD expectations concerning the double $\ln^2(1/x)$ resummation at low x for the integrated distributions. Complete Altarelli-Parisi splitting functions are included, that makes the formalism compatible with the LO Altarelli-Parisi evolution at large and moderately small values of x. The obtained modified polarized CCFM equation is shown to be partially diagonalized by the Fourier-Bessel transformation. Results of the numerical solution for this modifed polarized CCFM equation for the non-singlet quark distributions are presented.

1 Introduction

The basic, universal quantities which describe the inclusive cross-sections of hard processes within the QCD improved parton model are the scale dependent parton distributions. These distributions depend upon the longitudinal momentum fraction x and the hard scale Q^2 and correspond to the integrals over transverse momentum $k_\perp$ of the so called *unintegrated* distributions describing the (scale dependent) x and $k_\perp$ parton distributions. The unintegrated distributions are needed in the description of less inclusive quantities which are sensitive to the transverse momenta of the partons [1] - [5].

The unintegrated parton distributions are described in perturbative QCD by the Catani, Ciafaloni, Fiorani, Marchesini (CCFM) equation [6, 7, 8] which is based on color coherence which implies angular ordering. It embodies in a unified way the (LO) Altarelli-Parisi evolution at large and moderately small values of x with the BFKL dynamics at small x.

The CCFM equation which was originally formulated for unpolarized parton densities has recently been generalized to spin dependent unintegrated parton distributions [9]. Novel feature of the spin dependent case is its different structure at small x and in particular the absence of the non-eikonal (or non-Sudakov) form-factors. We include here also the (spin dependent) quark distributions, leading first to a system of CCFM equations for unintegrated spin dependent quark and gluon distributions. We shall then extend the analysis of Ref. [9] along the following lines:

1. We shall incorporate theoretical QCD expectations concerning the double logarithmic small x effects. We observe in particular that both the angular ordering constraint and the kernels of the corresponding system of the CCFM equations have to be appropriately modified.

2. We shall include complete splitting functions $P_{ab}(z)$ and not only their singular and finite parts in the limit $z \to 1$ and $z \to 0$ respectively.

3. We shall explore the relative simplicity of the system of the modified polarized CCFM equations due to absence of non-eikonal form-factors and utilize the transverse coordinate representation of those equations. The transverse coordinate $b_\perp$ is related to the transverse momentum of the partons through the Fourier-Bessel transformation and the system of the CCFM equation can be partially diagonalized by this transformation.

The content of our paper is as follows: in the next section we recall the original formulation of the CCFM equation developed in [9] and include the polarized quark distributions, while in sections 3 and 4 we formulate the modifications of the

CCFM framework, which will incorporate the QCD expectations concerning the double $\ln^2(1/x)$ resummation at low x and the complete Altarelli-Parisi evolution of integrated densities in moderately small and large values of x. We observe that the modifed CCFM equations, including their extension discussed in sections 4 and 5 can be partially diagonalized by the Fourier-Bessel transformation. Section 5 is devoted to the numerical analysis of the modified CCFM equations and for simplicity we limit ourselves to the non-singlet quark distributions. Finally, in Section 6 we summarize our main results and give our conclusions.

2 The original CCFM formulation for longitudinally polarized unintegrated parton distributions

Along the lines of [6][7][8] a version of the CCFM equation has been derived for the longitudinally polarized gluon distribution [9]. For the corresponding expressions including quarks one has to take into consideration that the soft emission occurs predominantly from a ladder of gluons because of their larger spin quantum number. Therefore, quarks appear only as initial states playing automatically the role of the hardest emission which enters in the Altarelli Parisi splitting function, see Fig. 1. Therefore, the arguments presented in [9] are valid in the case of quarks as well. The polarization does not enter into the soft emission. Consequently, again a non-eikonal form factor is absent and the eikonal form factor is identical to the one in the unpolarized case. Furthermore, in its original formulation the CCFM equation is an evolution equation valid only in the limits $x \to 0$ and $x \to 1$, and therefore, the Altarelli Parisi kernels enter only in interpolated form, taking the limits $z \to 0$ and $z \to 1$ into account. The complete polarized CCFM equations take the following general form:

$$\begin{aligned} \Delta\mathbf{f}(x,k_\perp^2,Q^2) &= \left(\Delta\mathbf{P}^{\mathrm{CCFM}} \otimes \Delta\mathbf{f}\right)(x,k_\perp^2,Q^2) \\ \Delta f_k(x,k_\perp^2,Q^2) &= \left(\Delta P_{qq}^{\mathrm{CCFM}} \otimes \Delta f_k\right)(x,k_\perp^2,Q^2) \quad . \end{aligned} \tag{1}$$

Here we have used:

$$\Delta\mathbf{f} = \begin{pmatrix} \Delta f_\Sigma \\ \\ \Delta f_g \end{pmatrix}, \quad \text{and} \quad \Delta\mathbf{P}^{\mathrm{CCFM}} = \begin{pmatrix} \Delta P_{qq}^{\mathrm{CCFM}} & \Delta P_{qg}^{\mathrm{CCFM}} \\ \\ \Delta P_{gq}^{\mathrm{CCFM}} & \Delta P_{gg}^{\mathrm{CCFM}} \end{pmatrix} . \tag{2}$$

Here Δf_g is the unintegrated polarized gluon distribution function, Δf_Σ the unintegrated polarized singlet quark parton distribution function and Δf_k denotes any polarized non-singlet combination of the quark parton distribution functions like the triplet (Δf_3) and the octet contribution (Δf_8). For the convolution one has the structure:

$$
\begin{aligned}
(\Delta P^{\rm CCFM}\otimes\Delta f)(x,k_\perp^2,Q^2) &= \int_0^{2\pi}\frac{d\theta_{q_\perp}}{2\pi}\int_{Q_0^2}^{\infty}\frac{dq_\perp^2}{q_\perp^2}\int_x^{1-Q_0/q}\frac{dz}{z}\Theta(Q-z|\mathbf{q}_\perp|)\\
&\quad\times\Delta P^{\rm CCFM}(z,q_\perp^2,Q^2)\Delta f(x/z,{k'_\perp}^2,q_\perp^2)\ , \qquad (3)
\end{aligned}
$$

using $\vec{k}'_\perp=\vec{k}_\perp+(1-z)\vec{q}_\perp$. In terms of the principles discussed above one obtains for the CCFM splitting kernels:

$$
\begin{aligned}
\Delta P_{gg}^{\rm CCFM}(z,q_\perp^2,Q^2) &= \Delta_{\rm e}^{(g)}(Q^2,(z\vec{q}_\perp)^2)\frac{\alpha_s(q_\perp^2(1-z)^2)}{2\pi}\Delta P_{gg}^{\rm AP0}(z)\\
\Delta P_{gq}^{\rm CCFM}(z,q_\perp^2,Q^2) &= \Delta_{\rm e}^{(g)}(Q^2,(z\vec{q}_\perp)^2)\frac{\alpha_s(q_\perp^2(1-z)^2)}{2\pi}\Delta P_{gq}^{\rm AP0}(z)\\
\Delta P_{qq}^{\rm CCFM}(z,q_\perp^2,Q^2) &= \Delta_{\rm e}^{(q)}(Q^2,(z\vec{q}_\perp)^2)\frac{\alpha_s(q_\perp^2(1-z)^2)}{2\pi}\Delta P_{qq}^{\rm AP0}(z)\\
\Delta P_{qg}^{\rm CCFM}(z,q_\perp^2,Q^2) &= \Delta_{\rm e}^{(q)}(Q^2,(z\vec{q}_\perp)^2)\frac{\alpha_s(q_\perp^2(1-z)^2)}{2\pi}\Delta P_{qg}^{\rm AP0}(z)\ . \qquad (4)
\end{aligned}
$$

The eikonal form factors have the form:

$$
\Delta_{\rm e}^{(g,q)}(q^2,(zq)^2) = \exp\left(-C_{A,F}\int_{(zq)^2}^{q^2}\frac{dq'^2}{q'^2}\int_0^{1-Q_0/q'}\frac{dz'}{1-z'}\frac{\alpha_s(q'^2(1-z')^2)}{\pi}\right) \qquad (5)
$$

and the interpolating Altarelli-Parisi kernels read then (c. f. [10]):

$$
\begin{aligned}
\Delta P_{gg}^{\rm AP0}(z) &= 2C_A\frac{2-z}{1-z}\\
\Delta P_{qg}^{\rm AP0}(z) &= \left(z-\frac{1}{2}\right)\\
\Delta P_{qq}^{\rm AP0}(z) &= C_F\frac{1+z^2}{1-z}\\
\Delta P_{gq}^{\rm AP0}(z) &= C_F(2-z)\ . \qquad (6)
\end{aligned}
$$

The fact that in the polarized case the CCFM splitting kernels are independent of $k_\perp$ allows a factorization in terms of the Fourier-Bessel transformation:

$$
\begin{aligned}
(\Delta P^{\rm CCFM}\otimes\Delta f)(x,b_\perp^2,Q^2) &= \int d^2\mathbf{k}_\perp e^{-i\mathbf{b}_\perp\cdot\mathbf{k}_\perp}(\Delta P^{\rm CCFM}\otimes\Delta f)(x,k_\perp^2,Q^2)\\
&= \int_x^1\frac{dz}{z}\int_{Q_0^2}^{Q^2/z^2}\frac{dq_\perp^2}{q_\perp^2}J_0((1-z)|\mathbf{b}_\perp||\mathbf{q}_\perp|)\\
&\quad\times\Delta P^{\rm CCFM}(z,q_\perp^2,Q^2)\Delta\bar{f}(x/z,b_\perp^2,q_\perp^2)\ . \qquad (7)
\end{aligned}
$$

One should note that $f(x,Q^2)=\bar{f}(x,0,Q^2)$ is just the $k_\perp$ integrated parton distribution.

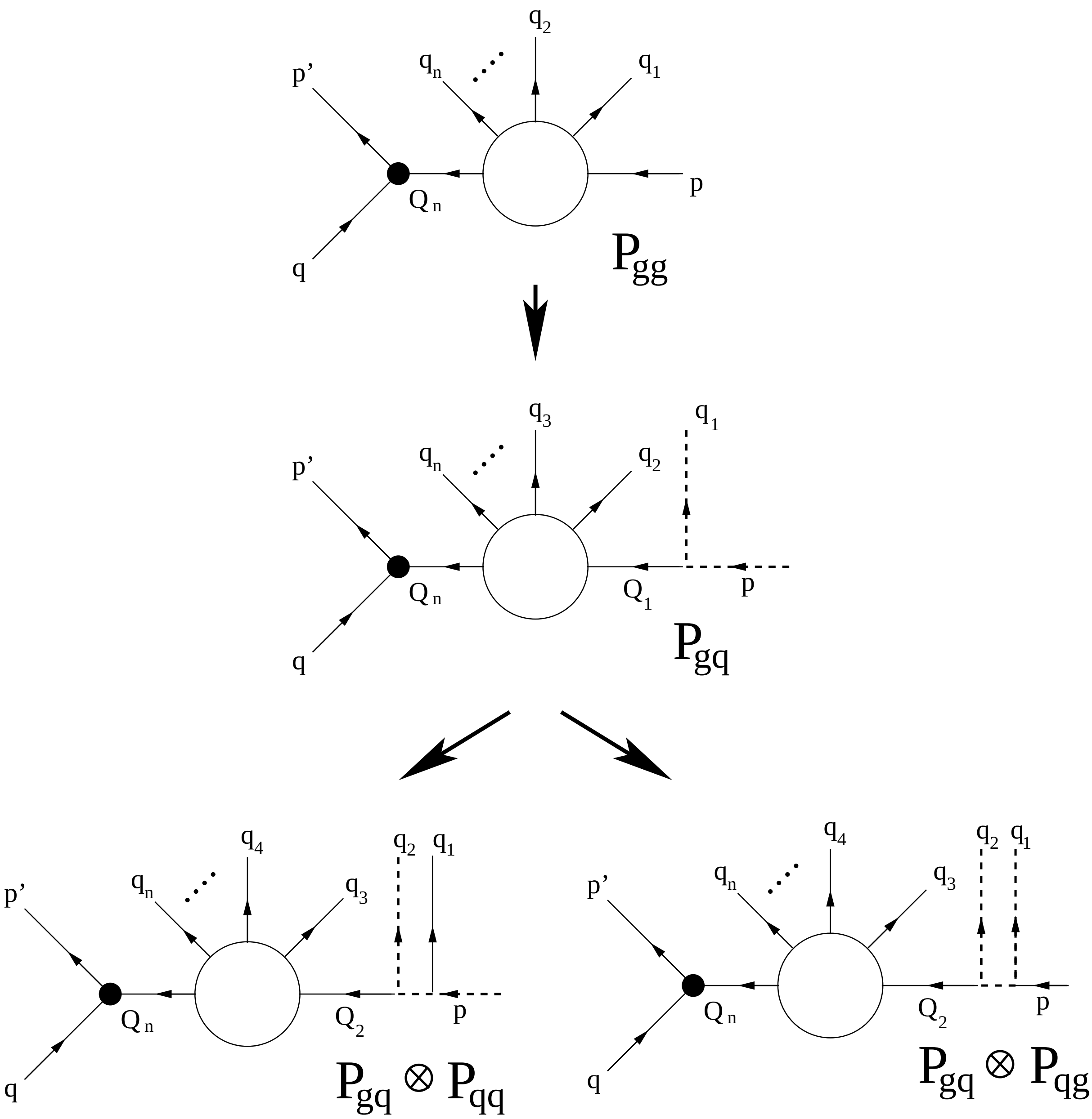

Figure 1: Construction of the polarized CCFM equation for quarks from the one for gluons. The kernels for qq, qg and gq splitting are subsequently constructed from the gg soft emission by amending the corresponding hard initial states. Here the solid line describes gluons while the dashed line denotes quarks.

3 Modifications to make contact with ladder diagrams

It may easily be observed that the angular ordering constraint which is embodied within the CCFM equation generates the double $\ln^2(1/x)$ terms for the integrated parton distributions. The result concerning resummation of those terms turns out to be, however, different from the QCD expectations discussed in [11], [12],[13],[14]. The double $ln^2(1/x)$ effects in QCD are generated by the ladder and non-ladder bremsstrahlung diagrams.
In order to get the expected double logarithmic limit of the integrated distributions corresponding to the ladder diagrams contribution it is sufficient to replace the angular ordering constraint $\Theta(Q - z|\mathbf{q}_\perp|)$ by the stronger constraint $\Theta(Q^2 - zq_\perp^2)$ in the corresponding evolution equations for *integrated* distributions. The latter just correspond to the Fourier transformed unintegrated evolution equations at $b_\perp = 0$. We assume that this replacement can be done for arbitrary values of the transverse coordinate $b_\perp$. Due to this replacement we will obtain an equation which incorporates the known collinear (LO Altarelli-Parisi) evolution for a not too small x and the double logarithmic asymptotics for $x \ll 1$. However, the equation does not sum up the single $\ln(1/x)$ contributions as it was done by the original CCFM/BFKL (unpolarized) equation. Having this point in mind we will henceforward call the evolution equation for the polarized unintegrated parton distributions which we derive starting from the CCFM formulation and where we are including our modfications a modified polarized CCFM evolution equation. Apart from this substitution in the argument of the theta function to include the double logarithmic contributions, we shall also make the following modifications of the original CCFM equations proposed in [9]:

1. The argument of α_s will be set equal to $q_\perp^2$ instead of $q_\perp^2(1-z)^2$.

2. The non-singular parts of the splitting function(s) will be included in the definition of the Sudakov form-factor(s).

3. Following Ref. [14] we include complete splitting functions $P_{ab}(z)$ and not only their singular parts at $z = 1$ and constant contributions at $z = 0$.

4. We represent the splitting functions $\Delta P_{ab}(z)$ as $\Delta P_{ab}(z) = \Delta P_{ab}(0) + \Delta\bar{P}_{ab}(z)$ where $\Delta\bar{P}_{ab}(0) = 0$. Following [14] we shall multiply $\Delta P_{ab}(0)$ and $\Delta\bar{P}_{ab}(z)$ by $\Theta(Q^2 - zq_\perp^2)$ and $\Theta(Q^2 - q_\perp^2)$ respectively in the integrands of the corresponding integral equations. Following the terminology of Ref. [14] we call the corresponding contributions to the evolution kernels the 'ladder' and 'Altarelli - Parisi ' contributions respectively.

5. We shall 'unfold' the eikonal form factors in order to treat real emission and virtual correction terms on equal footing.

Using those prescriptions we get the following unfolded and modified polarized CCFM equations:

$$\Delta f_g(x,\vec{k}_\perp^2,Q^2) = \Delta \tilde{f}_g^0(x,\vec{k}_\perp^2) + \int \frac{d^2\vec{q}_\perp}{\pi q_\perp^2}\frac{\alpha_s(q_\perp^2)}{2\pi}\Theta(q_\perp^2 - Q_0^2)\int_0^1 \frac{dz}{z}$$

$$\times\Bigg\{\Theta(z-x)\Bigg[\left(12\Theta(Q^2 - zq_\perp^2) + 6\Theta(Q^2 - q_\perp^2)\left(\frac{z}{1-z} - 2z\right)\right)\Delta f_g(x/z,\vec{k}_\perp'^2,q_\perp^2)$$

$$+\left(\frac{8}{3}\Theta(Q^2 - zq_\perp^2) - \frac{4}{3}z\Theta(Q^2 - q_\perp^2)\right)\Delta f_\Sigma(x/z,\vec{k}_\perp'^2,q_\perp^2)\Bigg]$$

$$-z\Theta(Q^2 - q_\perp^2)\left(\frac{6}{1-z} - \frac{11}{2} + \frac{N_f}{3}\right)\Delta f_g(x,\vec{k}_\perp^2,q_\perp^2)\Bigg\} \tag{8}$$

$$\Delta f_\Sigma(x,\vec{k}_\perp^2,Q^2) = \Delta \tilde{f}_\Sigma^0(x,\vec{k}_\perp^2) + \int \frac{d^2\vec{q}_\perp}{\pi q^2}\frac{\alpha_s(q_\perp^2)}{2\pi}\Theta(q_\perp^2 - Q_0^2)\int_0^1 \frac{dz}{z}$$

$$\times\Bigg\{\Theta(z-x)\Bigg[(-N_F\Theta(Q^2 - zq_\perp^2) + 2zN_F\Theta(Q^2 - q_\perp^2))\Delta f_g(x/z,\vec{k}_\perp'^2,q_\perp^2)$$

$$+\frac{4}{3}\left(\Theta(Q^2 - zq_\perp^2) + \frac{z+z^2}{1-z}\Theta(Q^2 - q_\perp^2)\right)\Delta f_\Sigma(x/z,\vec{k}_\perp'^2,q_\perp^2)\Bigg]$$

$$-z\Theta(Q^2 - q_\perp^2)\left(\frac{8}{3(1-z)} - 2\right)\Delta f_\Sigma(x,\vec{k}_\perp^2,q_\perp^2)\Bigg\}\ , \tag{9}$$

where we have put explicit numbers for the factors C_A and C_F, and also introduced the singlet spin dependent unintegrated quark distributions:

$$\Delta f_\Sigma = \Sigma_{i=1}^{N_F}(\Delta f_{q_i} + \Delta f_{\bar{q}_i})\ . \tag{10}$$

The 'non-singlet' quark distributions evolve as Δf_Σ but without the gluon contribution on the r.h.s. of the integral equation.

In equations (8) and (9) we set the upper limit of integration over dz equal to 1 instead of $1 - Q_0/q$ since the integrands are free from singularities at $z = 1$. It should be noted that $\vec{k}_\perp' = \vec{k}_\perp + (1-z)\vec{q}_\perp$. It should also be noted that the inhomogeneous terms $\Delta\tilde{f}_g^0$ and $\Delta\tilde{f}_\Sigma^0$ in equations (8) and (9) do not contain the Sudakov form-factors. They can be chosen to have (for instance) the Gaussian form in $k_\perp$ normalized to unity and multiplied by the input (integrated) distributions at the scale $Q^2 = Q_0^2$. The latter could be taken from one of the existing

(LO) QCD analysis of spin dependent parton distributions. To be precise the inhomogeneous terms are related to the starting distributions at the reference scale Q_0^2 in the 'single loop' approximation [15, 16] corresponding to the replacement $\Theta(Q^2 - zq_\perp^2)$ by $\Theta(Q^2 - q_\perp^2)$ in the integrals in equations (8,9), since in general the integrals containing the function $\Theta(Q^2 - zq_\perp^2)$ do not vanish at $Q^2 = Q_0^2$. Parameterization of the driving term in terms of the parton distribution may be regarded as a reasonable approximation, particularly in the region of large and moderately small values of x which is dominated by the single loop approximation.

Taking the Fourier-Bessel transformation on both sides of equations (8) and (9) we get the following equations for the distributions $\bar{f}_g(x, b_\perp^2, Q^2)$ and $\bar{f}_\Sigma(x, b_\perp^2, Q^2)$:

$$\bar{f}_g(x, b_\perp^2, Q^2) = \bar{f}_g^0(x, b_\perp^2) + \int \frac{dq_\perp^2}{\pi q_\perp^2} \frac{\alpha_s(q_\perp^2)}{2\pi} \Theta(q_\perp^2 - Q_0^2) \int_0^1 \frac{dz}{z}$$

$$\times \Bigg\{ J_0[b_\perp(1-z)q_\perp]\Theta(z-x) \Bigg[\left(12\Theta(Q^2 - zq_\perp^2) + 6\Theta(Q^2 - q_\perp^2) \left(\frac{z}{1-z} - 2z \right) \right) \bar{f}_g(x/z, b_\perp^2, q_\perp^2)$$

$$+ \left(\frac{8}{3}\Theta(Q^2 - zq_\perp^2) - \frac{4}{3} z\Theta(Q^2 - q_\perp^2) \right) \bar{f}_\Sigma(x/z, b_\perp^2, q_\perp^2) \Bigg]$$

$$-z\Theta(Q^2 - q_\perp^2) \left(\frac{6}{1-z} - \frac{11}{2} + \frac{N_f}{3} \right) \bar{f}_g(x, b_\perp^2, q_\perp^2) \Bigg\} \tag{11}$$

$$\bar{f}_\Sigma(x, b_\perp^2, Q^2) = \bar{f}_\Sigma^0(x, b_\perp^2) + \int \frac{dq_\perp^2}{q_\perp^2} \frac{\alpha_s(q_\perp^2)}{2\pi} \Theta(q_\perp^2 - Q_0^2) \int_0^1 \frac{dz}{z}$$

$$\times \Bigg\{ J_0[b_\perp(1-z)q_\perp]\Theta(z-x) \Bigg[(-N_F\Theta(Q^2 - zq_\perp^2) + 2zN_F\Theta(Q^2 - q_\perp^2)) \bar{f}_g(x/z, b_\perp^2, q_\perp^2)$$

$$+\frac{4}{3} \left(\Theta(Q^2 - zq_\perp^2) + \frac{z+z^2}{1-z}\Theta(Q^2 - q_\perp^2) \right) \bar{f}_\Sigma(x/z, b_\perp^2, q_\perp^2) \Bigg]$$

$$-z\Theta(Q^2 - q_\perp^2) \left(\frac{8}{3(1-z)} - 2 \right) \bar{f}_\Sigma(x, b_\perp^2, q_\perp^2) \Bigg\} , \tag{12}$$

where:

$$\begin{aligned} \bar{f}_i(x, b_\perp^2, Q^2) &= \int d^2\vec{k}_\perp \exp\left(-i\vec{k}_\perp \cdot b_\perp \right) \Delta f_i(x, k_\perp^2, Q^2) \\ &= 2\pi \int_0^\infty k_\perp dk_\perp J_0(k_\perp b_\perp) \Delta f_i(x, k_\perp^2, Q^2) , \end{aligned} \tag{13}$$

and

$$\begin{aligned}
\Delta f_i(x,k_\perp^2,Q^2) &= \int \frac{d^2\vec{b}_\perp}{(2\pi)^2}\exp\left(i\vec{k}_\perp\cdot b_\perp\right)\bar{f}_i(x,b_\perp^2,Q^2) \\
&= \frac{1}{2\pi}\int_0^\infty b_\perp db_\perp J_0(k_\perp b_\perp)\Delta f_i(x,b_\perp^2,Q^2)\ . \qquad (14)
\end{aligned}$$

From a physical point of view one can interpret $b_\perp$ as an 'impact parameter' giving the transverse distance of the partonic probe. Then the following expressions are obtained:

$$\begin{aligned}
&\overline{f}_g(x,b_\perp^2,Q^2) = \overline{f}_g^0(x,b_\perp^2) \\
&+\int_x^1\frac{dz}{z}\int_{Q_0^2}^{Q^2/z}\frac{dq_\perp^2}{q_\perp^2}\frac{\alpha_s(q_\perp^2)}{2\pi}\overline{f}_\Sigma(x/z,b_\perp^2,q_\perp^2)\frac{8}{3}J_0(|\mathbf{b}_\perp||\mathbf{q}_\perp|(1-z)) \\
&+\int_x^1\frac{dz}{z}\int_{Q_0^2}^{Q^2/z}\frac{dq_\perp^2}{q_\perp^2}\frac{\alpha_s(q_\perp^2)}{2\pi}\overline{f}_g(x/z,b_\perp^2,q_\perp^2)12J_0(|\mathbf{b}_\perp||\mathbf{q}_\perp|(1-z)) \\
&\qquad\qquad (\textbf{ladder}) \\
&+\int_x^1\frac{dz}{z}\int_{Q_0^2}^{Q^2}\frac{dq_\perp^2}{q_\perp^2}\frac{\alpha_s(q_\perp^2)}{2\pi}\overline{f}_\Sigma(x/z,b_\perp^2,q_\perp^2)\left(-\frac{4}{3}z\right)J_0(|\mathbf{b}_\perp||\mathbf{q}_\perp|(1-z)) \\
&+\int_x^1\frac{dz}{z}\int_{Q_0^2}^{Q^2}\frac{dq_\perp^2}{q_\perp^2}\frac{\alpha_s(q_\perp^2)}{2\pi}6z\left[\frac{\overline{f}_g(x/z,b_\perp^2,q_\perp^2)-\overline{f}_g(x,b_\perp^2,q_\perp^2)}{(1-z)}\right. \\
&\qquad\qquad\left.-2\overline{f}_g(x/z,b_\perp^2,q_\perp^2)\right]J_0(|\mathbf{b}_\perp||\mathbf{q}_\perp|(1-z)) \\
&+\int_{Q_0^2}^{Q^2}\frac{dq_\perp^2}{q_\perp^2}\frac{\alpha_s(q_\perp^2)}{2\pi}\overline{f}_g(x,b_\perp^2,q_\perp^2)\left[\frac{11}{2}-\frac{N_F}{3}+6\ln(1-x)\right] \\
&\qquad\qquad (\textbf{Altarelli Parisi}) \qquad (15)
\end{aligned}$$

$$\begin{aligned}
&\overline{f}_\Sigma(x,b_\perp^2,Q^2) = \overline{f}_\Sigma^0(x,b_\perp^2) \\
&+\int_x^1\frac{dz}{z}\int_{Q_0^2}^{Q^2/z}\frac{dq_\perp^2}{q_\perp^2}\frac{\alpha_s(q_\perp^2)}{2\pi}\overline{f}_\Sigma(x/z,b_\perp^2,q_\perp^2)\frac{4}{3}J_0(|\mathbf{b}_\perp||\mathbf{q}_\perp|(1-z)) \\
&-\int_x^1\frac{dz}{z}\int_{Q_0^2}^{Q^2/z}\frac{dq_\perp^2}{q_\perp^2}\frac{\alpha_s(q_\perp^2)}{2\pi}\overline{f}_g(x/z,b_\perp^2,q_\perp^2)N_F J_0(|\mathbf{b}_\perp||\mathbf{q}_\perp|(1-z))
\end{aligned}$$

$$(\textbf{ladder})$$

$$+\int_x^1 \frac{dz}{z}\int_{Q_0^2}^{Q^2}\frac{dq_\perp^2}{q_\perp^2}\frac{\alpha_s(q_\perp^2)}{2\pi}J_0(|\mathbf{b}_\perp||\mathbf{q}_\perp|(1-z))$$
$$\times\frac{4}{3}\left[\frac{(z+z^2)\overline{f}_\Sigma(x/z,b_\perp^2,q_\perp^2)-2z\overline{f}_\Sigma(x,b_\perp^2,q_\perp^2)}{(1-z)}\right]$$
$$+\int_{Q_0^2}^{Q^2}\frac{dq_\perp^2}{q_\perp^2}\frac{\alpha_s(q_\perp^2)}{2\pi}\overline{f}_\Sigma(x,b_\perp^2,q_\perp^2)\left[2+\frac{8}{3}\ln(1-x)\right]$$
$$+\int_x^1 \frac{dz}{z}\int_{Q_0^2}^{Q^2}\frac{dq_\perp^2}{q_\perp^2}\frac{\alpha_s(q_\perp^2)}{2\pi}\overline{f}_g(x/z,b_\perp^2,q_\perp^2)2zN_F J_0(|\mathbf{b}_\perp||\mathbf{q}_\perp|(1-z))$$
$$(\textbf{Altarelli Parisi}) \tag{16}$$

The contribution for the quark non-singlet part can be simply obtained from the expressions for the singlet part leaving simply out all gluonic contributions:

$$\overline{f}_{q\,NS}(x,b_\perp^2,Q^2)=\overline{f}^0_{q\,NS}(x,b_\perp^2)$$
$$+\int_x^1 \frac{dz}{z}\int_{Q_0^2}^{Q^2/z}\frac{dq_\perp^2}{q_\perp^2}\frac{\alpha_s(q_\perp^2)}{2\pi}\overline{f}_{q\,NS}(x/z,b_\perp^2,q_\perp^2)\frac{4}{3}J_0(|\mathbf{b}_\perp||\mathbf{q}_\perp|(1-z))$$
$$(\textbf{ladder})$$

$$+\int_x^1 \frac{dz}{z}\int_{Q_0^2}^{Q^2}\frac{dq_\perp^2}{q_\perp^2}\frac{\alpha_s(q_\perp^2)}{2\pi}J_0(|\mathbf{b}_\perp||\mathbf{q}_\perp|(1-z))$$
$$\times\frac{4}{3}\left[\frac{(z+z^2)\overline{f}_{q\,NS}(x/z,b_\perp^2,q_\perp^2)-2z\overline{f}_{q\,NS}(x,b_\perp^2,q_\perp^2)}{(1-z)}\right]$$
$$+\int_{Q_0^2}^{Q^2}\frac{dq_\perp^2}{q_\perp^2}\frac{\alpha_s(q_\perp^2)}{2\pi}\overline{f}_{q\,NS}(x,b_\perp^2,q_\perp^2)\left[2+\frac{8}{3}\ln(1-x)\right]$$
$$(\textbf{Altarelli Parisi}) \tag{17}$$

The expressions show that except for the scale of α_s and the occurrence of the Bessel function J_0 the expression match exactly the ones derived in [14] calculating ladder diagrams and combining this with the Altarelli Parisi evolution.

This shows the tight relationship that can be shown between modified CCFM and the ladder contributions.

4 Inclusion of non-ladder diagrams

There is a third contribution to the evolution of unintegrated parton distributions which is not covered by the 'Altarelli - Parisi +ladder' approximation of the modified polarized CCFM equation, these are the non-ladder bremsstrahlung contributions. The method of implementing the non-ladder bremsstrahlung corrections in general into the double logarithmic resummation was proposed by Kirschner and Lipatov [13]. For integrated polarized parton distributions they have been implemented in Ref. [14]. The method developed in Ref. [13] is based on the infrared equations for the partial waves $\mathbf{F_{0,8}}(\omega, \alpha_\mathbf{s})$:

$$\mathbf{F}_{0,\mathbf{8}} = \begin{pmatrix} F_{0,8}^{qq} & F_{0,8}^{qg} \\ & \\ F_{0,8}^{gq} & F_{0,8}^{gg} \end{pmatrix} . \tag{18}$$

The infrared equations for the partial waves read:

$$\mathbf{F_0}(\omega, \alpha_s) = \frac{4\pi\alpha_s}{\omega}\mathbf{M_0} - \frac{2\alpha_s}{\pi\omega^2}\mathbf{F_8}(\omega, \alpha_s)\mathbf{G_0} + \frac{1}{8\pi^2\omega}\mathbf{F_0^2}(\omega, \alpha_s) \tag{19}$$

$$\mathbf{F_8}(\omega, \alpha_s) = \frac{4\pi\alpha_s}{\omega}\mathbf{M_8} + \frac{\alpha_s N}{2\pi\omega}\frac{d}{d\omega}\mathbf{F_8}(\omega, \alpha_s) + \frac{1}{8\pi^2\omega}\mathbf{F_8^2}(\omega, \alpha_s) , \tag{20}$$

where the matrices $\mathbf{M_0}, \mathbf{M_8}$ and $\mathbf{G_0}$ are given by:

$$\mathbf{M_8} = \begin{pmatrix} \frac{-1}{2N} & -\frac{N_F}{2} \\ & \\ N & 2N \end{pmatrix} \tag{21}$$

$$\mathbf{M_0} = \begin{pmatrix} \Delta P_{qq}(0) & \Delta P_{qg}(0) \\ & \\ \Delta P_{gq}(0) & \Delta P_{gg}(0) \end{pmatrix} \tag{22}$$

$$\mathbf{G_0} = \begin{pmatrix} \frac{N^2-1}{2N} & 0 \\ & \\ 0 & N \end{pmatrix} . \tag{23}$$

The singlet partial wave matrix $\mathbf{F_0}$ is linked with the anomalous dimension matrix $\gamma_S^{RES}(\omega, \alpha_s)$ controlling the evolution of the moments of the integrated spin dependent distributions:

$$\mathbf{F_0} = 8\pi^2\gamma_S^{RES}(\omega, \alpha_s) . \tag{24}$$

The anomalous dimension matrix corresponding to the solution of equation (19) is given by:

$$\gamma_S^{RES}(\omega,\alpha_s)=\frac{\omega}{2}\left(1-\sqrt{1-\frac{2\alpha_s}{\pi\omega}\left(\frac{\mathbf{M_0}}{\omega}-\frac{\mathbf{F_8}(\omega,\alpha_s)\mathbf{G_0}}{2\pi^2\omega^2}\right)}\right)\ . \tag{25}$$

The anomalous dimension matrix contains the resummation of the double logarithmic $\ln^2(1/x)$ effects which correspond to the sum of powers of α_s/ω^2 in the ω space.
It should be noted that the inhomogeneous term in the nonlinear equation (19) which is proportional to $\mathbf{M_0}$ is also proportional to the kernel matrix defining the ladder diagram contributions in the double logarithmic approximation to the evolution equation for the (integrated) parton distributions (cf. equations (8, 9) at $b_\perp=0$). The fact that in equation (19) $\mathbf{M_0}$ appears in the inhomogeneous term while in equations (8, 9) it appears in the kernel matrix is linked with the fact that equation (19) defines the qq-scattering amplitude in the ω representation while equations (8, 9) define the parton distributions. The double logarithmic approximation of equations (8, 9) at $b_\perp=0$ would generate the anomalous dimension matrix $\gamma_{\rm ladder}^{RES}$ corresponding to ladder diagrams in the double logarithmic approximation which is given by:

$$\gamma_{\rm ladder}^{RES}(\omega,\alpha_s)=\frac{\omega}{2}\left(1-\sqrt{1-\frac{2\alpha_s\mathbf{M_0}}{\pi\omega^2}}\right)\ . \tag{26}$$

It has been observed in Ref. [14] that in order to get complete anomalous dimension given by equation (25) one has to add the corresponding terms proportional to $\mathbf{F_8}(\omega,\alpha_s)\mathbf{G_0}$ in the kernel matrix defining contribution of ladder diagrams. In the two-scale unintegrated case one can simply add them by analogy to the Altarelli - Parisi and ladder contribution by inserting the factor $J_0(b_\perp q_\perp(1-z))$. The results are given below:

$$\overline{f}_g(x,b_\perp^2,Q^2)=\overline{f}_g^0(x,b_\perp^2)$$

$$+\int_x^1\frac{dz}{z}\int_{Q_0^2}^{Q^2/z}\frac{dq_\perp^2}{q_\perp^2}\frac{\alpha_s(q_\perp^2)}{2\pi}\overline{f}_\Sigma(x,b_\perp^2,q_\perp^2)\frac{8}{3}J_0(|\mathbf{b}_\perp||\mathbf{q}_\perp|(1-z))$$

$$+\int_x^1\frac{dz}{z}\int_{Q_0^2}^{Q^2/z}\frac{dq_\perp^2}{q_\perp^2}\frac{\alpha_s(q_\perp^2)}{2\pi}\overline{f}_g(x/z,b_\perp^2,q_\perp^2)12J_0(|\mathbf{b}_\perp||\mathbf{q}_\perp|(1-z))$$

(ladder)

$$+\int_x^1\frac{dz}{z}\int_{Q_0^2}^{Q^2}\frac{dq_\perp^2}{q_\perp^2}\frac{\alpha_s(q_\perp^2)}{2\pi}\overline{f}_\Sigma(x/z,b_\perp^2,q_\perp^2)\left(-\frac{4}{3}z\right)J_0(|\mathbf{b}_\perp||\mathbf{q}_\perp|(1-z))$$

$$+\int_x^1 \frac{dz}{z}\int_{Q_0^2}^{Q^2}\frac{dq_\perp^2}{q_\perp^2}\frac{\alpha_s(q_\perp^2)}{2\pi}6z\Bigg[\frac{\overline{f}_g(x/z,b_\perp^2,q_\perp^2)-\overline{f}_g(x,b_\perp^2,q_\perp^2)}{(1-z)}$$

$$-2\overline{f}_g(x/z,b_\perp^2,q_\perp^2)\Bigg]J_0(|\mathbf{b}_\perp||\mathbf{q}_\perp|(1-z))$$

$$+\int_{Q_0^2}^{Q^2}\frac{dq_\perp^2}{q_\perp^2}\frac{\alpha_s(q_\perp^2)}{2\pi}\overline{f}_g(x,b_\perp^2,q_\perp^2)\left[\frac{11}{2}-\frac{N_F}{3}+6\ln(1-x)\right]$$

$$(\textbf{Altarelli Parisi})$$

$$-\int_x^1 \frac{dz}{z}\int_{Q_0^2}^{Q^2}\frac{dq_\perp^2}{q_\perp^2}\frac{\alpha_s(q_\perp^2)}{2\pi}J_0(|\mathbf{b}_\perp||\mathbf{q}_\perp|(1-z))\left(\left[\frac{\tilde{\mathbf{F}}_\mathbf{8}}{\tilde{\omega}^2}\right](z)\frac{\mathbf{G_0}}{2\pi^2}\right)_{gq}\overline{f}_\Sigma(x/z,b_\perp^2,q_\perp^2)$$

$$-\int_x^1 \frac{dz}{z}\int_{Q^2}^{Q^2/z}\frac{dq_\perp^2}{q_\perp^2}\frac{\alpha_s(q_\perp^2)}{2\pi}J_0(|\mathbf{b}_\perp||\mathbf{q}_\perp|(1-z))\left(\left[\frac{\tilde{\mathbf{F}}_\mathbf{8}}{\tilde{\omega}^2}\right]\left(\frac{q_\perp^2}{Q^2}z\right)\frac{\mathbf{G_0}}{2\pi^2}\right)_{gg}\overline{f}_g(x/z,b_\perp^2,q_\perp^2)$$

$$(\textbf{non}-\textbf{ladder}) \tag{27}$$

$$\overline{f}_\Sigma(x,b_\perp^2,Q^2)=\overline{f}^0_\Sigma(x,b_\perp^2)$$

$$+\int_x^1 \frac{dz}{z}\int_{Q_0^2}^{Q^2/z}\frac{dq_\perp^2}{q_\perp^2}\frac{\alpha_s(q_\perp^2)}{2\pi}\overline{f}_\Sigma(x/z,b_\perp^2,q_\perp^2)\frac{4}{3}J_0(|\mathbf{b}_\perp||\mathbf{q}_\perp|(1-z))$$

$$-\int_x^1 \frac{dz}{z}\int_{Q_0^2}^{Q^2/z}\frac{dq_\perp^2}{q_\perp^2}\frac{\alpha_s(q_\perp^2)}{2\pi}\overline{f}_g(x/z,b_\perp^2,q_\perp^2)N_F J_0(|\mathbf{b}_\perp||\mathbf{q}_\perp|(1-z))$$

$$(\textbf{ladder})$$

$$+\int_x^1 \frac{dz}{z}\int_{Q_0^2}^{Q^2}\frac{dq_\perp^2}{q_\perp^2}\frac{\alpha_s(q_\perp^2)}{2\pi}J_0(|\mathbf{b}_\perp||\mathbf{q}_\perp|(1-z))$$

$$\times\frac{4}{3}\left[\frac{(z+z^2)\overline{f}_\Sigma(x/z,b_\perp^2,q_\perp^2)-2z\overline{f}_\Sigma(x,b_\perp^2,q_\perp^2)}{(1-z)}\right]$$

$$+\int_{Q_0^2}^{Q^2}\frac{dq_\perp^2}{q_\perp^2}\frac{\alpha_s(q_\perp^2)}{2\pi}\overline{f}_\Sigma(x,b_\perp^2,q_\perp^2)\left[2+\frac{8}{3}\ln(1-x)\right]$$

$$+\int_x^1 \frac{dz}{z}\int_{Q_0^2}^{Q^2}\frac{dq_\perp^2}{q_\perp^2}\frac{\alpha_s(q_\perp^2)}{2\pi}\overline{f}_g(x/z,b_\perp^2,q_\perp^2)2zN_F J_0(|\mathbf{b}_\perp||\mathbf{q}_\perp|(1-z))$$

$$(\mathbf{Altarelli\ Parisi})$$

$$-\int_x^1 \frac{dz}{z}\int_{Q_0^2}^{Q^2}\frac{dq_\perp^2}{q_\perp^2}\frac{\alpha_s(q_\perp^2)}{2\pi}J_0(|\mathbf{b}_\perp||\mathbf{q}_\perp|(1-z))\left(\left[\frac{\tilde{\mathbf{F}}_\mathbf{8}}{\tilde{\omega}^2}\right](z)\frac{\mathbf{G_0}}{2\pi^2}\right)_{qq}\overline{f}_\Sigma(x/z,b_\perp^2,q_\perp^2)$$

$$-\int_x^1 \frac{dz}{z}\int_{Q^2}^{Q^2/z}\frac{dq_\perp^2}{q_\perp^2}\frac{\alpha_s(q_\perp^2)}{2\pi}J_0(|\mathbf{b}_\perp||\mathbf{q}_\perp|(1-z))\left(\left[\frac{\tilde{\mathbf{F}}_\mathbf{8}}{\tilde{\omega}^2}\right]\left(\frac{q_\perp^2}{Q^2}z\right)\frac{\mathbf{G_0}}{2\pi^2}\right)_{qg}\overline{f}_g(x/z,b_\perp^2,q_\perp^2)$$

$$(\mathbf{non-ladder}) \tag{28}$$

The contribution for the quark non-singlet part can be simply obtained from the expressions for the singlet part leaving simply out all gluonic contributions:

$$\overline{f}_{q\,NS}(x,b_\perp^2,Q^2)=\overline{f}^0_{q\,NS}(x,b_\perp^2)$$

$$+\int_x^1 \frac{dz}{z}\int_{Q_0^2}^{Q^2/z}\frac{dq_\perp^2}{q_\perp^2}\frac{\alpha_s(q_\perp^2)}{2\pi}\overline{f}_{q\,NS}(x/z,b_\perp^2,q_\perp^2)\frac{4}{3}J_0(|\mathbf{b}_\perp||\mathbf{q}_\perp|(1-z))$$

$$(\mathbf{ladder})$$

$$+\int_x^1 \frac{dz}{z}\int_{Q_0^2}^{Q^2}\frac{dq_\perp^2}{q_\perp^2}\frac{\alpha_s(q_\perp^2)}{2\pi}J_0(|\mathbf{b}_\perp||\mathbf{q}_\perp|(1-z))$$
$$\times\frac{4}{3}\left[\frac{(z+z^2)\overline{f}_{q\,NS}(x/z,b_\perp^2,q_\perp^2)-2z\overline{f}_{q\,NS}(x,b_\perp^2,q_\perp^2)}{(1-z)}\right]$$
$$+\int_{Q_0^2}^{Q^2}\frac{dq_\perp^2}{q_\perp^2}\frac{\alpha_s(q_\perp^2)}{2\pi}\overline{f}_{q\,NS}(x,b_\perp^2,q_\perp^2)\left[2+\frac{8}{3}\ln(1-x)\right]$$

$$(\mathbf{Altarelli\ Parisi})$$

$$-\int_x^1 \frac{dz}{z}\int_{Q_0^2}^{Q^2}\frac{dq_\perp^2}{q_\perp^2}\frac{\alpha_s(q_\perp^2)}{2\pi}J_0(|\mathbf{b}_\perp||\mathbf{q}_\perp|(1-z))\left(\left[\frac{\tilde{\mathbf{F}}_\mathbf{8}}{\tilde{\omega}^2}\right](z)\frac{\mathbf{G_0}}{2\pi^2}\right)_{qq}\overline{f}_{q\,NS}(x/z,b_\perp^2,q_\perp^2)$$

$$-\int_x^1 \frac{dz}{z}\int_{Q^2}^{Q^2/z}\frac{dq_\perp^2}{q_\perp^2}\frac{\alpha_s(q_\perp^2)}{2\pi}J_0(|\mathbf{b}_\perp||\mathbf{q}_\perp|(1-z))\left(\left[\frac{\tilde{\mathbf{F}}_\mathbf{8}}{\tilde{\omega}^2}\right]\left(\frac{q_\perp^2}{Q^2}z\right)\frac{\mathbf{G_0}}{2\pi^2}\right)_{qq}\overline{f}_{q\,NS}(x/z,b_\perp^2,q_\perp^2)$$

$$(\mathbf{non-ladder})$$

(29)

Here $\left(\left[\frac{\tilde{\mathbf{F}}_{\mathbf{8}}}{\omega^2}\right](z)\right)$ is the inverse Mellin transformation of $\mathbf{F_8}(\omega)/\omega^2$. Derived for large N and fixed α_s, one can use an approximate form [11]:

$$\left[\frac{\tilde{\mathbf{F}}_{\mathbf{8}}^{\mathrm{Born}}}{\omega}\right](z) = 2\pi\alpha_s \mathbf{M_8} \ln^2(z) \; , \tag{30}$$

so that one gets in our case e.g.:

$$\left(\left[\frac{\tilde{\mathbf{F}}_{\mathbf{8}}}{\tilde{\omega}^2}\right](z)\frac{\mathbf{G_0}}{2\pi^2}\right)_{qq} \approx -\frac{N^2-1}{4\pi N^2}\alpha_s(q_\perp^2)\ln^2(z) \; . \tag{31}$$

5 Numerical studies

5.1 The input distributions

For the polarized input distributions we are going to use the LO Standard Scenario parameterization given in [17]. The structure of the integrated input distributions is a factor multiplied with the unpolarized input distributions given in [18]. As we have 'de-exponentiated' the eikonal form factor in the evolution equations, the input distributions have to be Q^2 independent. This is different from the usual unpolarized CCFM-input distributions, where one uses for the Q^2 dependence the corresponding eikonal form factor [19]. For the $k_\perp$ dependence of the input distributions a Gaussian Ansatz is common [19]. In this way the input distribution as defined at an input scale Q_0^2 have the following general scheme:

$$\begin{aligned}
\Delta\tilde{f}_i^0(x,k_\perp^2,Q^2) &= \Delta p_i(x,Q_0^2)\exp\left(-\frac{k_\perp^2}{\sigma^2}\right)\frac{1}{\pi\sigma^2} \\
\Rightarrow \Delta\bar{f}_i^0(x,b_\perp^2,Q^2) &= \Delta p_i(x,Q_0^2)\exp\left(-\frac{b_\perp^2\sigma^2}{4}\right) \; .
\end{aligned} \tag{32}$$

where $\Delta p_i(x,Q_0^2)$ are the input integrated spin dependent parton distributions at the reference scale Q_0^2.

5.2 Features of the evolution

Setting $b_\perp = 0$ in Eqs. (12) and (11), one obtains evolution equations for the integrated polarized parton distributions equivalent to those given in [14]. Therefore, the x and Q^2 dependence of the unintegrated parton distributions will be exactly the same as given by the Altarelli Parisi evolution equations supplemented by ladder and non-ladder contributions. In this way the modified polarized CCFM equation as discussed here is consistent with the standard evolution of the integrated polarized parton distributions. In fact the whole transverse momentum dependence is governed by the inclusion of the factor $J_0(b_\perp q_\perp(1-z))$. In principle, the only genuine new feature of the modified CCFM equation is the $k_\perp$ (or $b_\perp$) dependence. At large and moderately small values of x one can make the 'single-loop' approximation [15, 16] corresponding to the replacement $\Theta(Q^2 - zq_\perp^2)$ by just $\Theta(Q^2 - q_\perp^2)$ in the 'ladder' contribution and to neglecting the 'non-ladder' contribution. At $b_\perp = 0$ the modified polarized CCFM equation in the single loop approximation equation reduces then to the LO Altarelli-Parisi equation in the integral form.

Numerically, we perform the evolution using the Chebyshev approximation technique as discussed in the Appendix. The calculation is very time consuming, therefore, we take only 8 polynomials in all cases into account. As the Bessel function J_0 is oscillating the corresponding integration is numerically quite problematic. As a pragmatical solution the integration is only performed up to the fourth zero. We have checked that this procedure provides stable results. As the mathematical structure is the same for singlet and non-singlet contributions we can restrict ourselves for the discussion of the $k_\perp$ dependence to the simple non-singlet case. In Fig. 2 we show the evolution of the $k_\perp$ dependence for the triplet contribution $\Delta f_3 = \frac{1}{6}\left(\Delta u + \Delta\bar{u} - \Delta d - \Delta\bar{d}\right)$. The input distributions at $Q_0^2 = 0.26$ GeV^2 taken from the GRVS LO Standard Scenario set [17] are compared to the evolved distributions at $Q^2 = 10.0$ GeV^2 and $Q^2 = 100.0$ GeV^2. The width of the initial transverse momentum dependence σ has been chosen to be 1 GeV. For the simulation the full content of the equation, the Altarelli-Parisi, ladder and non-ladder contributions all have been included. It is seen that due to the evolution the $k_\perp$ dependence is broadening away from a Gaussian behavior to a more purely exponential decay. Such a feature has already been seen in the purely gluonic formulation of the genuine polarized CCFM equation as discussed in [9].

6 Summary and conclusions

In this paper we have derived an integral equation of the unintegrated polarized parton distributions starting from a genuine polarized CCFM (pCCFM) formulation and including some modifications incorporating the known collinear (LO Altarelli-Parisi) evolution for a not too small x and the double log asymptotics for $x \ll 1$. This evolution equation which we call modified polarized CCFM equation does not sum up the single $\ln(1/x)$ as it was done by the CCFM/BFKL unpolarized equation. An inclusion of the single log contributions would be very interesting, but is unfortunately beyond the scope of this article. The modified CCFM equation yields an approximate description, which contains contributions

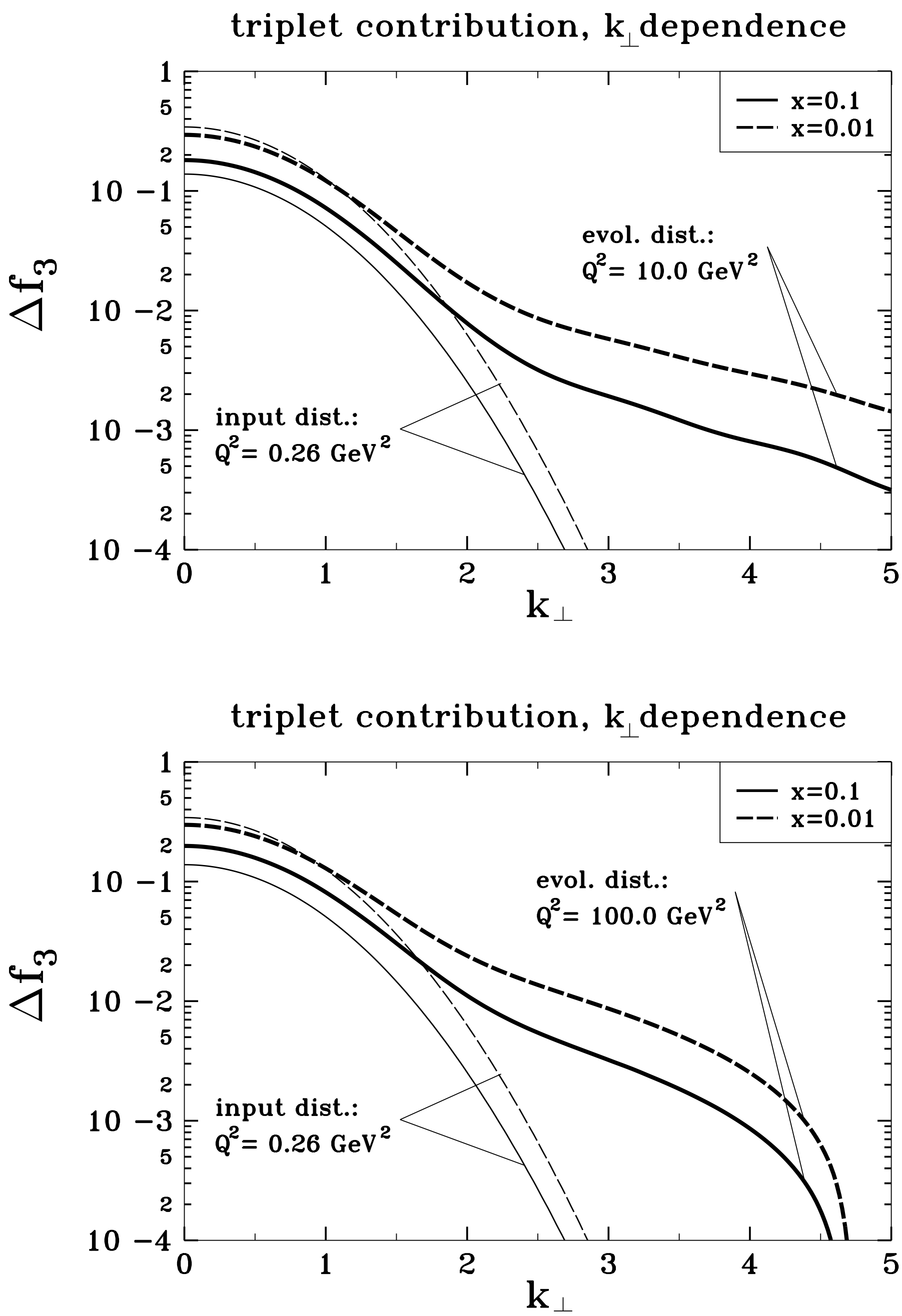

Figure 2: Transverse momentum dependence for the triplet contribution $\Delta f_3 = \frac{1}{6}\left(\Delta u + \Delta\bar{u} - \Delta d - \Delta\bar{d}\right)$ of the full modified pCCFM evolution (approximate form: Altarelli Parisi + ladder + non-ladder). The thin lines show the input distributions GRSV LO Standard Scenario [17] for $Q_0^2 = 0.26$ GeV^2 for $x = 0.1$ (solid) and $x = 0.01$ (dashed), while the bold lines show the same distribution evolved to $Q^2 = 10$ GeV^2 (top) and $Q^2 = 100$ GeV^2 (bottom). It can be seen from the logarithmical scale that the evolution leads to a $k_\perp$ broadening away from the Gaussian shape to a mere simple exponential decay.

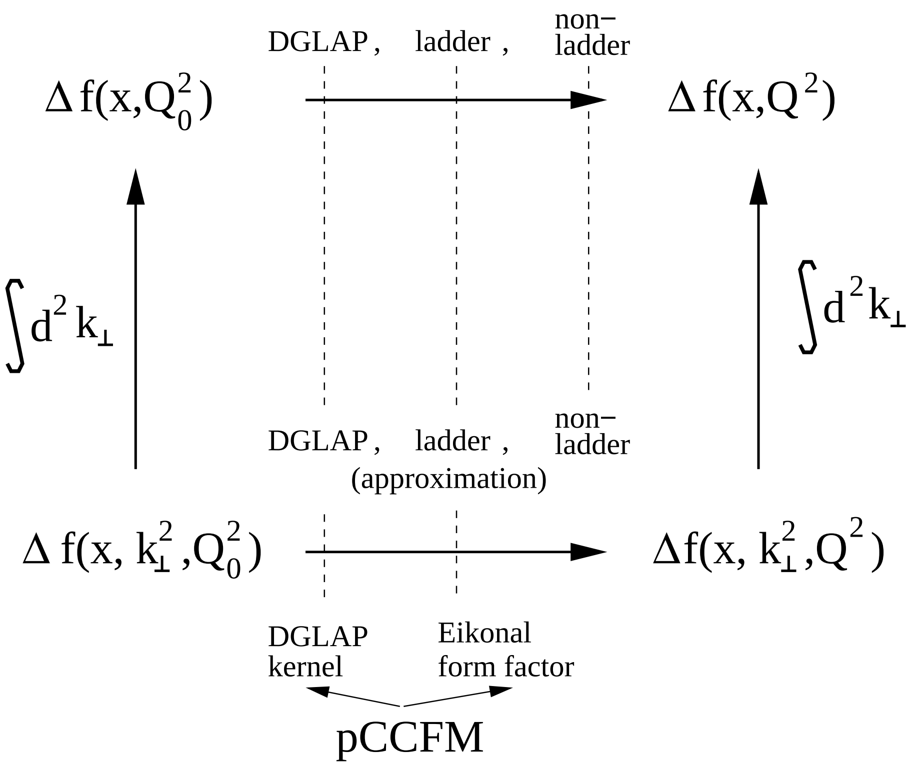

Figure 3: Relationship between the pCCFM evolution and its modified approximate form for unintegrated parton distributions on the one hand and the evolution of integrated parton distributions on the other hand.

of the type of 'Altarelli Parisi', 'ladder' and 'non-ladder' and which correspond to respective contributions in the integrated case. We have utilized the fact that these equations can be diagonalized using the Fourier-Bessel transformation with the 'impact parameter' $b_\perp$. The nice feature of this transformation is that by simply setting $b_\perp = 0$ one obtains already the corresponding expressions for the integrated parton distributions. In fact, the difference between both is only a factor $J_0(b_\perp q_\perp(1-z))$. As to the $b_\perp$ dependence the mathematical structure is the same for the non-singlet and singlet contributions to the unintegrated parton distributions. Using the technique of the Chebyshev approximation we have performed the evolution for the triplet distribution Δf_3 and observed a characteristic $k_\perp$ broadening away from the Gaussian form to a more exponential decay. The relationship between the genuine and the modified pCCFM for unintegrated parton distributions on the one hand and the evolution of integrated parton distributions on the other hand is displayed in Fig. 3. The modifications to the genuine pCCFM equation correspond to the Altarelli-Parisi part plus ladder contributions in the evolution of the unintegrated parton distributions. The non-ladder contributions have to be added by hand. As the only difference in the evolution kernels for the integrated and the unintegrated parton distributions in Fourier space is the factor $J_0(b_\perp q_\perp(1-z))$, which becomes unity as the transverse impact parameter $b_\perp$ goes to zero, it is clear that the diagram between transverse integration and evolution shown in Fig. 3 commutes.

The next step will be to use the modified polarized CCFM equation as presented here to construct a consistent set of unintegrated polarized parton distributions from existing data.

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Appendix

Approximation by Chebyshev polynomials

A possible diagonalization of the problem from a mathematical point of view would be the transformation of the pCCFM equation (here using the weaker constraint $\Theta(Q - zq_\perp))$ into Mellin space:

$$
\begin{aligned}
&\overline{(\Delta P^{\mathrm{CCFM}} \otimes \Delta f)}(\omega, b_\perp^2, Q^2) = \int_0^1 dx x^{\omega-1} (\Delta P^{\mathrm{CCFM}} \otimes \Delta f)(x, b_\perp^2, Q^2) \\
= &\int_{Q_0^2}^{Q^2} \frac{dq_\perp^2}{q_\perp^2} \Delta \bar{f}(\omega, b_\perp^2, q_\perp^2) \\
&\times \left(\int_0^1 dz z^{\omega-1} J_0((1-z)|\mathbf{b}_\perp||\mathbf{q}_\perp|) \Delta P^{\mathrm{CCFM}}(z, q_\perp^2, Q^2) \right) \\
&+ \int_{Q^2}^{\infty} \frac{dq_\perp^2}{q_\perp^2} \Delta \bar{f}(\omega, b_\perp^2, q_\perp^2) \\
&\times \left(\int_0^{\left(q_\perp^2/Q^2\right)^{-1/2}} dz z^{\omega-1} J_0((1-z)|\mathbf{b}_\perp||\mathbf{q}_\perp|) \Delta P^{\mathrm{CCFM}}(z, q_\perp^2, Q^2) \right) \ .
\end{aligned}
\tag{33}
$$

Unfortunately, it turns out that the back transformation after evolution, especially as regards the small-x region, is numerically quite problematic. Therefore, we take a different approach here, where both the $q_\perp$ and the x dependence is expanded into properly chosen Chebyshev polynomials. These are defined in the following way:

$$
\begin{aligned}
T_n(x) &= \cos(n \arccos(x)) \\
T_0(x) &= 1 \\
T_1(x) &= x \\
T_{n+1}(x) &= 2xT_n(x) - T_{n-1}(x), \quad n \geq 1
\end{aligned}
\tag{34}
$$

$$
\int_{-1}^{1} \frac{T_i(x) T_j(x)}{\sqrt{1-x^2}} dx = \begin{cases} 0 & i \neq j \\ \frac{\pi}{2} & i = j \neq 0 \\ \pi & i = j = 0 \end{cases} \ . \tag{35}
$$

The generic advantage of the Chebyshev polynomials is now that there exists also a discrete orthogonality relation. Let

$$
x_k^{(N)} := \cos\left[\pi \frac{\left(k - \frac{1}{2}\right)}{N}\right] \tag{36}
$$

be the k-th zero of $T_N(x)$ then one has as a discrete orthogonality relation ($i, j < N$):

$$\sum_{k=1}^{N} T_i(x_k)T_j(x_k) = \begin{cases} 0 & i \neq j \\ \frac{N}{2} & i = j \neq 0 \\ N & i = j = 0 \end{cases} \quad . \tag{37}$$

The central point is that for any arbitrary function f(x) in the interval [-1,1] the N coefficients c_j given by:

$$\begin{aligned} c_j &= \frac{2}{N}\sum_{k=1}^{N} f(x_k^{(N)})T_{j-1}(x_k^{(N)}) \\ &= \frac{2}{N}\sum_{k=1}^{N} f\left[\cos\left(\pi\frac{k-\frac{1}{2}}{N}\right)\right]\cos\left(\pi(j-1)\frac{k-\frac{1}{2}}{N}\right) \quad . \end{aligned} \tag{38}$$

yield an approximation formula:

$$f(x) \approx \left[\sum_{k=1}^{N} c_k T_{k-1}(x)\right] - \frac{c_1}{2} \ , \tag{39}$$

which is exact on all $x_k^{(N)}, k = 1, \ldots, N$. Therefore, one can always substitute the continuous integral expression for the isolation of the coefficients c_j by a discrete one:

$$\begin{aligned} c_j &= \frac{2}{\pi}\int_{-1}^{1}\frac{dx}{\sqrt{1-x^2}}T_{k-1}(x)f(x) \\ &\to \frac{2}{N}\sum_{k=1}^{N} f\left[\cos\left(\pi\frac{k-\frac{1}{2}}{N}\right)\right]\cos\left(\pi(j-1)\frac{k-\frac{1}{2}}{N}\right) \quad . \end{aligned} \tag{40}$$

So, choosing a cutoff $Q_{\max}$ large enough, one can define a set of variables:

$$\begin{aligned} t' &= 2\frac{\ln(q_\perp^2/Q_0^2)}{\ln(Q_{\max}^2/Q_0^2)} - 1 \\ t &= 2\frac{\ln(Q^2/Q_0^2)}{\ln(Q_{\max}^2/Q_0^2)} - 1 \ . \end{aligned} \tag{41}$$

Correspondingly for an $x_{\min}$ small enough one can define a second set of variables:

$$\begin{aligned} y &= 1 - 2\frac{\ln x}{\ln x_{\min}} \\ y' &= 1 - 2\frac{\ln z}{\ln x_{\min}} \\ T_n(y') &= T_n\left(1 - 2\frac{\ln z}{\ln x_{\min}}\right) = T_n^*(z) \ . \end{aligned} \tag{42}$$

In this way one can expand:

$$\Delta\bar{f}(x,b_\perp,q_\perp^2)=\sum_{ij}\Delta\bar{f}_{ij}(b_\perp)c_iT_{i-1}(t')c_jT^*_{j-1}(x)\ , \qquad (43)$$

where $c_1=1/2$ and $c_k=1$ in all other cases. In this way the CCFM equation transforms to a simple set of linear equations:

$$\begin{aligned}
\Delta\bar{f}(x,b_\perp^2,Q^2) &= \Delta\bar{f}^0(x,b_\perp^2)+(\Delta P^{\rm CCFM}\otimes\Delta\bar{f})(x,b_\perp^2,Q^2)\\
\Rightarrow \Delta\bar{f}_{ij}(b_\perp^2) &= \Delta\bar{f}^0_{ij}(b_\perp^2)+a_{ij\,i'j'}(b_\perp^2)\bar{f}_{i'j'}(b_\perp^2)\\
a_{ij\,i'j'}(b_\perp^2) &= -\frac{\ln(x_{\rm min})\ln(Q^2_{\rm max}/Q_0^2)}{\pi^2}c_{i'}c_{j'}\int_{-1}^1\frac{T_{i-1}(t)dt}{\sqrt{1-t^2}}\int_{-1}^1\frac{T_{j-1}(y)dy}{\sqrt{1-y^2}}\\
&\quad\times\int_y^1 dy'\int_{-1}^{t-2(1-y')\frac{\ln x_{\rm max}}{\ln(Q^2_{\rm max}/Q_0^2)}}dt'\\
&\quad\times J_0((1-z)|\mathbf{b}_\perp||\mathbf{q}_\perp|)\Delta P^{\rm CCFM}(z,t',t)T_{i'-1}(t')\\
&\quad\times T^*_{j'-1}\left[\exp\left(\frac{\ln x_{\rm min}}{2}(y'-y)\right)\right]\\
&\rightarrow -\frac{\ln(x_{\rm min})\ln(Q^2_{\rm max}/Q_0^2)}{N^2}c_{i'}c_{j'}\Bigg\{\sum_{k=1}^N\sum_{k'=1}^N T_{i-1}(t_k)T_{j-1}(y_{k'})\\
&\quad\times\int_{y_{k'}}^1 dy'\int_{-1}^{t_k-2(1-y')\frac{\ln x_{\rm max}}{\ln(Q^2_{\rm max}/Q_0^2)}}dt'J_0((1-z)|\mathbf{b}_\perp||\mathbf{q}_\perp|)\\
&\quad\times\Delta P^{\rm CCFM}(z,q_\perp^2,t_k)T_{i'-1}(t')T_{j'-1}(1-(y'-y_{k'}))\Bigg\}\ . \qquad (44)
\end{aligned}$$

Here we used:

$$t_k=y_k=\cos\left(\pi\frac{k-\frac{1}{2}}{N}\right)\ . \qquad (45)$$

The generalization for the gluon and quark singlet matrix valued expression is straightforward. The form of the master equation (44) has now the advantage that one has to handle only a simple two dimensional integration. For some contributions of the Altarelli-Parisi like type the expressions are diagonal in x and $q_\perp^2$. In those cases more simple expressions are possible:

$$\begin{aligned}
\Delta\bar{f}(x,Q^2,b_\perp^2) &= \ldots+\int_{Q_0^2}^{Q^2}\frac{dq_\perp^2}{q_\perp^2}\frac{\alpha_s(q_\perp^2)}{2\pi}\\
&\quad\times\left[2+\frac{8}{3}\ln(1-x)\right]\Delta\bar{f}(x,q_\perp,b_\perp^2)\\
\Rightarrow\sum_{kl}c_kc_lf_{kl}(b_\perp^2)T_{k-1}(t_x)T_{l-1}(t_{Q^2}) &= \ldots+\sum_{k'l'}c_{k'}c_{l'}f_{k'l'}(b_\perp^2)\int_{Q_0^2}^{Q^2}\frac{dq_\perp^2}{q_\perp^2}\frac{\alpha_s(q_\perp^2)}{2\pi}T_{l'-1}(t_{q_\perp^2})\\
&\quad\times\left[2+\frac{8}{3}\ln(1-x)\right]T_{k'-1}(t_x) \qquad (46)
\end{aligned}$$

$$
\begin{aligned}
\Rightarrow f_{kl}(b_\perp^2) &= \ldots + \sum_{l'k'} b_{kk'} d_{ll'} f_{k'l'}(b_\perp^2) \\
b_{kk'} &= \sum_j c_{k'} \frac{2}{N} \left[2 + \frac{8}{3} \ln(1 - x_j) \right] T_{k'-1}(t_j) T_{k-1}(t_j) \ , \quad x_j = x_{\rm min}^{(1-t_j)/2} \\
d_{ll'} &= \sum_j c_{l'} \frac{2}{N} T_{l-1}(t_j) \int_{Q_0^2}^{Q_j^2} \frac{dq_\perp^2}{q_\perp^2} \frac{\alpha_s(q_\perp^2)}{2\pi} T_{l'-1}(t_{q_\perp^2}) \ , \quad Q_j^2 = Q_{\rm max}^2 \left(\frac{Q_0^2}{Q_{\rm max}^2} \right)^{(1-t_j)/2} \ ,
\end{aligned}
\tag{47}
$$

where t_j is again the j-th zero of $T_N(t)$.

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