Some comments on the current status of event generators for small-x

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Abstract. We discuss some aspects of the current status of event generators for small-x DIS. In particular we report on recent results using the CCFM evolution implemented in the SMALLX and CASCADE program concerning the importance of the so-called consistency constraint, and compare to the Linked Dipole Chain Model implemented in the LDC program. We point out some potential problems with the *resolved virtual photon* approach to small-x final-states. We do not, however, discuss the fact that the ARIADNE program, implementing the soft-radiation dipole cascade, reproduces the bulk of small-x final-state data from HERA.

1. Introduction

Small-x final states is a key ingredient to the understanding of small-x evolution in general. To understand final-state properties such as jet rates and transverse energy flow, it is important to have event generators which give a good description of the experimental data. It has, however, turned out to be very difficult to produce such an event generator. Although the ARIADNE[1] program seems to be giving a fairly good description, it has turned out to be difficult to extract more than a qualitative understanding of the underlying evolution from the essentially semi-classical model[2] implemented in this program.

To leading-log accuracy, the CCFM[3] evolution should be the best way of describing small-x final states, and some attempts to implement CCFM into an event generator has been made e.g. SMALLX[4] and LDCMC[5]. Up until recently, none of these programs have been able to reproduce e.g. forward jet rates, which may be the most striking feature of the small-x final states measured at HERA. Below we will discuss some recent developments of the SMALLX and LDCMC generators as well as the new CASCADE[6] generator for CCFM, which hopefully will solve this problem.

An alternative way to view small-x final states is to consider the incoming virtual photon as a quasi-real particle which can be resolved into partons which may then scatter on partons in the proton to create hard forward jets. This approach is implemented in the RAPGAP[7] and PYTHIA[8] programs, which indeed are able to reproduce e.g. forward jet rates as measured at HERA. There are, however, some questionable features in this approach, especially related to the choice of scale of the hard parton–parton sub-process, which we will discuss below.

2. CCFM and the consistency constraint

The implementation of the CCFM [3] parton evolution in the forward evolution Monte Carlo program SMALLX is described in detail in [4]. Here only the basic ideas and the treatment of the non-Sudakov form factor is discussed.

The initial state gluon cascade is generated in a forward evolution approach from a starting distribution of the k_t unintegrated gluon distribution. Gluons then can branch into a virtual (t-channel) gluon k_{i+1} and a final gluon q_{i+1} according to the CCFM splitting function [3]:

$$dP_i = \tilde{P}_g^i(z_i, q_{ti}^2, k_{ti}^2) \cdot \Delta_s dz_i \frac{d^2 q'_{ti}}{\pi q'^2_{ti}} \cdot \Theta(q'_{ti} - z_i q'_{t\, i-1}) \cdot \Theta(1 - z_i - \epsilon_i) \tag{1}$$

with $q'_{ti} = q_{ti}/(1-z_i)$ being the rescaled transverse momentum, $z_i = x_i/x_{i-1}$, $\epsilon_i = Q_0/q'_{ti}$ being a collinear cutoff to avoid the $1/(1-z)$ singularity and Δ_s being the Sudakov form factor which, at an inclusive level, cancels against the $1/(1-z)$ collinear singularity and is used to generate q'_{ti}. The gluon splitting function $\tilde{P}_g^i$ is given by:

$$\tilde{P}_g^i = \frac{\bar{\alpha}_s(q_{ti}^2)}{1-z_i} + \frac{\bar{\alpha}_s(k_{ti}^2)}{z_i} \Delta_{ns}(z_i, q_{ti}^2, k_{ti}^2) \tag{2}$$

with the non-Sudakov form factor Δ_{ns} being defined as:

$$\log \Delta_{ns} = -\bar{\alpha}_s(k_{ti}^2) \int \frac{dz'}{z'} \int \frac{dq^2}{q^2} \cdot \Theta(k_{ti} - q)\Theta(q - z' q_{ti}) \tag{3}$$

which gives for the region $k_{ti}^2 > z_i q_{ti}^2$:

$$\log \Delta_{ns} = -\bar{\alpha}_s(k_{ti}^2) \log\left(\frac{1}{z_i}\right) \log\left(\frac{k_{ti}^2}{z_i q_{ti}^2}\right) \tag{4}$$

The constraint $k_{ti}^2 > z_i q_{ti}^2$ is often referred to as the "consistency constraint" [12]. However the upper limit of the z' integral is constrained by the Θ functions in eq.(3) by: $z_i \leq z' \leq \min(1, k_{ti}/q_{ti})$, which results in the following form of the non-Sudakov form factor [13]:

$$\log \Delta_{ns} = -\bar{\alpha}_s(k_{ti}^2) \log\left(\frac{z_0}{z_i}\right) \log\left(\frac{k_{ti}^2}{z_0 z_i q_{ti}^2}\right), z_0 = \begin{cases} 1 & \text{if } k_{ti}/q_{ti} > 1 \\ k_{ti}/q_{ti} & \text{if } z < k_{ti}/q_{ti} \leq 1 \\ z & \text{if } k_{ti}/q_{ti} \leq z \end{cases} \tag{5}$$

giving no suppression in the region $k_{ti}/q_{ti} \leq z$ where then $\Delta_{ns} = 1$. The Monte Carlo program SMALLX[4] has been modified to include the non-Sudakov form factor according to eq.(5). To avoid problems at small k_t^2, $\alpha_s(k_t^2)$ is restricted to $\alpha_s(k_t^2) \leq 0.6$.

The "consistency constraint" was introduced to account for next-to-leading effects in the BFKL equation, and which was found [12] to simulate about 70% of the full next-to-leading corrections to the BFKL equation. Since in LO BFKL, the true kinematics of the branchings are neglected, they can be interpreted as next-to-leading effects, and this constraint is often also called "kinematic constraint". In the CCFM equation energy and momentum conservation is already included at LO, and it is not clear whether the arguments coming from BFKL also apply to CCFM. In the following the effects of the "consistency constraint" are studied in more detail.

In Fig. 1a the prediction for the forward jets is shown. The data are nicely described. Including the "consistency constraint" the x dependence of the total cross section changes, but a similarly good description of F_2 is obtained by changing Q_0. However the forward jet cross section becomes smaller, as shown with the dotted line

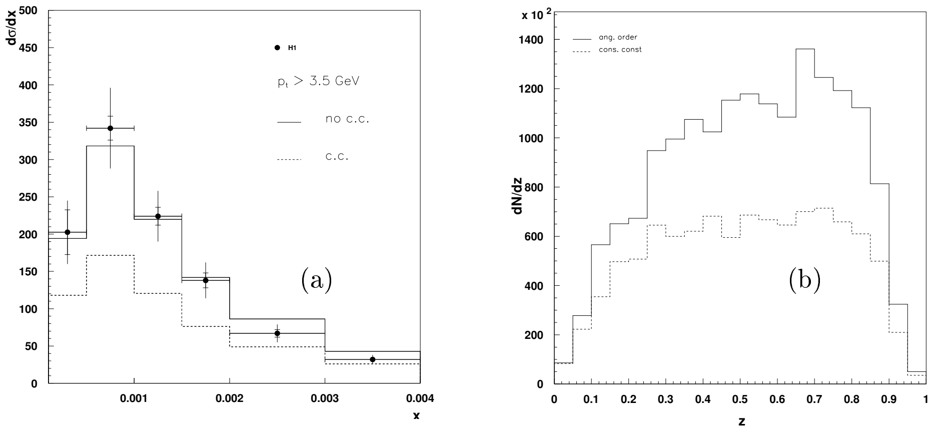

Figure 1. (a) The cross section for forward jet production as a function of x, compared to H1 data [15]. (b) The values of z in the initial state cascade for events satisfying the selection of forward jet production. The solid (dashed) lines are the prediction of the Monte Carlo without (with) applying the "consistency constraint" (c.c.).

in Fig. 1a. It is interesting to note, that this prediction is very similar to the one obtained from the BFKL equation as shown in [12] with k_t^2 for the scale in α_s.

The effect of the "consistency constraint" is best seen in the spectrum of the values of the splitting variable z for events that satisfy the criteria of forward jet production (Fig. 1b). Only about half of the events satisfy the "consistency constraint", and especially medium and large values of z are rejected. Furthermore we notice that in general z is not very small thus showing that even in forward jet production we are far away from the asymptotic region, where the small x approximation is valid.

In general, large z values and therefore more branchings are needed to build up the forward jet, followed by smaller z to go from the forward jet to the photon. Therefore it is understandable that when applying the "consistency constraint" (removing mainly large z values) the forward jet cross section becomes much smaller.

3. LDC

The Linked Dipole Chain Model[10], LDC, is a reformulation of the CCFM evolution. The main idea is to reinterpret the non-Sudakov form factor in eq. (3) as a normal Sudakov for the non-emission probability in the phase-space region given by the integration limits. By redefining the division between initial- and final-state splittings, which is done by angular ordering in CCFM, requiring the transverse momentum of a gluon emitted in the initial-state to be larger than the minimum transverse momenta of the connecting propagators,

$$q_{i\perp}^2 > \min(k_{i\perp}^2, k_{i-1\perp}^2), \tag{6}$$

an extra weight is given to each splitting corresponding to the sum of all emissions which in this way are treated as final-state. In the limit of emissions which are strongly ordered both in longitudinal and transverse momenta of the propagating gluon, it can then be shown that this sum exactly cancels the non-Sudakov form-factor, but only if the kinematical constraint is applied.

It is intriguing to note that the LDCMC gives approximately the same result for forward jet rates as does the SMALLX program with the kinematical constraint, i.e. roughly a factor 2 below the data. This could suggest that the reason LDC fails is the use of the kinematical constraint in the cancellation of the non-Sudakov. It is, however, conceivable that the cancellation will still hold even if the consistency constraint is relaxed, as long as the integration limits in the non-Sudakov is the same as in the sum of emissions failing the constraint in eq. (6). This has yet to be proven.

Since this workshop, much effort has been put into the understanding of the differences between SMALLX and CASCADE without the kinematical constraint on one hand and LDCMC on the other. One difference is in the treatment of the quark-box closest to the virtual photon. In LDCMC, quark emissions are treated on the same footing as gluon emission, and the quark-box is hence treated as an ordinary emission, but with a correction to reproduce the $\gamma^* g \to q\bar{q}$ matrix element. In particular it means that the transverse momenta of the quarks (in the $\gamma^* p$ rest frame) is required to be above some cutoff. This is in contrast to the SMALLX and CASCADE where no cuts are put on the kinematics of the quark-box. Here the divergency at small transverse momenta is instead regulated by freezing α_S at some small scale. This procedure can, of course, also be implemented in LDCMC. Some care has to be taken in the resulting redefinition of the non-perturbative input quark- and gluon-distributions which are used in the program, but preliminary investigations shows that such a change not only enables LDCMC to give a near-perfect description of F_2, but also increases the perturbative activity in the forward region to approximately the right level. Much more studies are required to understand why this modest change in the treatment of the quark-box can have such a large effect on the evolution.

4. Resolved virtual photons

In the resolved virtual photon approach, the virtual photon is treated as a real particle which may be resolved into quarks and gluons. The parton densities of the the virtual photon can in principle be calculated within perturbative QCD and the cross section for jet production is obtained by convoluting these densities with the parton density parameterization of the proton and hard parton-parton matrix elements.

This approach is implemented in RAPGAP and PYTHIA and it is indeed possible to reproduce e.g. forward jet rates in DIS. However, this can only be done if the scale for the hard parton-parton interaction chosen to be $Q^2 + p_\perp^2$ or $4p_\perp^2$, which is different to the standard scales used normally in the hard parton-parton matrix element. Typically one would expect the scale in the hard parton-parton sub-process to be given by $\hat{t}$ or $p_\perp^2$ or, as argued by Friberg and Sjöstrand [17] $p_\perp^2(\hat{s} + Q^2)/\hat{s}$, which they use as the preferred scale in their implementation of resolved virtual photons in PYTHIA.

However, in leading order the scales are not defined. In a full next-to leading order calculation including direct and resolved virtual photon contributions in a consistent way, Kramer and Pötter [18] have used the scale $Q^2 + p_\perp^2$ and have shown, that the di-jet rate and the forward jet cross section can be described well. They obtain the same answer as from RAPGAP using the same scale, together with initial state parton showers. Thus using $Q^2 + p_\perp^2$ as the scale, the full NLO calculation is well simulated by the leading order Monte Carlo RAPGAP which gives confidence in that choice of the scale.

The reason that the more *natural* scales does not give enough forward jets is that the virtual photon densities are evolved with with DGLAP evolution, assuming increasing transverse momenta in the emissions summed over. In particular it means that if the scale of the hard sub-process is less than or approximately equal to Q^2, there is no evolution at all. But it is clear that also emissions with very small $p_\perp$ will contribute – in fact it seems that these are very important for the LDC model – and to get a reasonable evolution in a resolved virtual photon scenario a *fake* scale which is always larger than Q^2 must be used to artificially obtain a reasonable evolution of the parton densities in the virtual photon.

5. Conclusions

There are now three different models implemented in event generators which give a reasonable description of small-x final state observables measured at HERA. As reported here, there are unresolved questions related to each of these models, but it is clear that our understanding of small-x parton dynamics is steadily improving.

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